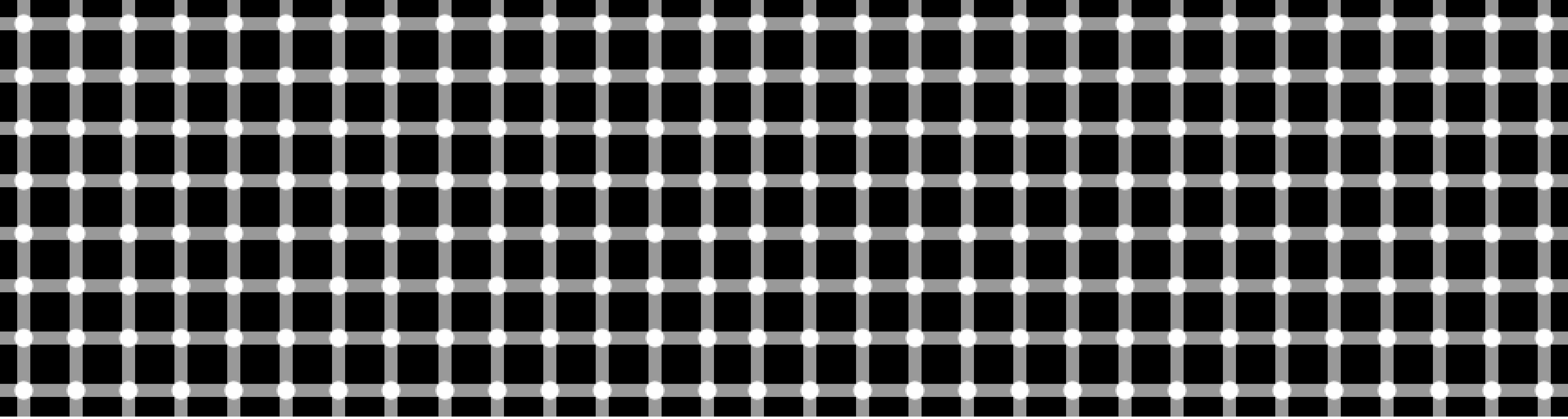

Radio propagation and scintillation

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Talk outline (in approximate order)

- Plasma properties
  - Non-magnetised plasmas
  - Magnetised plasmas
  - Real life plasmas
    - Ionosphere
    - Interplanetary medium
    - Interstellar medium
    - Intergalactic medium
- Propagation through a uniform plasma
  - Plasma frequency
  - Faraday rotation
  - Delays and dispersion

Talk outline (in approximate order)

- Non-uniform plasmas
  - Blob approximation
  - Kolmogorov spectrum
- Scintillation and scattering
  - Thin screen approximation
  - Fresnel scale
  - Weak and strong scattering
- Angular broadening
- Temporal broadening
- Refractive and diffractive scintillation
- Scintillation as a probe
  - Pulsar scintillation (interstellar)
  - Interplanetary scintillation and solar weather

Plasma properties – non-magnetised plasmas

- Space is not empty. *All* signals from astrophysical sources travel through ionised media before they reach Earth:
  - Intergalactic medium (~10^{-5} -10^{-3} electrons cm^{-3})
  - Intestellar medium (~10^{-2} cm^{-3})
  - Galactic HII regions (~10^{2}-10^{4} cm^{-3})
  - Interplanetary medium (1-100 cm^{-3} at 1 AU)
  - Ionosphere (10^{4}-10^{6} cm^{-3})

Plasma properties – non-magnetised plasmas

- Each type of plasma has its own characteristic turbulent structure, but the electron column density contributions (called the dispersion measure)

$$\int n_e(z)dz$$

to a source at 1 Mpc are interesting:

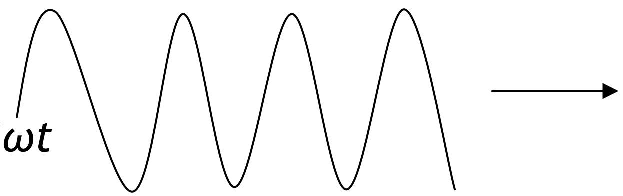
- Intergalactic medium (~10^{19} - 10^{20} electrons cm^{-2}) } **these dominate**
- Intestellar medium (~10^{19} - 10^{20} cm^{-2})
- Interplanetary medium (10^{14} - 10^{16} cm^{-2})
- Ionosphere (10^{11}-10^{13} cm^{-2}). Here, the integral is called the *total electron content* (TEC)

(see later for details: the usual units are pc cm^{-3})

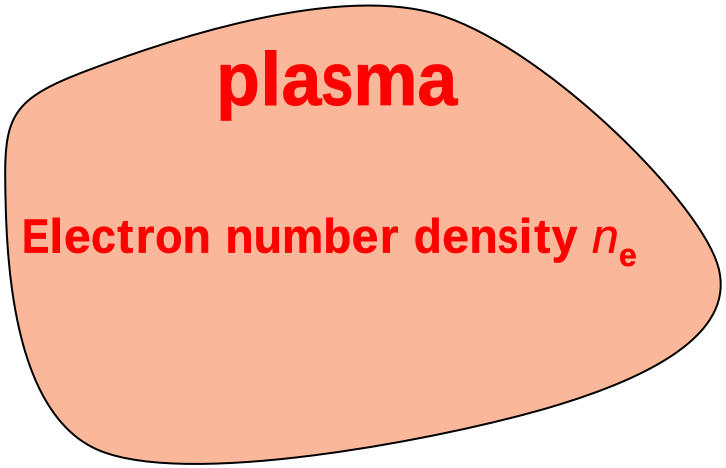
Plasma properties – non-magnetised plasmas

- Propagation:

$$E = E_0 e^{i\omega t}$$



electromagnetic wave, angular frequency ω



- Electrons oscillate around (quasistationary) protons as

$$x = x_0 e^{i\omega t}$$

- Equation of motion:

$$Ee = m_e \ddot{x} = -m_e \omega^2 x$$

Plasma properties – non-magnetised plasmas

- This appears as a bulk polarisation, defining the relative permittivity of the plasma, ε_r :

$$P = n_e p = (\varepsilon_r - 1)\varepsilon_0 E$$

where $p = xe$ is the dipole moment for one electron/proton pair

- Comparing with the equation of motion we see that

$$\varepsilon_r = 1 - \frac{n_e e^2}{\varepsilon_0 m_e \omega^2}$$

so that the refractive index of the plasma, η , is

$$\eta = \sqrt{\varepsilon_r} = \left(1 - \frac{f_p^2}{f^2}\right)^{1/2}, \text{ where } f_p = \frac{1}{2\pi}\left(\frac{n_e e^2}{\varepsilon_0 m_e}\right)^{1/2}$$

Plasma properties – non-magnetised plasmas

- $$f_p = \frac{1}{2\pi}\left(\frac{n_e e^2}{\varepsilon_0 m_e}\right)^{1/2} \approx 9(n_e/\mathrm{cm}^3)^{1/2}\ \mathrm{kHz}$$

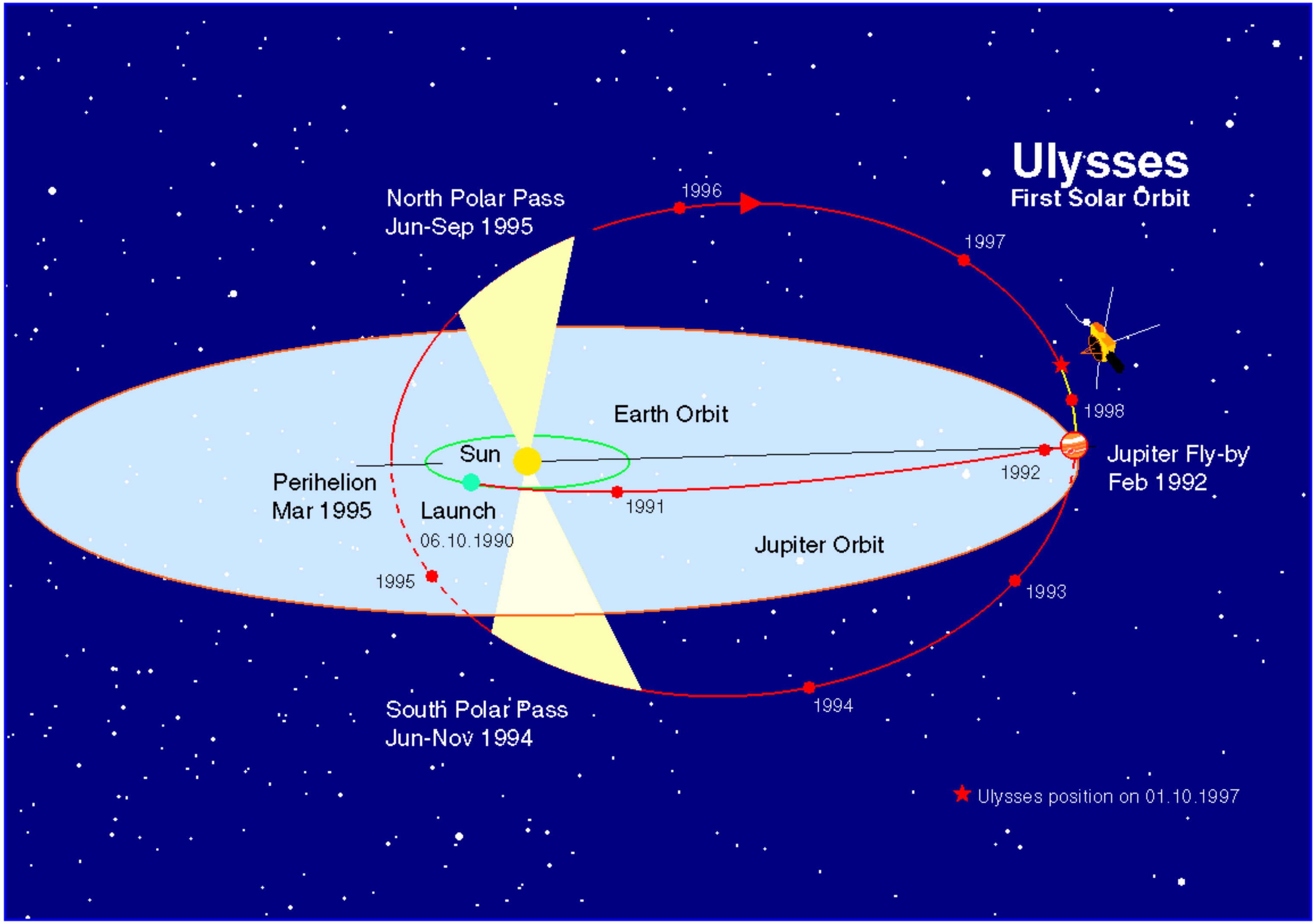
  is the *plasma frequency* – the natural oscillatory frequency of the plasma

- For $f < f_p$, there are no TEM propagating modes

- For $f > f_p$, phase velocity = c/η (>c)
  group velocity = $c\eta$ (<c)

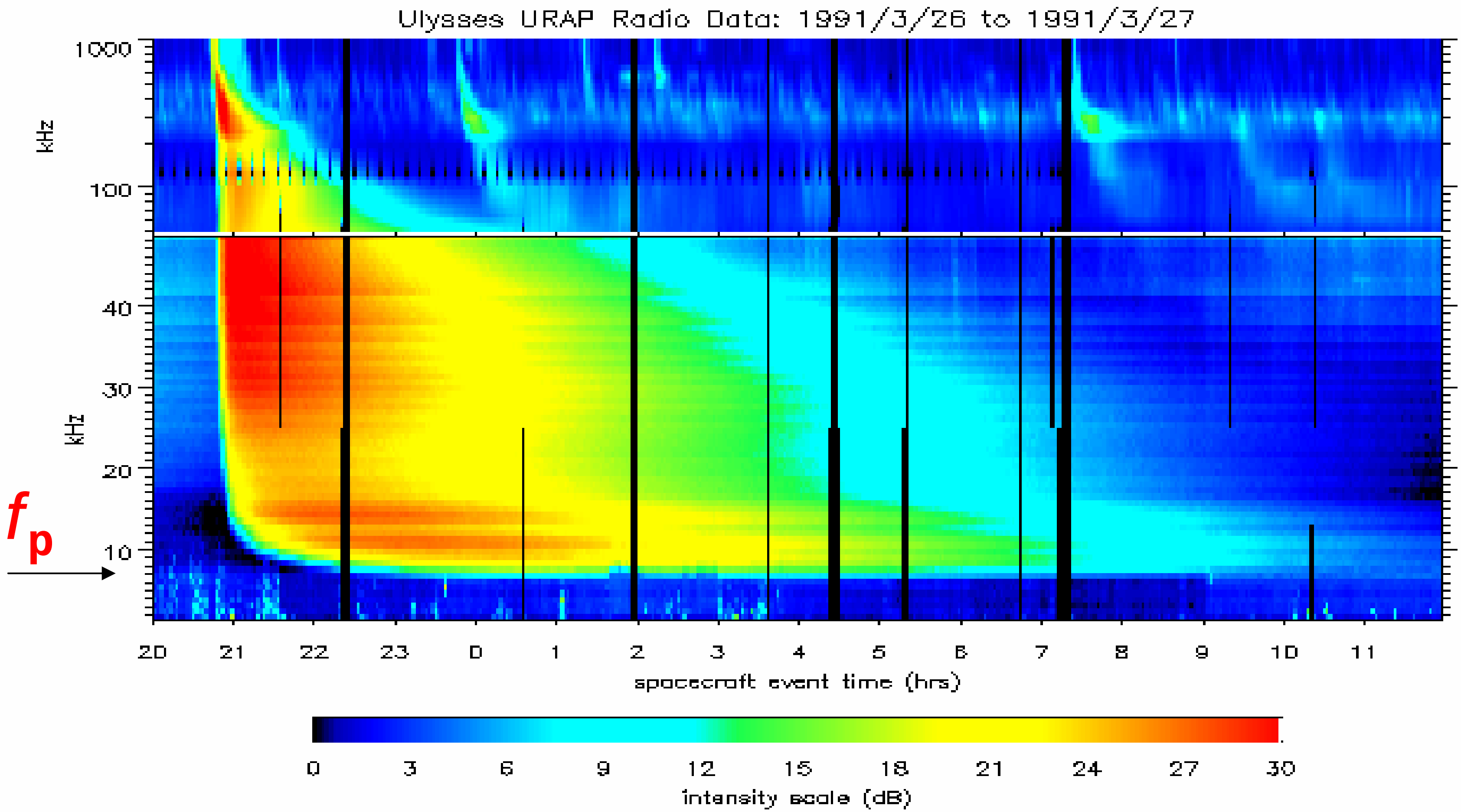
- Spatial variations in electron density will give a non-uniform refractive index, leading to refractive and diffractive scattering

Plasma properties – non-magnetised plasmas



Plasma properties – non-magnetised plasmas

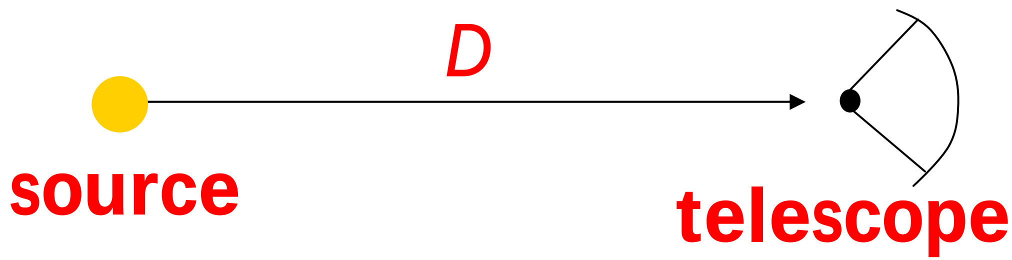
Plasma frequency in the IPM ~7 kHz (type III burst)



Plasma properties – non-magnetised plasmas

- The refractive index depends on frequency, so the travel-time of a signal is also frequency dependent:

$$t = \int_0^D \frac{dz}{\eta(z)c}$$



- If $f_p \ll f$ the extra delay relative to the travel time at *c* is

$$\tau_D = \frac{e^2}{2\pi m_e c}\frac{1}{f^2}\int_0^D n_e(x)dx$$

Plasma properties – non-magnetised plasmas

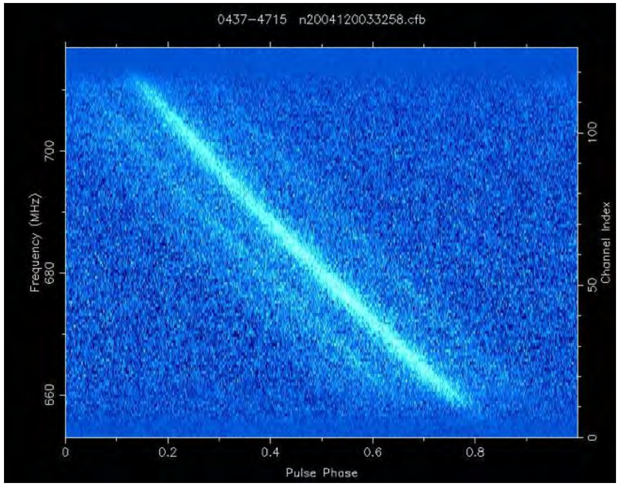
- This is usually written

$$\tau_D = 4.15 \times 10^3 \frac{1}{f_{MHz}^2} DM \text{ seconds}$$

where $DM = \int_0^D n_{e,cm^{-3}}(x)dx_{pc}$ is the dispersion measure

- Radio dispersion transforms the pulses from pulsars into chirps:

frequency



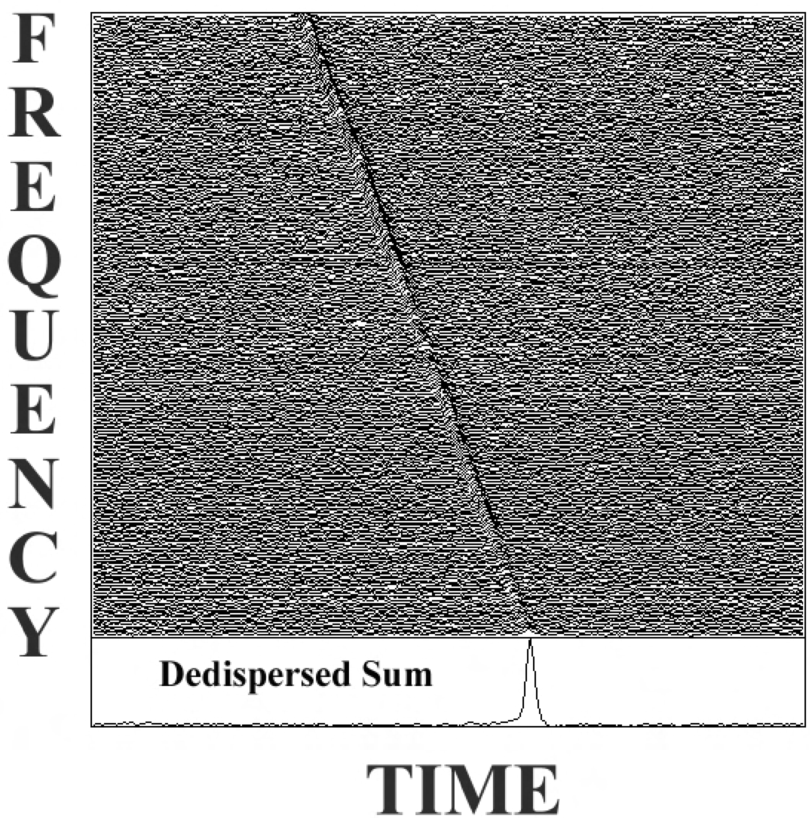
Swinburne

The highest frequencies arrive first

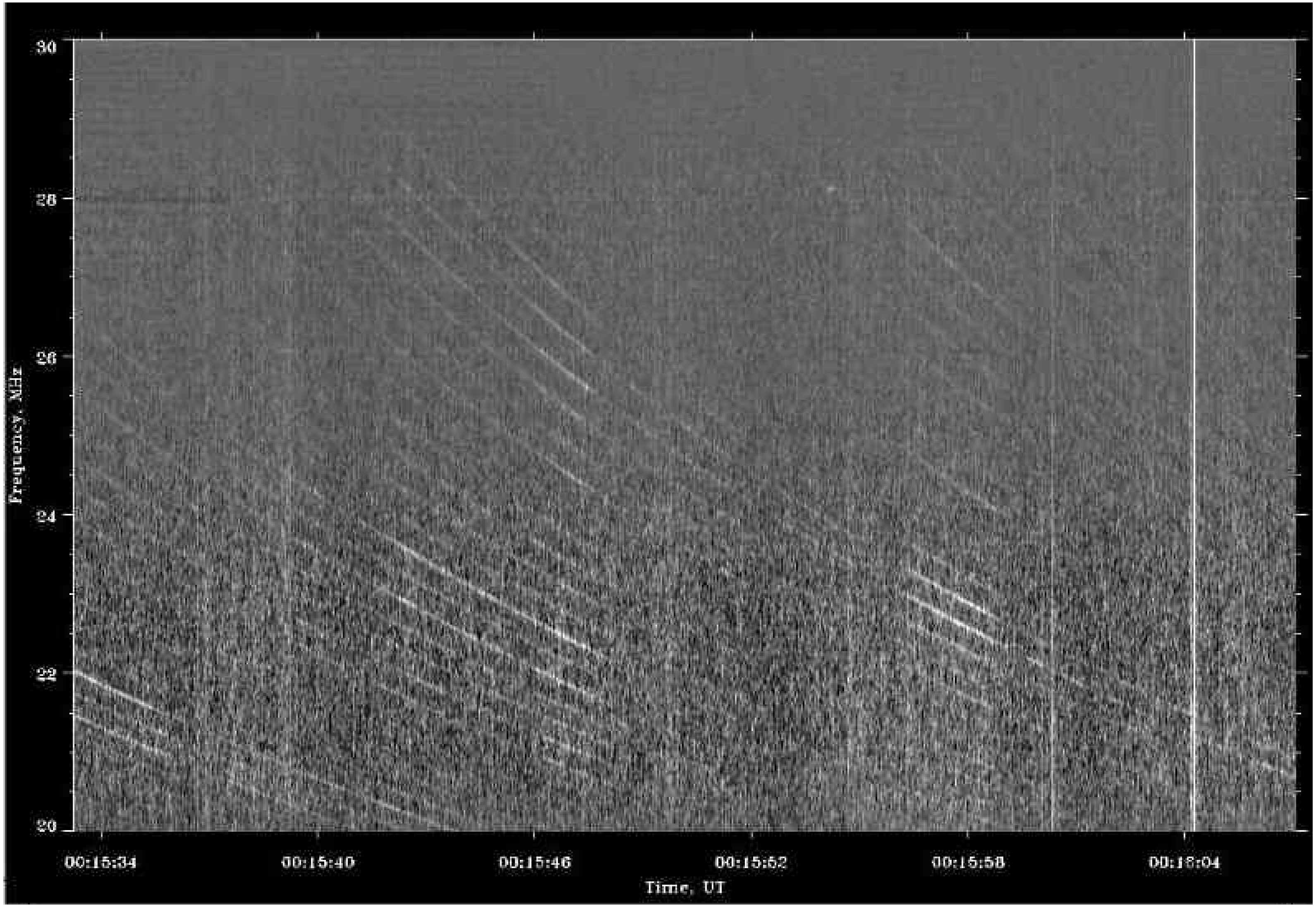
Arrival time

Plasma properties – non-magnetised plasmas

$$\tau_D = 4.15 \times 10^3 \frac{1}{f_{MHz}^2} DM \text{ seconds}$$

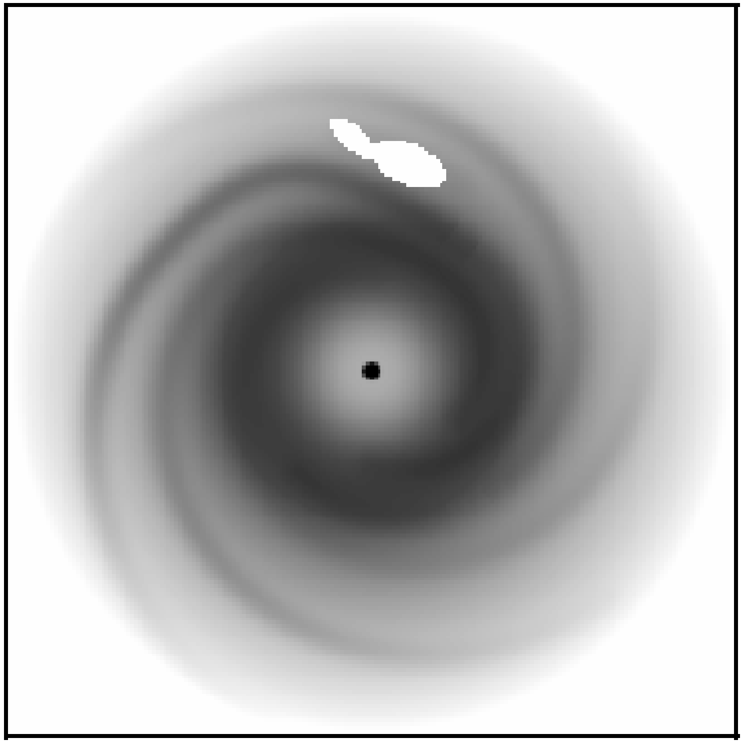


Low frequency dispersion



Plasma properties – non-magnetised plasmas

- Dispersion to pulsar can be used to help map the galactic electron density



Cordes-Lazio NE2001 Galactic Free Electron Density Model (2002)

typically, $n_e \approx 0.03\ \mathrm{cm}^{-3}$
but there is much variation

Plasma properties – magnetised plasmas

- A magnetic field component parallel to the direction of travel gives a different refractive index for left- and right-handed circular polarisation,

$$\eta = \left(1 - \frac{f_p^2}{f(f \pm f_B)}\right)^{1/2}, \text{ where } \quad f_B = \frac{eB}{2\pi m_e}$$

rotating linearly polarised radiation by an angle

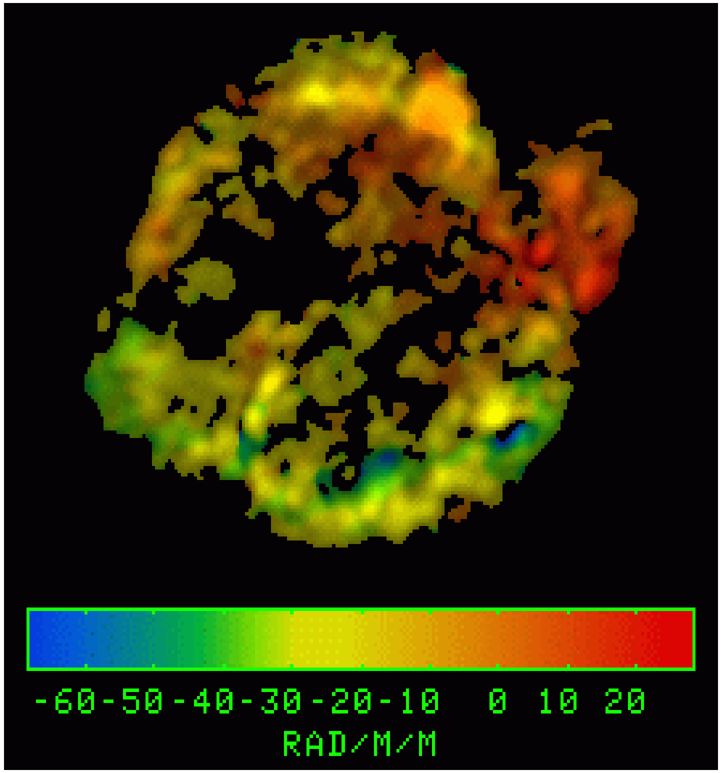
$$\Psi = \frac{e^3}{2\pi m_e^2 c^2 f^2} \int_0^D n_e(z) B_{||}(z) dz = \lambda^2 RM$$

where RM defines the rotation measure

Plasma properties – magnetised plasmas

The ratio RM/DM gives the electron-density-weighted mean magnetic field along the line of sight

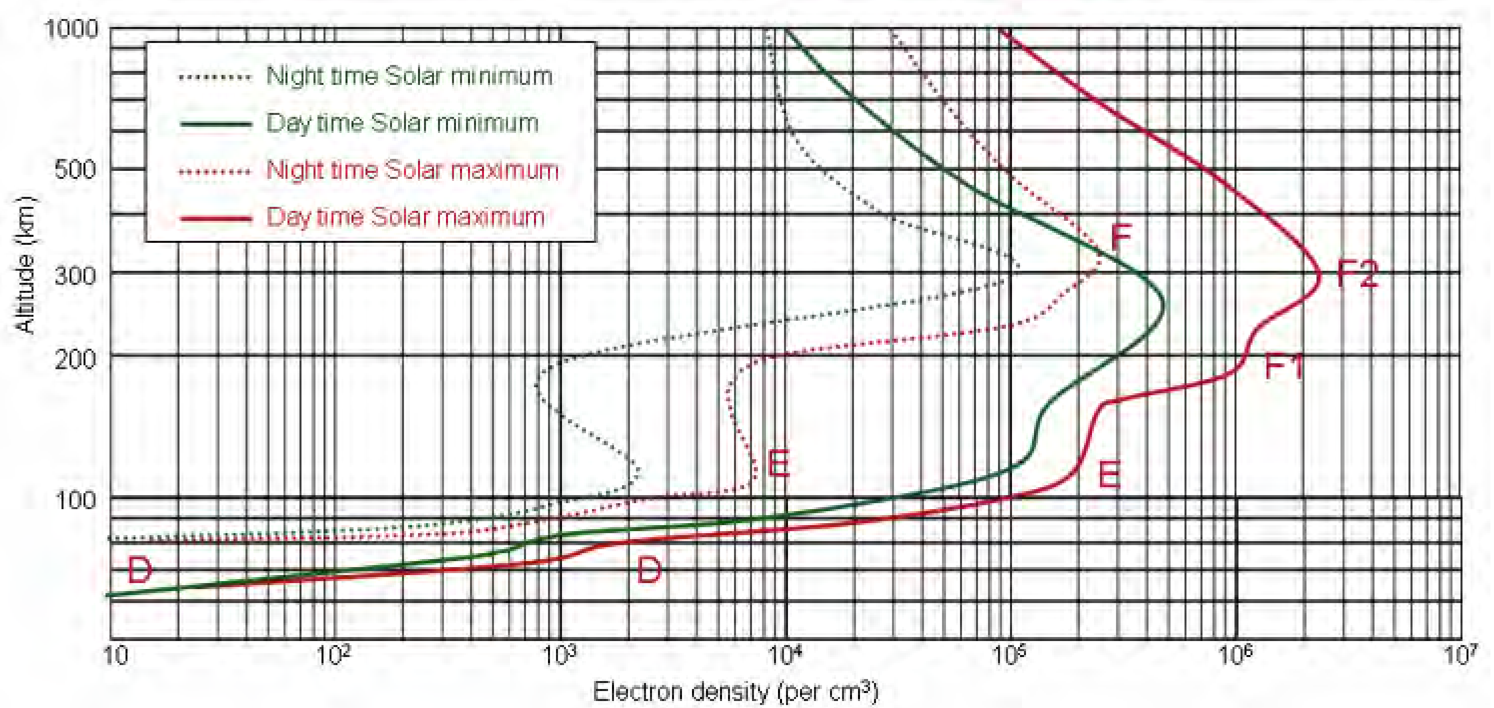
$$\frac{RM}{DM} = \frac{\int n_e B_{||} dz}{\int n_e dz} = \langle B_{||} \rangle$$



Rotation measure map of
Kepler's supernova remnant
Between 6 and 20 cm
(Delaney et al Astrophys.J. 580 (2002) 914-927)

Real plasmas -- ionosphere

- Variability:
  - Diurnal variations (EUV)
  - Ionospheric/thermospheric winds and gravity waves
  - Geomagnetic storms

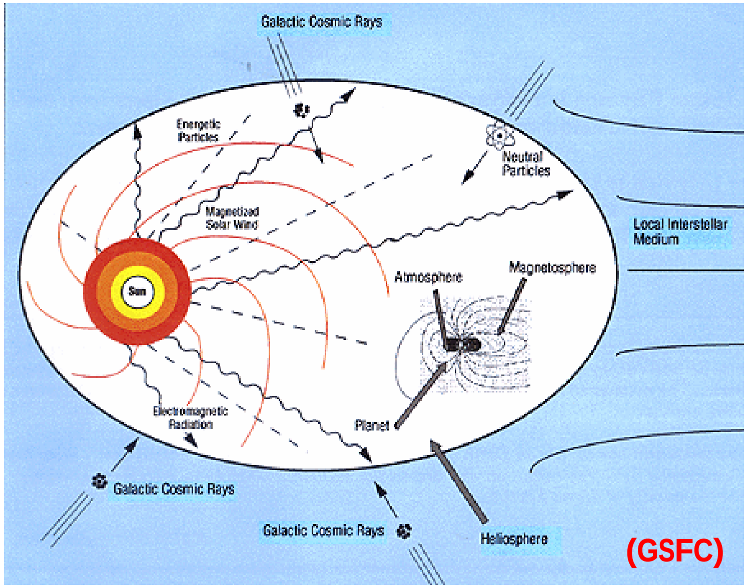


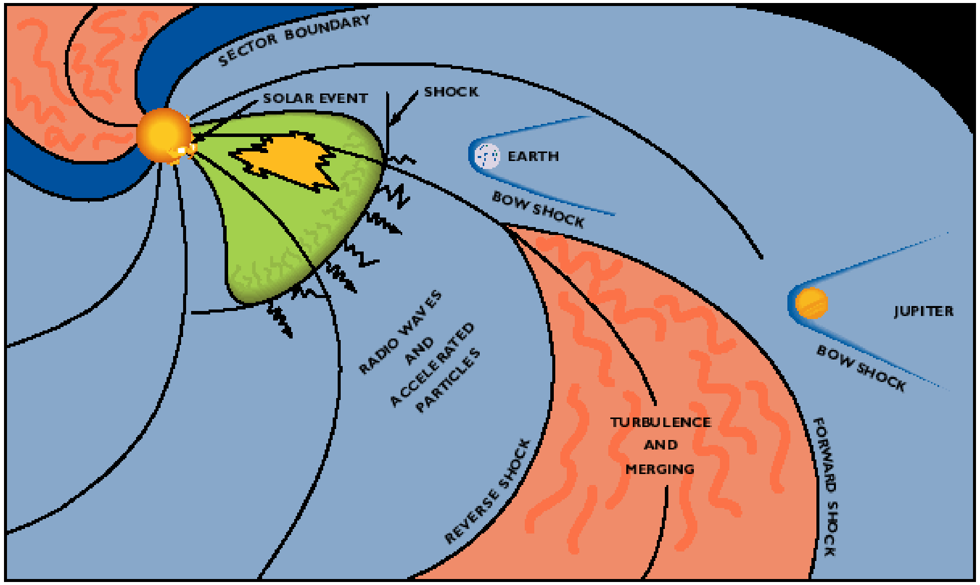
Real plasmas -- ionosphere

- Scintillation (much more on this later!):
  - Ionospheric irregularities cause random fluctuations in the amplitude of radio signals that propagate through them (“scintillations”)
  - Common around the magnetic pole, pre-midnight at magnetic equator and nightime in auroral zone
  - Wide range of scale sizes (~cm to ~100 km)
  - Timescale depends on many factors, but is ~10s at ~100 MHz (see later for theory!)
  - Can severely affect low-frequency radio astronomy, particularly around solar maximum

Real plasmas – interplanetary medium

- The solar wind is a turbulent, slightly magnetised, plasma with mean electron density dropping as ~$1/r^2$,





- Interplanetary density transients from coronal mass ejections or corotating dense streams cause mean density fluctuations up to x100 on timescales of hours
- Smaller-scale inhomogeneities give interplanetary scintillation (timescale ~1 s at ~100 MHz) and other propagation effects

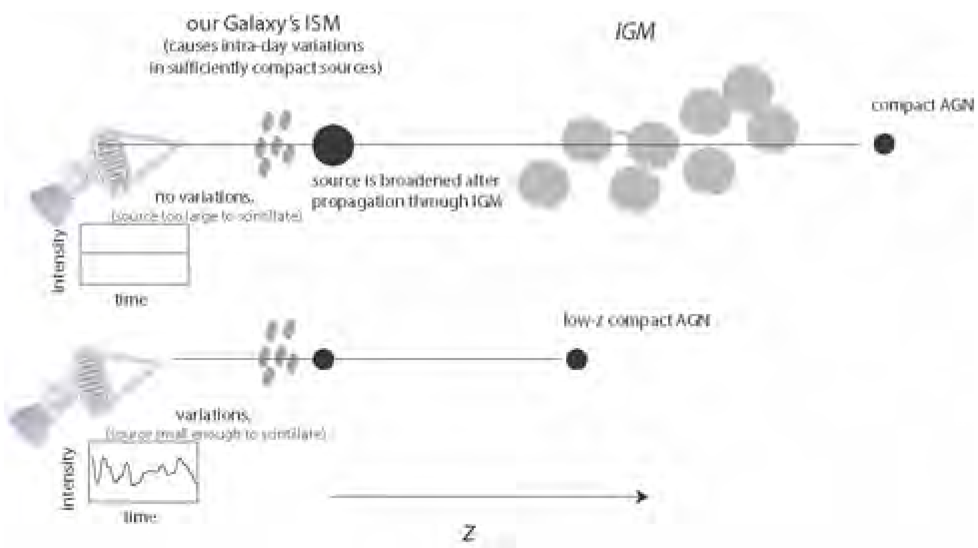
Real plasmas – interstellar medium

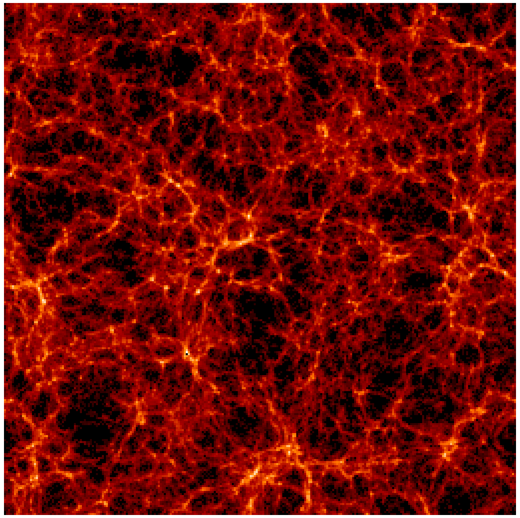
- The ISM is a complex mixture of neutral and ionised components (99% gas, 1% dust):

Component	Temp (K)	Volume fraction	Number density (cm^{-3})	species
Molecular clouds	20-50	<1%	10^3-10^6	Molecular hydrogen
Cold neutral medium	50-100	1-5%	1-10^3	Atomic hydrogen
Warm neutral medium	10^3-10^4	10-20%	10^{-1}-10	Atomic hydrogen
Warm ionised medium	**10^3-10^4**	**20-50%**	**10^{-2}**	**Electrons/protons**
HII regions	**10^4**	**10%**	**10^2-10^4**	**Electrons/protons**
Hot ionised medium	**10^6-10^7**	**30-70%**	**10^{-4}-10^{-2}**	**Electrons/protons/metallic ions**

Real plasmas – intergalactic medium

- Least well understood component
- Intracluster gas at ~10^7-10^8 K, n_e~10^{-3} cm^{-3}
- Cluster gas should produce quasar scintillations at 50-100 GHz on time scales ranging from days to months (Ferrara & Perna, 2001), and do give source broadening (Ojha et al 2007 -- more on this later!)





Benson et al. 2001, MNRAS, 320, 153

Uniform propagation – summary

- The line of sight to all radio sources passes through magnetised plasma (ionosphere, IPM, ISM, IGM, source plasma …)

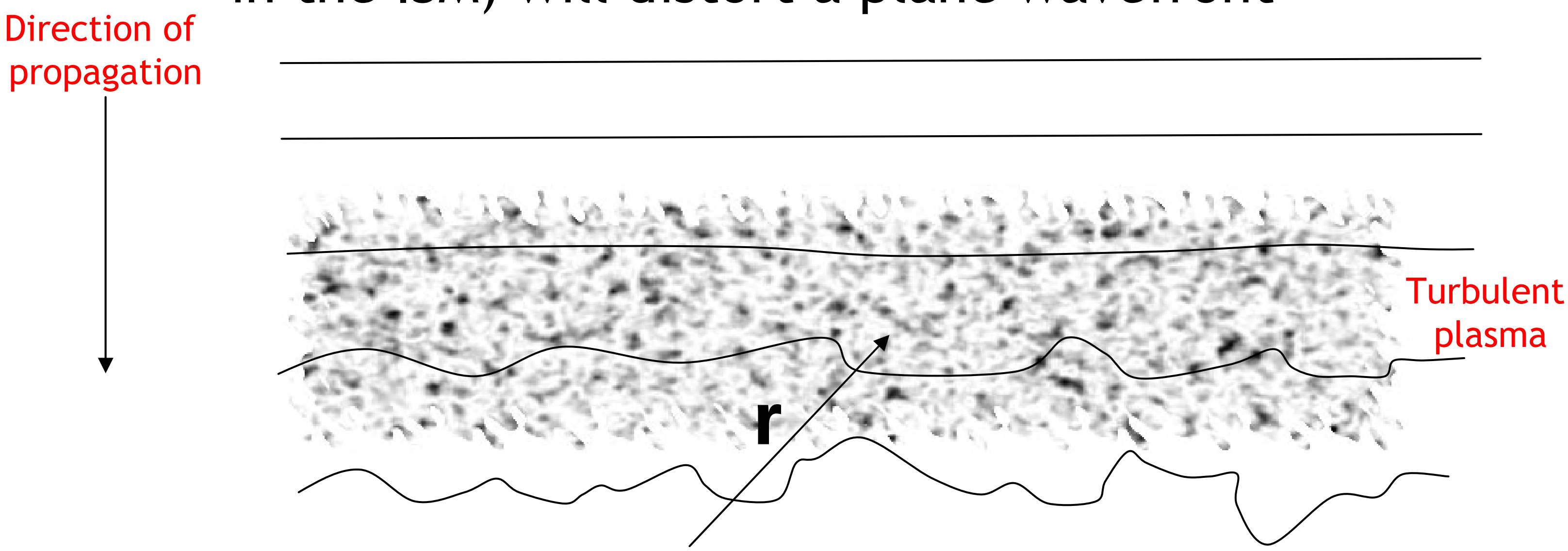
- At the very least these introduce:
  - Dispersive delays to the signal (delay $\propto \frac{DM}{f^2}$)
  - Faraday rotation (angle $\propto \frac{RM}{f^2}$)

  both of which are useful probes of the plasma

- But no plasma is entirely uniform. Spatial variations in n_e mean that different regions of wavefront see different delays/rotations…

Non-uniform plasmas

- A plasma with refractive index variations (typically 0.1% in the ISM) will distort a plane wavefront



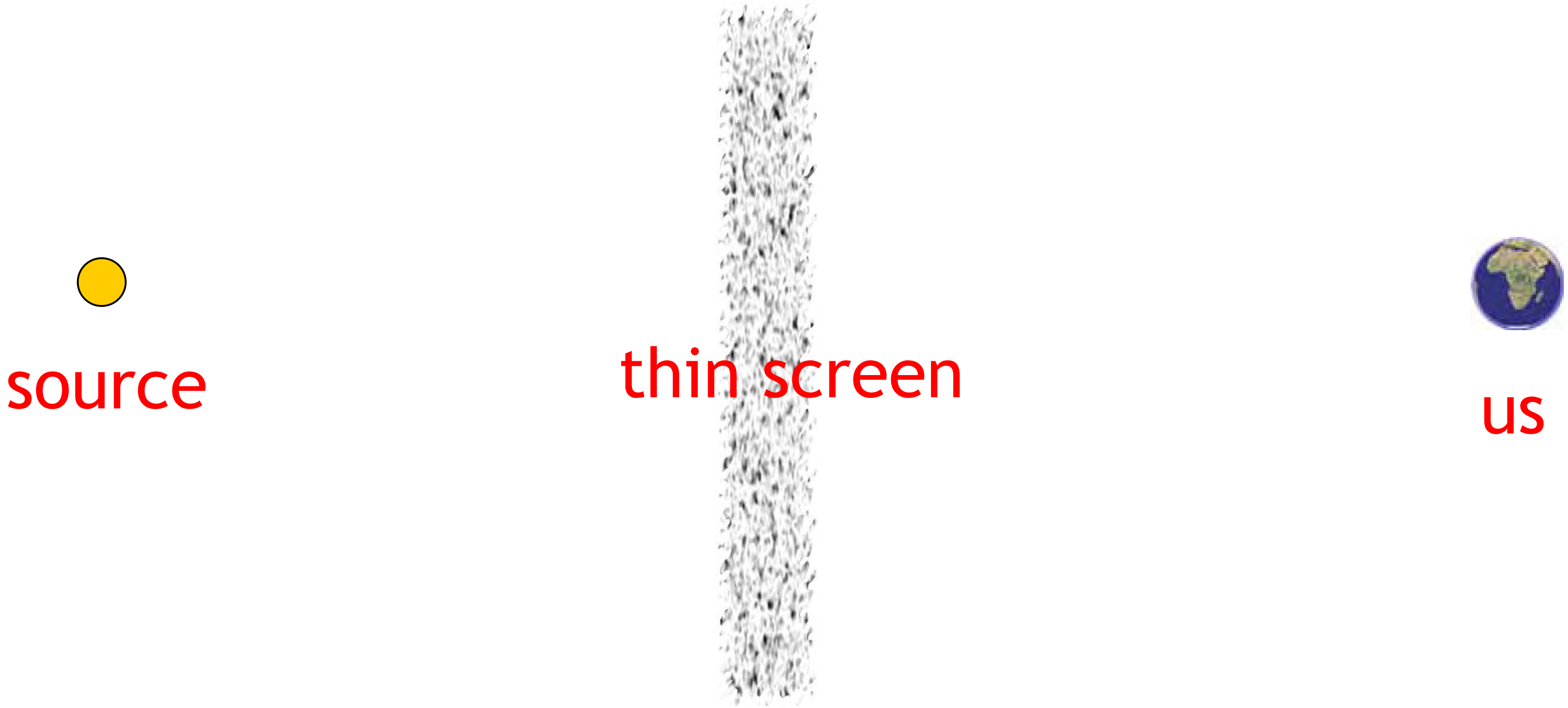
- Excess refractive index due to excess electron density at **r** is

Classical radius of the electron

$$\Delta\eta(\mathbf{r}) = \frac{e^2}{8\pi^2\varepsilon_0 m_e}\frac{\Delta n_e(\mathbf{r})}{f^2} = \frac{r_e}{2\pi}\lambda^2\Delta n_e(\mathbf{r})$$

The thin screen approximation

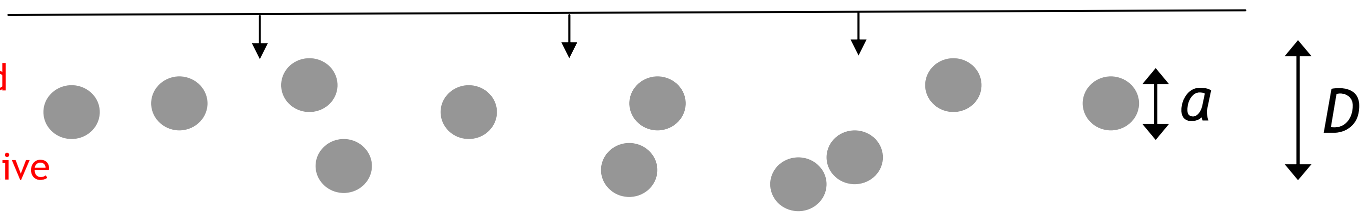
- It is usually not too much of an approximation to imagine the plasma confined to a thin screen, about half way to the source



Non-uniform plasmas – the blob approximation

- A simple and instructive way to model propagation through a random medium is to think of randomly placed, identical blobs of excess plasma density:

Randomly placed blobs of plasma with excess refractive index $\Delta\eta$

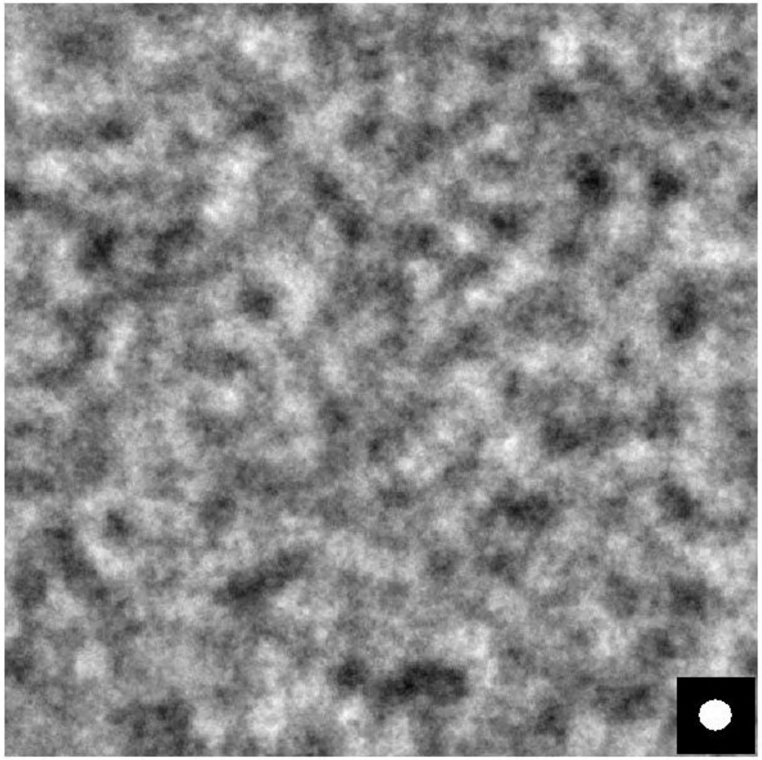


mean number of blobs encountered = D/a
rms variation in number encountered = $(D/a)^{1/2}$
each introduces $2\pi\Delta\eta a/\lambda$ of phase, so phase perturbations across the wavefront are

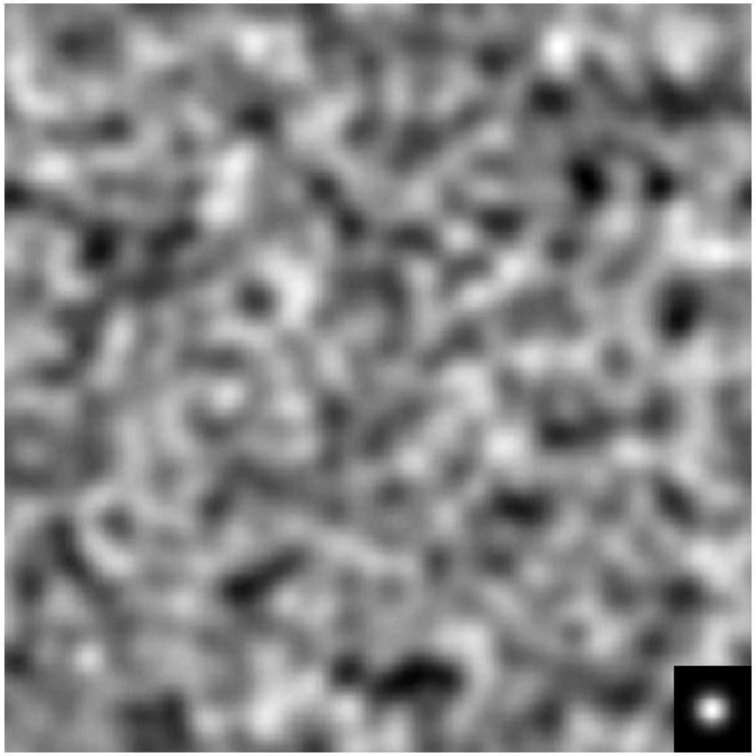
$$\boxed{\Delta\varphi = r_e\lambda(Da)^{1/2}\Delta n_e}$$

Simple phase screen – the blob approximation

$$\Delta\varphi = r_e \lambda (Da)^{1/2} \Delta n_e$$

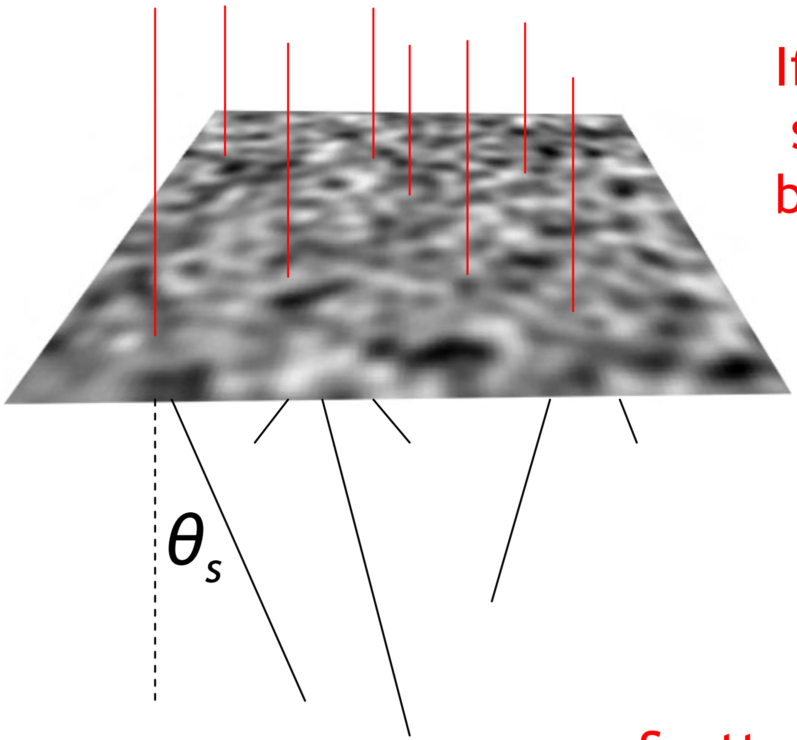


Circular blobs

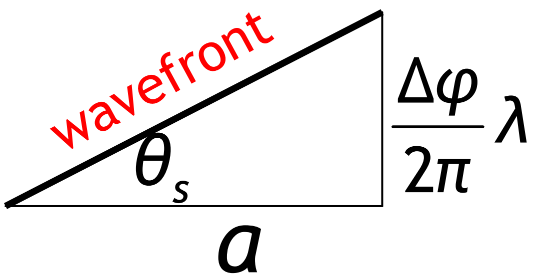


Gaussian blobs

Simple phase screen – refractive scattering



If $a \gg \lambda$, rays passing through the screen will be deflected (as if by little prisms)



Scattering angle is

$$\theta_s \approx \frac{\varphi}{2\pi}\frac{\lambda}{a} = \frac{1}{2\pi} r_e \lambda^2 \sqrt{D/a}\,\Delta n_e$$

More general angular broadening

- More generally, consider two adjacent rays from a point source passing through the screen, separated by *b* **in the observer's plane.** The mean square difference in the phase at the two points is called the phase structure function:

$$D_{\varphi}(\mathbf{b}) = \left\langle [\varphi(\mathbf{R}) - \varphi(\mathbf{R}+\mathbf{b})]^2 \right\rangle$$

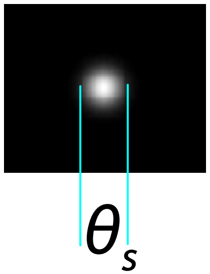
The coherence scale, r_0, is the separation for which

$$D_{\varphi}(\mathbf{r}_0) = 1$$

and we define the general scattering angle as

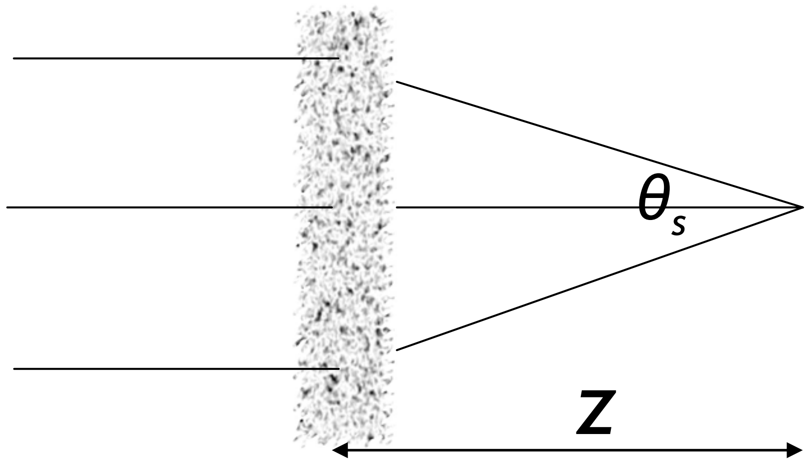
$$\theta_s \equiv \frac{1}{k r_0}$$

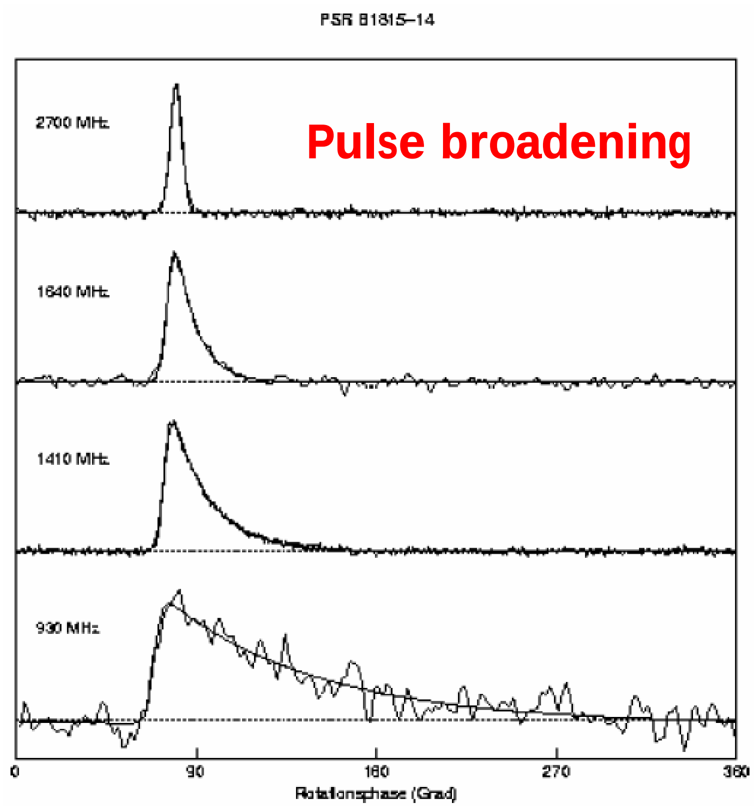
Point source →



Temporal broadening (refractive analysis)

- Different rays from the blurred source take different times to reach the observer:





$$\tau_s = \frac{z}{c}(1 - \cos\theta_s) \approx \frac{z\theta_s^2}{2c}$$

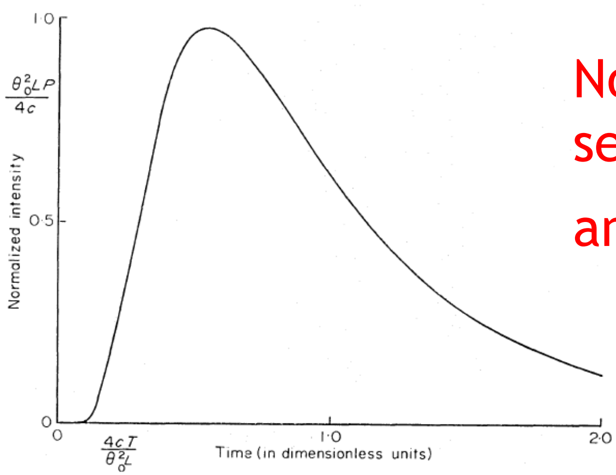
$$\text{if } \theta_s \propto \lambda^2 \quad \text{then} \quad \tau_s \propto \lambda^4$$

Temporal broadening (refractive analysis)

- For a thin screen, a short pulse is broadened to an exponential decay

$$I(t) \propto \exp(-t/\tau_s)$$

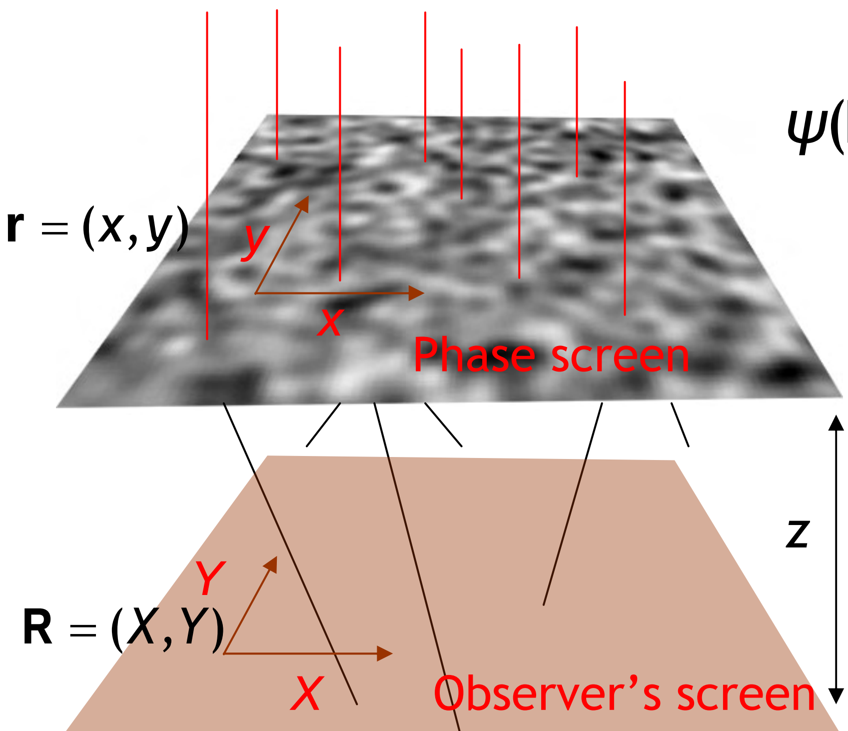
multiple scattering smooths this to



Note: $\tau_s \propto \lambda^4 z^2$
severe at low frequencies
and for distant pulsars

Simple phase screen - scattering

- Point sources therefore appear broadened in angle and time
- The full evolution of the wave can be computed using the Fresnel diffraction formula:

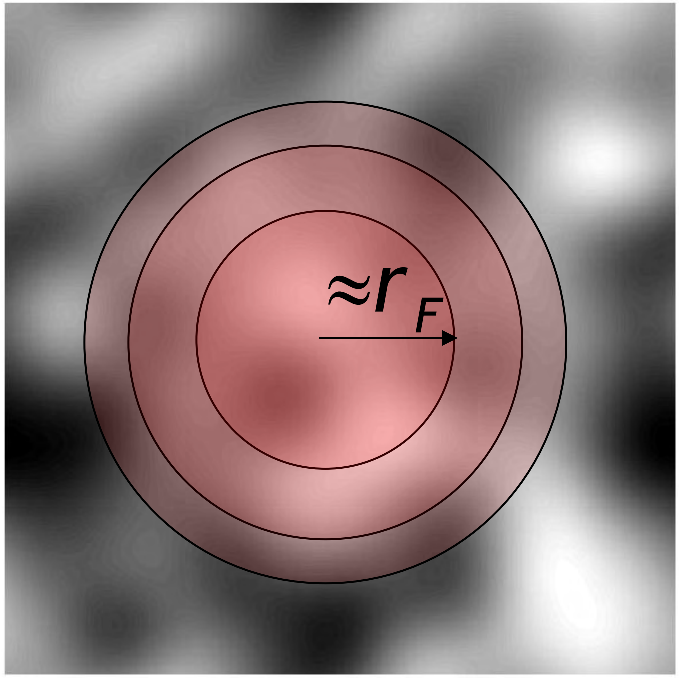


$$\psi(\mathbf{R}) = \frac{e^{-i\pi/2}}{2\pi r_F^2} \iint \exp\left(i\varphi(\mathbf{r}) + i\frac{|\mathbf{r}-\mathbf{R}|^2}{2r_F^2} \right) d^2 r$$

- $r_F = (\lambda z / 2\pi)^{1/2}$ is the Fresnel scale - the size of the centre patch of the screen within which all points are nearly equidistant from the observer

Simple phase screen – scattering

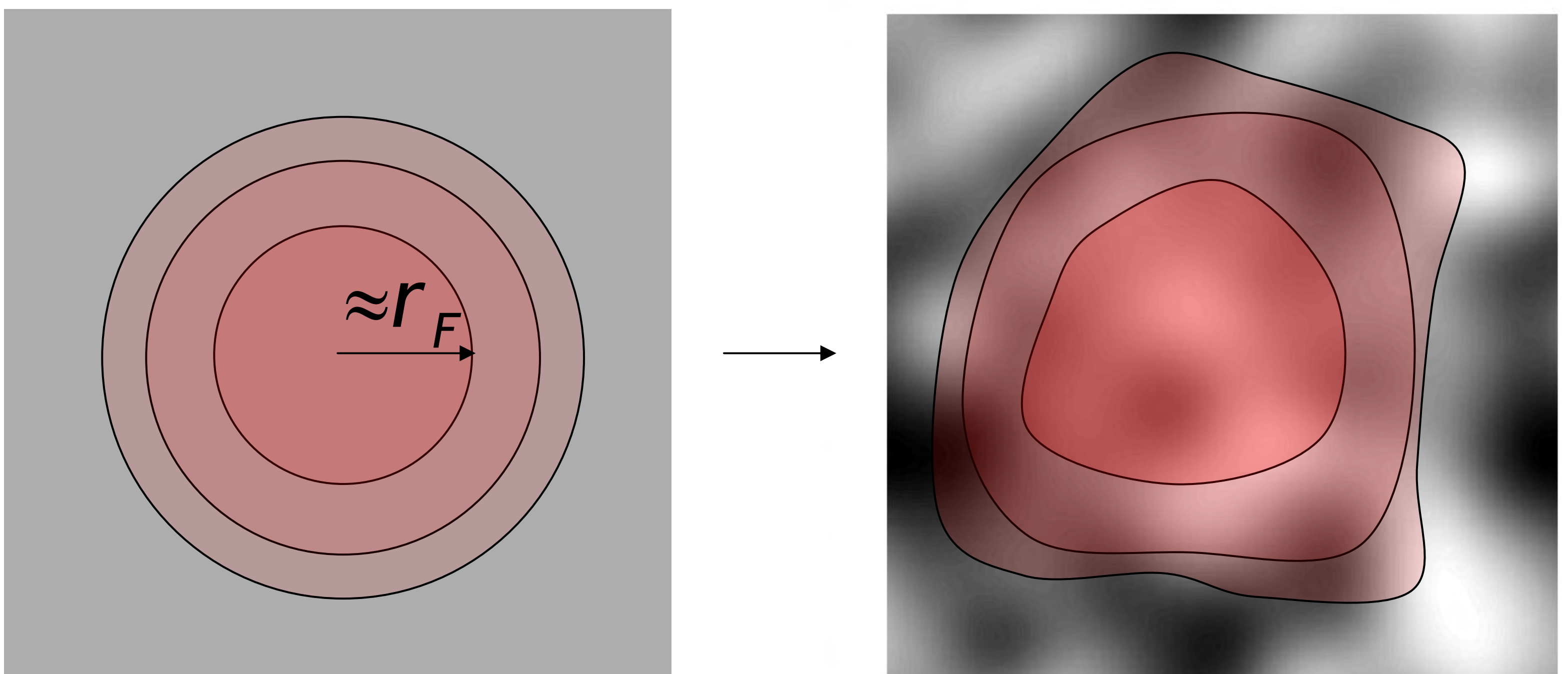
- In principle, the intergral is over the whole screen, but in practice the contribution from the first Fresnel zone ($\sim r_F$) dominates



- If the phase disturbance changes only a little ($\ll \pi$) over the Fresnel zone, we have weak scattering ($r_F \ll r_0$)
- If there are large changes over the zone, we have strong scattering ($r_F \gg r_0$)

Simple phase screen – scattering

- Another way to think about it: weak scattering corresponds to mild distortions of the first Fresnel zone, causing weak focusing/defocusing of the important rays



Scintillation – weak scattering

- Consider a 1-dimensional sinusoidal phase screen

$$\varphi(x,y) = \varphi_0 \sin qx$$

- In the weak scattering limit, $\varphi(x,y)$ is small over the Fresnel zone so

$$\exp[i\varphi(x,y)] \approx 1 + i\varphi(x,y)$$

and $$\psi(\mathbf{R}) = 1 + \frac{e^{-i\pi/2}}{2\pi r_F^2} \iint i\varphi(\mathbf{r}) \exp\left(i \frac{|\mathbf{r}-\mathbf{R}|^2}{2r_F^2} \right) d^2 r$$

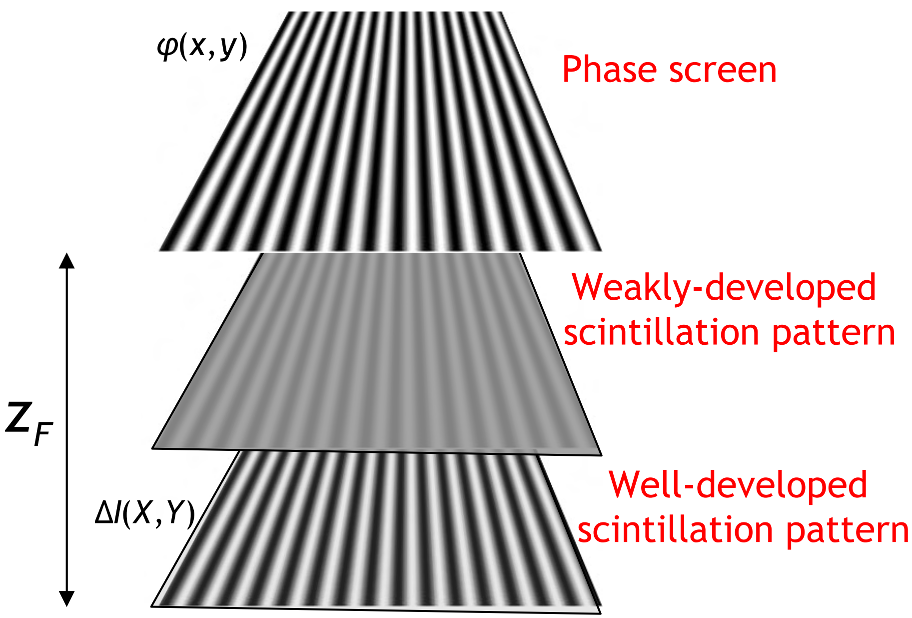
so that the fluctuation in the complex amplitude over the observer's plane is

$$\Delta\psi(X,Y) = \varphi_0 \left(\sin\frac{q^2 r_F^2}{2} + i\cos\frac{q^2 r_F^2}{2} \right) \sin qX$$

Scintillation – weak scattering

- To first order in $\Delta\psi$, the intensity variation on the observer's plane is

$$\Delta I(X) = 2\varphi_0 \sin qX \sin \frac{q^2 r_F^2}{2}$$



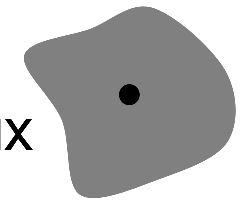
- so an identical intensity pattern appears in the observer's screen, becoming well-developed at the Fresnel distance

$$z_F = \frac{2\pi^2}{\lambda q^2}$$

where the scale of the phase fluctuations in the screen are smaller then the Fresnel zone

Scintillation – weak scattering

- Weak scattering therefore produces intensity fluctuations on the ground that are a linearly filtered version of the phase perturbations, allowing through only scales smaller than r_F (and usually dominated by scales at r_F)
- The source appears surrounded by a halo containing some (small) fraction of the flux



- If the screen or source is moving we see intensity fluctuations (twinkling, or scintillation) on timescales

$$\tau_{\text{scint}} = \frac{r_F}{v_\perp}$$

Scintillation – weak scattering

- An extended source (angular size θ) smears out the scintillation pattern on the ground on a scale of θz so that sources larger than r_F / z don't scintillate

- Weak scattering dominates in the interplanetary medium at elongations > ~30 degrees at ~100 MHz

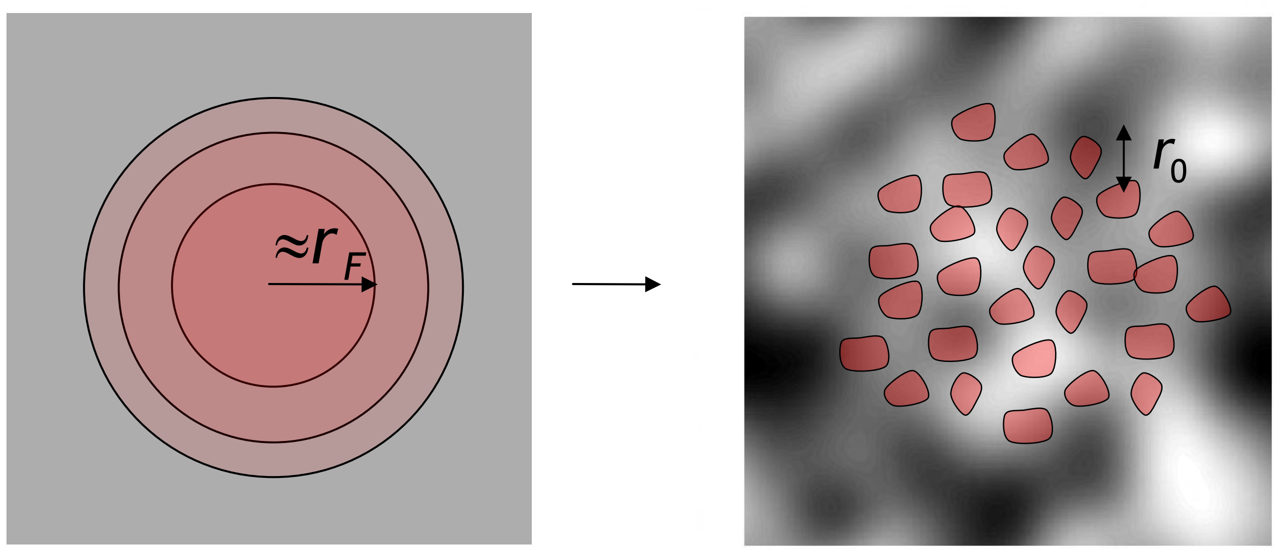
- Quite generally, the scintillation index, *m*, of a source of brightness profile *B* shining through a turbulent screen is

$$m^2 = \frac{\left\langle (\Delta I)^2 \right\rangle}{I^2} = \iint \frac{|\bar{B}(zq)|^2}{|\bar{B}(0)|^2} \Phi(q_x, q_y) 4 \sin^2 \frac{q^2 r_F^2}{2} dq_x dq_y$$

where $\bar{B}$ is the Fourier transform of *B* and Φ is the power spectrum of the phase fluctuations.

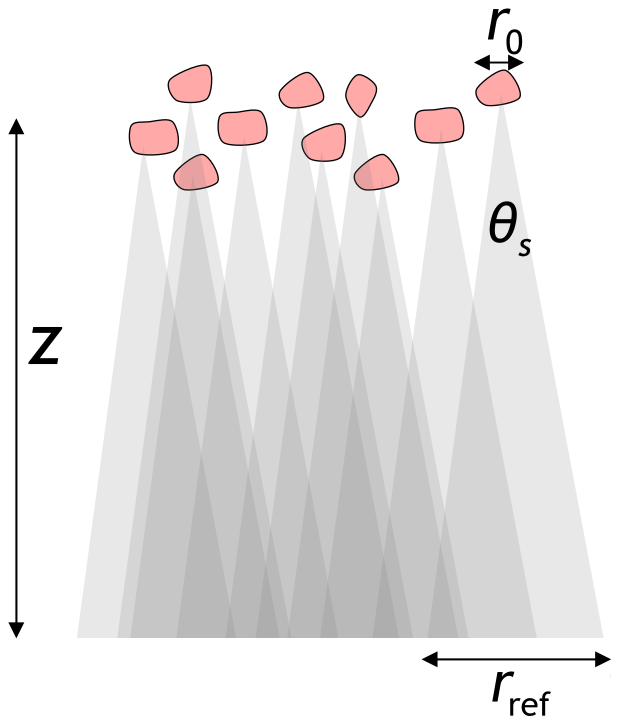
Strong scattering

- Strong scattering corresponds to the situation where the screen generates a large variation in phase over the Fresnel scale (so destroying its importance)
- The new phase-stationary scale is r_0 -- the coherence scale



Strong scattering

- Each patch diffracts radiation over a scattering angle $\theta_s \approx 2\pi\lambda / r_0$ and r_0 is sometimes called the diffractive scale, r_{diff}
- An observer sees radiation from patches over a scale $r_{\text{ref}} = z\theta_s$, called the refractive scale



- Note that $r_{\text{diff}} r_{\text{ref}} = r_F^2$. In weak scattering we are restricted to one scintillation mode, but in strong scattering we get diffractive scintillation and refractive scintillation

Strong scattering– diffractive scintillation

- If the radio source is sufficiently small (and band-limited), the phase screen is illuminated with spatially coherent radiation and the overlapping scattered waves from each phase stationary patch create a strong, random, interference pattern on the ground (scintillation index of ~1) with a scale size of r_{diff} (smeared out if $\theta_{\text{source}} > r_{\text{diff}} / z$)
- The radiation takes a range of paths to reach us. To maintain the interference pattern we must restrict the bandwidth to approximately the inverse of the temporal broadening time

$$\Delta f \approx \frac{1}{2\pi\tau_s}$$

This defines the decorrelation bandwidth of the scintillations.

Strong scattering – refractive scintillation

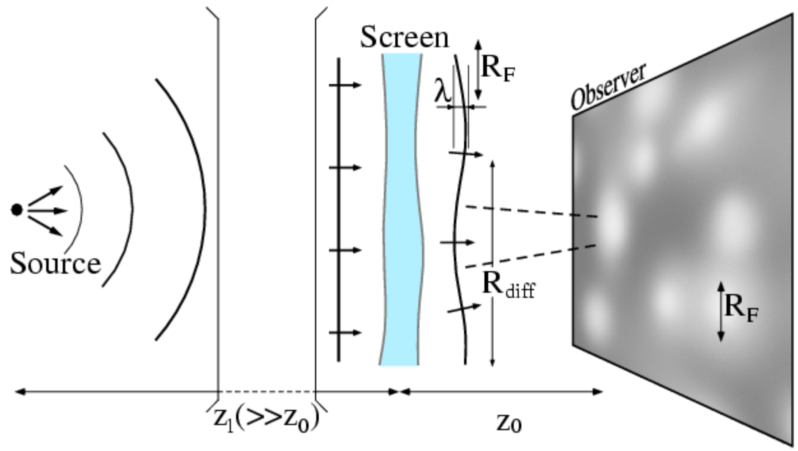
- The refractive scale defined the region of the phase screen that contributes to the intensity on the ground.
- Variations in the refractive index of the screen on > this scale will refract the scattering cones in/out of view, modulating the intensity. This is a broadband effect.

Weak and strong scattering – a summary

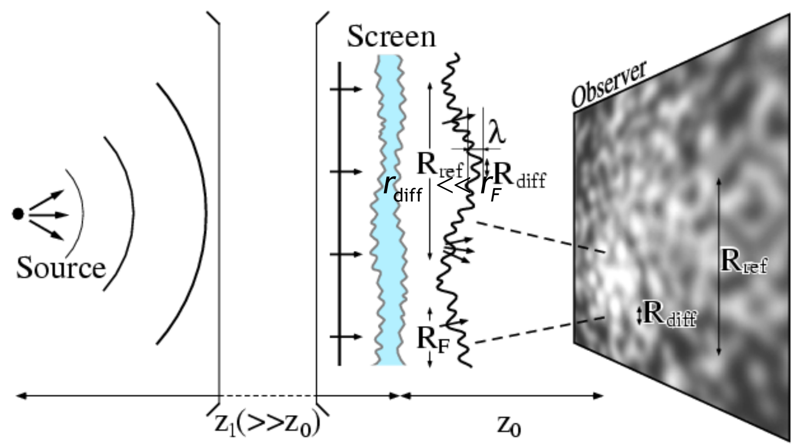
Top panel: $r_{\text{diff}} >> r_F$ The weakly distorted wavefront produces weak scintillation on a scale r_F in the observer's plane.

Bottom panel: $r_{\text{diff}} << r_F$ The strongly distorted wavefront produces strong scintillation on scales of r_{diff} (diffractive scintillation) and r_{ref} (refractive scintillation) in the observer's plane.

(From M. Moniez, 2003)



Weak scattering



strong scattering

Non-uniform plasmas – Kolmogorov spectra

- So far we have considered randomly positioned Gaussian blobs of plasma, all with the same scale size (a). The spatial power spectrum of the corresponding electron number density is

$$P(q_x, q_y, q_z) \propto \left\langle \left| \iiint n_e(\mathbf{r}) \exp(i\mathbf{q}.\mathbf{r}) d^3 r \right|^2 \right\rangle \propto \exp(-q^2 a^2)$$

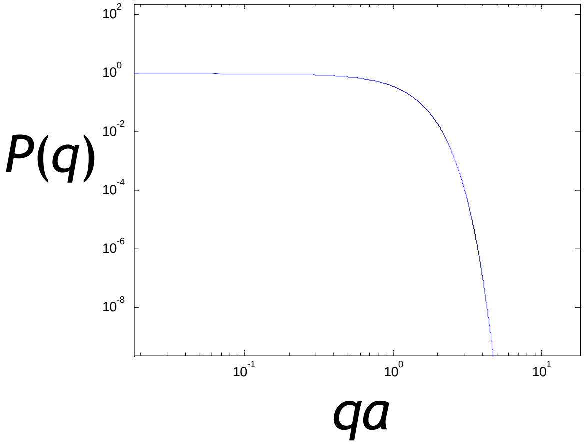
- It is conventional to define the spectral density in terms of wavenumber, q, such that

$$q^2 P(q) \Delta q \propto \text{power in interval } \Delta q$$

so that

$$P(q) \propto \exp(-q^2 a^2)$$

for our single scale-size model

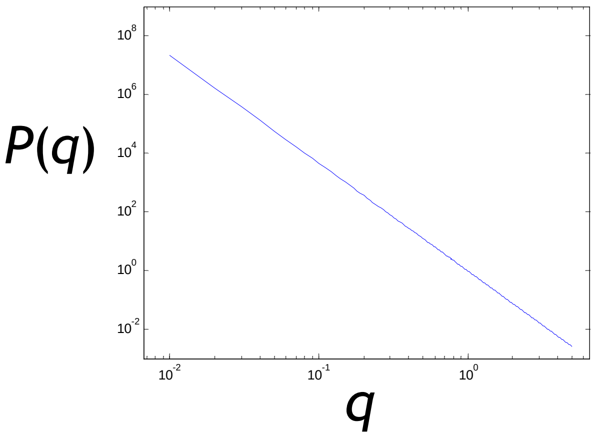


Non-uniform plasmas – Kolmogorov spectra

- A more realistic power spectrum for many turbulent fluids is a Kolmogorov spectrum, which is a power law:

$$P(q) \propto q^{-11/3}$$

- Basis: assume energy is dumped into the medium on large scale sizes, and diffuses to smaller scale sizes by some non-linear process, finally dissipating as heat at a uniform rate per unit volume. Dimensional analysis leads to the above.
- Clearly need to define the largest and smallest scale sizes too…



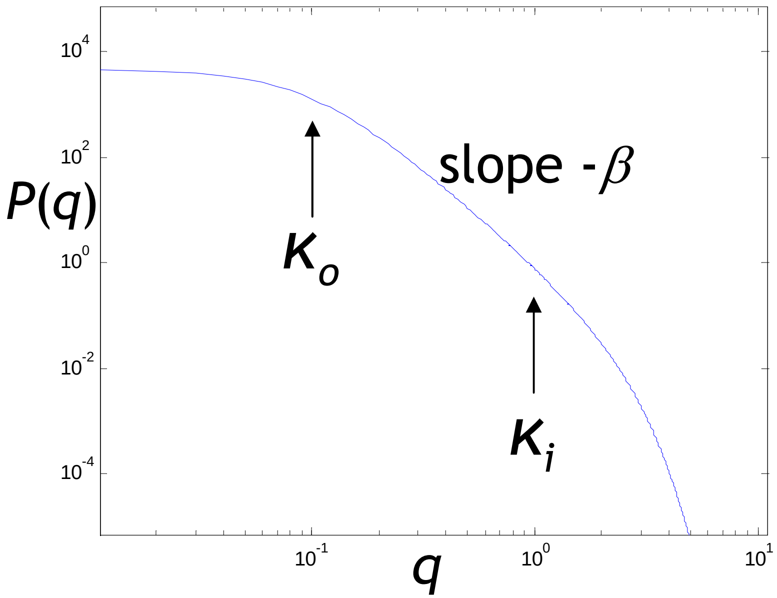
Non-uniform plasmas – Kolmogorov spectra

- We can think in terms of a generalised power law model

$$P(q) = \frac{C_{n_e}^2(z)}{(q^2 + \kappa_o^2)^{\beta/2}} \exp\left(-\frac{q^2}{4\kappa_i^2} \right)$$

with the Kolmogorov model corresponding to

$\beta = 11/3$



- $C_{n_e}^2$ is the scattering strength of the medium. The intergral

$$\mathrm{SM} = \int C_{n_e}^2(z)dz$$

is called the scattering measure, and indicates the magnitude of the line of sight electron density fluctuations.

Non-uniform plasmas – Kolmogorov spectra

- The effect of using a power law spectrum are relatively slight:
  - In weak scattering, the dominant scale size becomes the Fresnel scale (as it has more power in it than the smaller allowed scales)
  - Refractive scintillation is limited by the outer scale size
  - Kolmogorov models work quite well over a wide range of q for the ionosphere, IPM and ISM
  - Sharp-edged density variations (like our original disk model) correspond to $\beta = 4$ on small scales
  - For $\beta = 11/3$, angular broadening goes as $\lambda^{2.2}$ and temporal broadening goes as $\lambda^{4.4}$

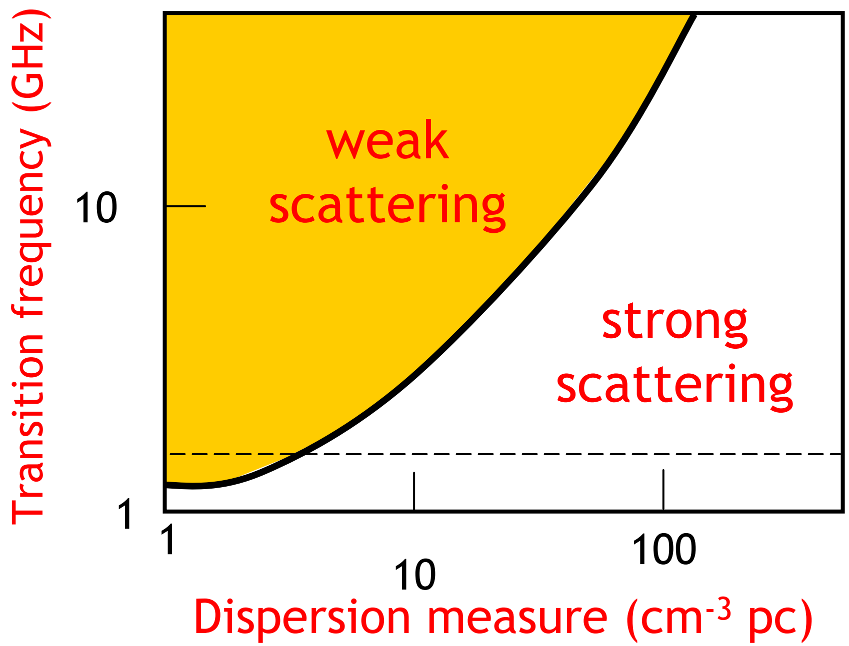
Real astrophysical plasmas

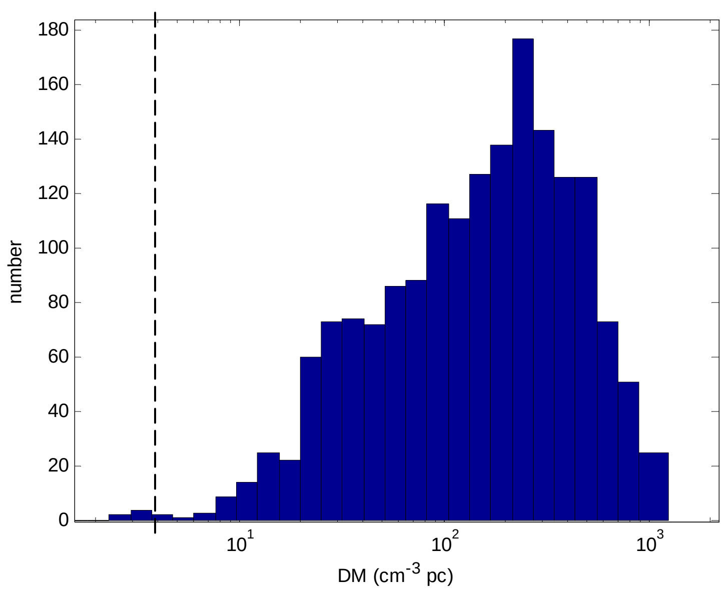
medium	Wavelength used (cm)	Distance (cm)	Fresnel scale (cm)	Diffractive scale (cm)	Scintillation timescales (s)	Scattering mode
troposphere	20	10^5	6×10^2	$\sim10^5$	10	weak
Ionosphere	300	3×10^7	4×10^4	$\sim10^5$	10	Usually weak
IPM	100	10^{13}	10^7	$>10^7$	1	Mostly weak, strong at small solar elongations
ISM	100	10^{21}	10^{11}	$\sim10^9$	DISS: 10^2-10^4 RISS: 10^5-10^7	strong

(after Narayan 1992)

Pulsar scintillation

- Most pulsar observations fall into the strong scattering regime (dashed lines corresponding to 1.4 GHz)





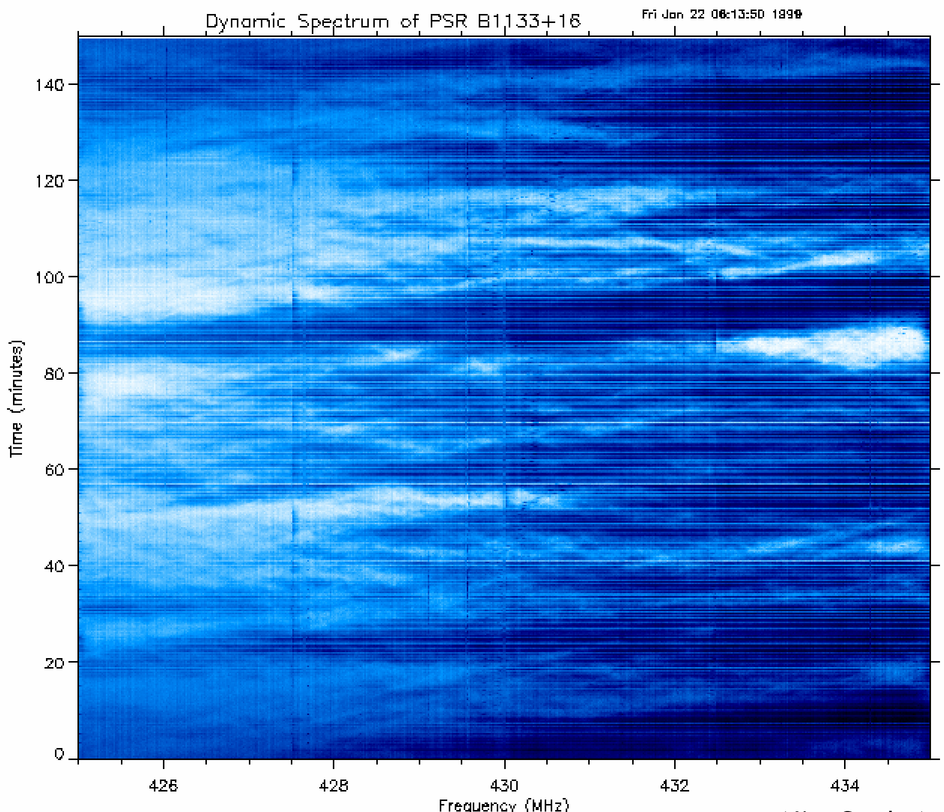
- For scintillation we need

$$\theta_{\text{source}} < r_{\text{diff}} / z$$

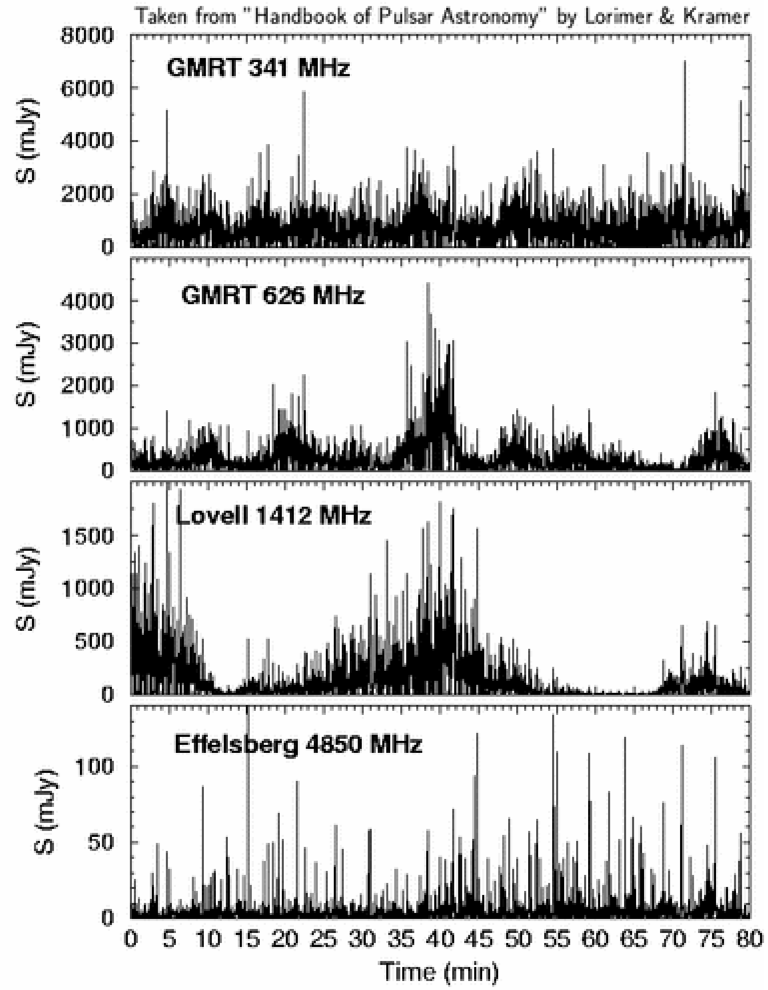
i.e., the source must be smaller than the diffractive scale (~10^4 km) – pulsars!

Pulsar scintillation

- Diffractive scintillation is clearly seen



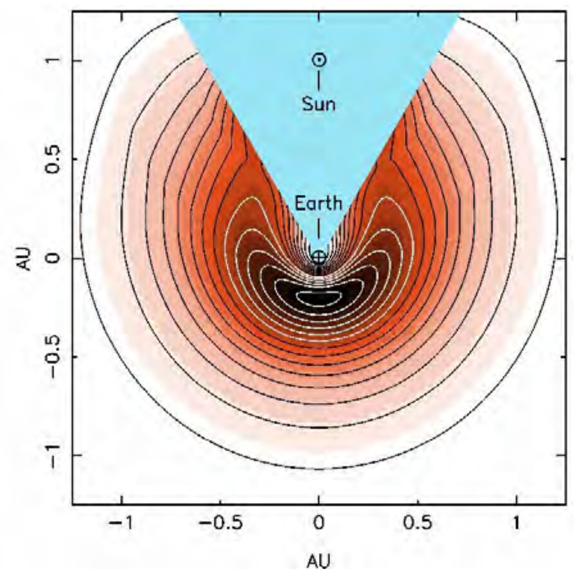
(Jim Cordes)



Interplanetary scintillation and solar weather

- Interplanetary scintillation is usually weak (above about 50 MHz and if not too close to the Sun)
- Scintillation timescale ~1 s,
- Critical source size $\theta_{\text{source}} \approx 0.5$ arcsecond
- Fresnel scale ~100 km
- Solar wind speed ~400 km/s
- Scintillation is seen to increase when CMEs or corotating streams pass across the line of sight to a source

Relative weighting of solar wind to scintillation (81.5 MHz)

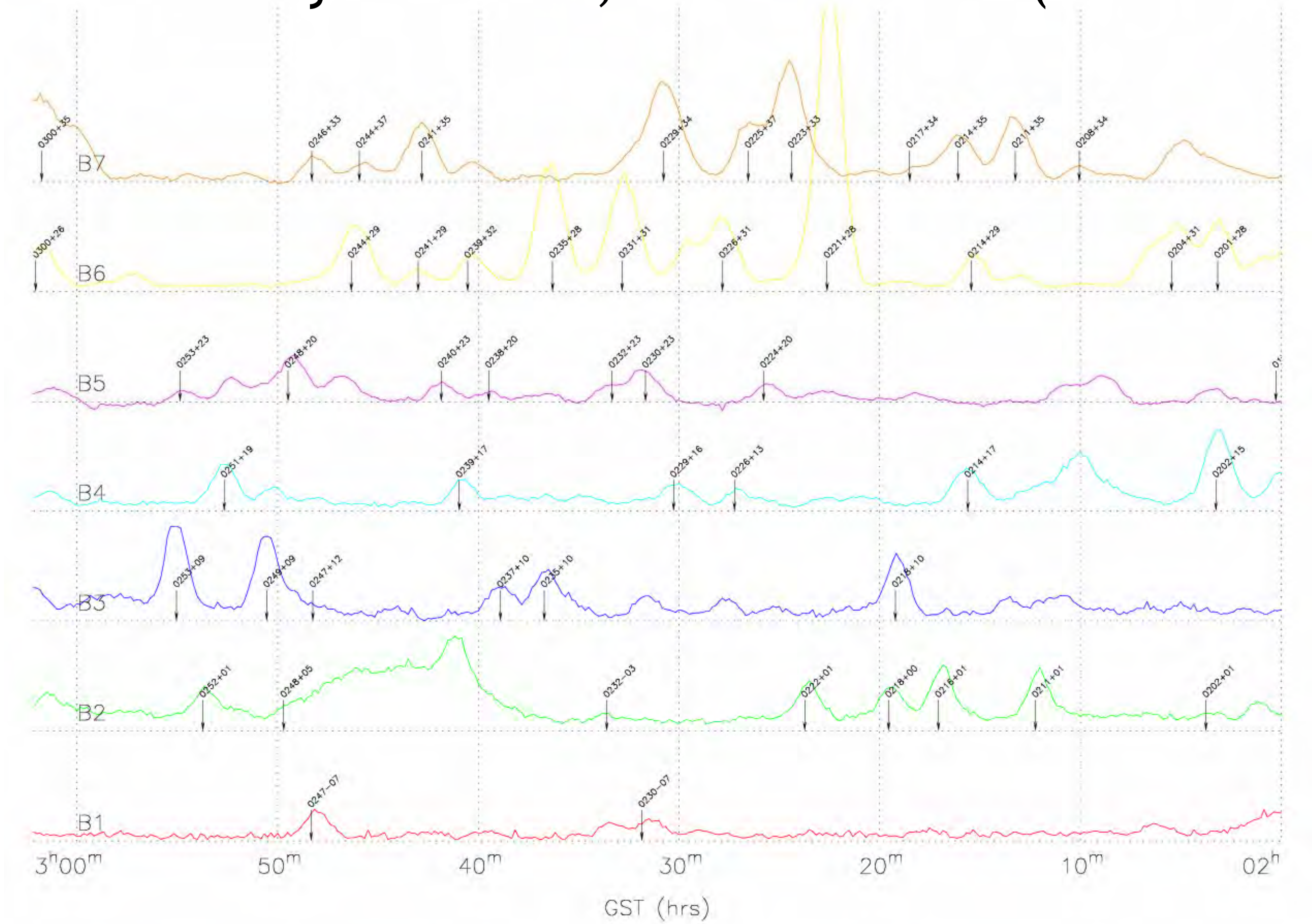


Interplanetary scintillation and solar weather

- The original ‘pulsar telescope’ (the 3.6 hectare array in Cambridge) was originally designed to measure interplanetary scintillation at 81.5 MHz

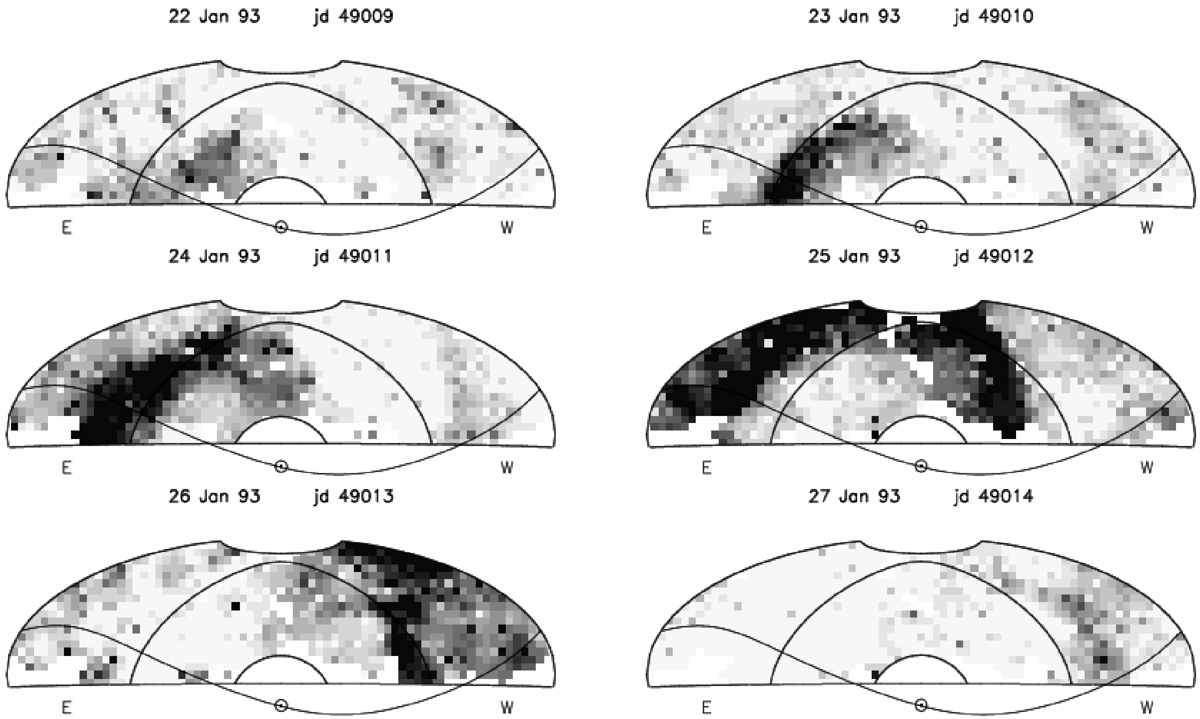
Interplanetary scintillation and solar weather

- The scintillation index (m) of compact sources could be measured daily at transit, with 16 beams (~800 sources).



Interplanetary scintillation and solar weather

- Variations in scintillation index can be used to map out interplanetary density structures.



Radio propagation and scintillation

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