Air showers and their radio component

Thierry Gousset (SubaTech, Nantes, France)

Air showers

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Refs:
— Allan, in: Progress in elementary particle and cosmic ray physics, p. 169 (North Holland, Amsterdam, 1971)

— Stanev, High energy cosmic rays (Springer, 2004)

— Gaisser, Cosmic rays and particle physics (Cambridge University Press, 1990)

— Nagano-Watson, Rev Mod Phys 72, 689 (2000)

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Air showers and the primary radiation

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Length in m $\rightarrow$ length in $\mathrm{g/cm^2}$

Interaction of rays with matter described by various lengths ℓ
$\rightarrow$ mean free path for something to happen (coll, abs)
probability of nothing to happen up to x:

$$\frac{dp}{p} = -\frac{dx}{\ell}$$

- $1/\ell = \text{particle density} \times \text{cross section} = n \times \sigma$
- $n \rightarrow n(x)$, useful to use depth of material X such that $dX = \rho(x)dx$

$$\frac{dp}{p} = -\frac{dX}{\lambda}$$

with λ in units of X, in practice $\mathrm{g/cm^2}$

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$85\ \mathrm{g/cm^2}$ **vs** $1000\ \mathrm{g/cm^2}$

- Earth's atmosphere $=$ cosmic-ray shield
- High energy protons have interaction length in air $\lambda_{pA} = 85\ \mathrm{g/cm^2}$
- Note: A for air (or 80% N$+$ 20% O)
- For a downward vertical path to sea level $\int dX \approx 1000\ \mathrm{g/cm^2}$

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Cosmic rays and high energy physics

$$p(E_p = 10^{17}\ \mathrm{eV}) + A \to X$$

$s_{NN} = 2m_N c^2 E_p = O\Big((10\ \mathrm{TeV})^2\Big)$, i.e. LHC

- Tevatron $\to (2\ \mathrm{TeV})^2$ and RHIC $\to (200\ \mathrm{GeV})^2$

$\to$ need to extrapolate

$\to$ hadronic models (more in Stanev sec 8.3)

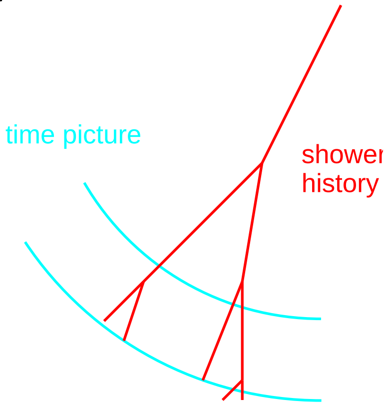
Secondary particles

$$p + A \to X$$

- X
  - $\sim 10^{\,2}$ pions (20% something else) $+$ target fragments $+$ “original” baryon with a fraction of the initial energy
  - pions are $\pi^{\,+}$, π^- and π^0
- $\pi^0 \overset{99\%}{\to} 2\gamma$
  - $c\tau_{\,\pi^0} = 25$ nm; π^0’s desintegrate before reinteracting
- γ’s initiate the electromagnetic component of the shower
  1. pair creation $\gamma + A \to e^+e^- + X$
  2. bremsstrahlung $e + A \to e + \gamma + X$
  3. repeat 1 and 2

Shower in space and time

- multiplicative process
- energy distributed among a vast number of secondary particles
- almost forward development



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Time development and energy distribution

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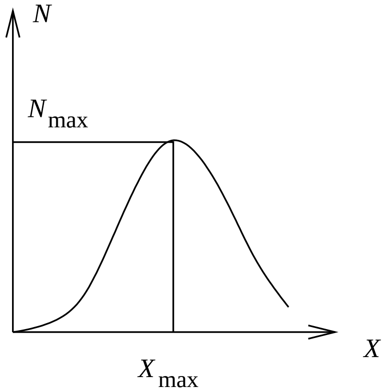
Time development

which N?

- electrons and positrons
- charges
- charges above an energy threshold (in practice that of particle detection)



$N(X)$ trend results from competition

- multiplicative processes $\Rightarrow dN > 0$ and $E \searrow$
- ionisation loss $\Rightarrow E \searrow$

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Heitler model

Toy model for cascade development

- $1 \to 2$ process, with daughter particles each carrying half the parent energy
- Branching at every step of length $X_{1/2}$
- After the kth branching $X = k \times X_{1/2}$, $N = 2^k$ and the energy per particle is E_p/N
- Assume branching process stops when $E \leq E_C$

$$N_{\max} = \frac{E_p}{E_C}, \quad X_{\max} = X_{1/2} \log_2(E_p/E_C)$$

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γ initiated shower

- $\gamma + A \to e^+e^- + X$ and $e + A \to e + \gamma + X$ are $1 \to 2$ processes
- $\approx$ same length scale 'radiation length'$= X_0 \approx 40\ \mathrm{g/cm^2}$
- $X_{1/2} \approx \ln 2 \times X_0 = 30\ \mathrm{g/cm^2}$
- these branchings dominate for $E > E_C$, with a critical energy in air ≈ 100 MeV

$$N_{\max} = \frac{E_\gamma}{100\ \mathrm{MeV}}, \quad X_{\max} = 100\ \mathrm{g/cm^2} \times \log_{10}(E_\gamma/100\ \mathrm{MeV})$$

model misses energy loss by ionisation $\Rightarrow N_{\max}$ overestimated

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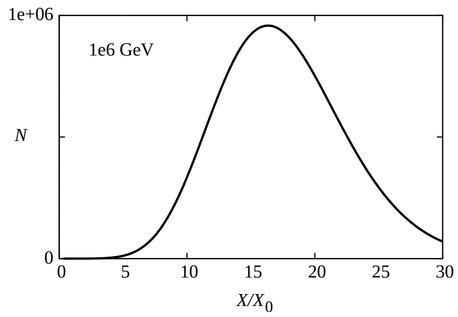
Greisen parametrization

(Stanev p. 175)

$$N_e^\gamma = \frac{0.31}{\sqrt{\ln E_\gamma/E_C}} \exp\left[(1 - \frac{3}{2}\ln s)X/X_0\right], \quad s = \frac{3X}{X + 2X_{\max}}$$



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$N_{\max}$ from total track length

- Energy dissipated in ionisation loss; for relativistic particle the rate is $dE/dX \approx -2$ MeV$/\mathrm{g}/\mathrm{cm}^2$
- $N(X)$ number of charged particles at depth X
- Energy dumped in $[X, X+dX]$ slice

$$dE = (2\ \mathrm{MeV}/\mathrm{g}/\mathrm{cm}^2) \times N(X)dX$$

$\Rightarrow$

$$\int N(X)dX \approx \frac{E_p}{2\ \mathrm{GeV}} \times 1000\ \mathrm{g}/\mathrm{cm}^2$$

($\rightarrow$ fluorescence method, more in Nagano-Watson)

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$N_{\max}$ from total track length (cont'd)

$\int N(X)dX = N_{\max} \times$characteristic shower length

taking 1 atmospheric thickness:

$$N_{\max} = \frac{E_p}{2\ \mathrm{GeV}}$$

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Energy spectrum (e-m component)

in Heitler model:

- total track length associated with particles of energy greater than $E = E_\gamma/2^k$ $= 30\ \mathrm{g}/\mathrm{cm}^2 \times 2^k(1/2 + 1/4 + \cdots) \approx (E_\gamma/E) \times 30\ \mathrm{g}/\mathrm{cm}^2$
- an electron with $E = E_C$ loses it in one radiation length
- total track length associated with particles of energy lower than $E_C = (E_\gamma/E_C) \times 40\ \mathrm{g}/\mathrm{cm}^2$
- more weight to low energy in actual fact

$$\int_{>E} N dX \approx 40\ \mathrm{g}/\mathrm{cm}^2 \times \frac{E_\gamma}{E_C} \times \frac{30\ \mathrm{MeV}}{E + 30\ \mathrm{MeV}}$$

this is for the whole shower
$\rightarrow$ at and around maximum $N(>E)/N$

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Hadronic component

- $\pi^{\pm}$: $c\tau = 8$ m
  - $\lambda_{\pi A} = 120\ \mathrm{g/cm^2} \rightarrow \ell_{\pi A}$ ≈1 km for $n(z=0) = 1$ mg/cm^3
  - at high energy pions reinteract
  - otherwise they decay $\rightarrow \mu\nu$; muons (only lose 2 MeV$/\mathrm{g/cm}^2$) $\rightarrow$ direct information on pions
- π^0: estimate of $X_{\max}$ and $N_{\max}$ for proton induced shower assuming that the e.m. showers are initiated by 1st generation π^0's

$$X_{\max} = \lambda_{pA} + X_0 \ln\left[\frac{(1-K)\,E_p}{2\langle m\rangle E_C}\right], \quad N_{\max} = \frac{(1-K)\,E_p}{3\,E_C}$$

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in practice

average behavior (adjusted with MC)
Gaisser-Hillas formula (Stanev p. 186)

$$N(X) = N_{\max}\left(\frac{X - X_1}{X_{\max} - \lambda}\right)^{X_{\max}/\lambda - 1} \exp - \left(\frac{X - X_1}{\lambda}\right)$$

+ fluctuations:

- on $X_1 \rightarrow X_{\max}$
- on shape and $N_{\max}$: individual realizations of first hadronic collisions (inelasticity, multiplicity, energy of secondaries)

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Nucleus vs proton initiated shower

- in the superposition approximation: nucleus $= A\times$ independent nucleons with energy E_p/A
- nucleus shower $= A\times$ nucleon showers
- shift of $X_{\max}$: $X_{\max}(E_p, A) = X_{\max}(E_p/A, p)$
- less shower to shower fluctuation

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Negative charge excess

- below E_C

$$e(\gamma) + A \to e(\gamma) + e^- + X$$

  delta rays (Compton recoil)
- positron annihilate in flight

$\to$ 10–20% e^- excess in the energy range below E_C

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Lateral spread and longitudinal dispersion

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Multiple scattering

- Emission $\to$ spread of hadrons
- Multiple scattering $\to$ spread of electrons

spread of hadrons limited to a few meters

electrons

- typical scattering angle $\theta \sim 1/\gamma$
- $\theta^2(n) = n \times (1/\gamma)^2$
- proportion of scatterings with radiation $\sim 1/\alpha$

$\Rightarrow$ $d\theta^2 = (E_s/E)^2 dX/X_0$, with $E_s = 4\pi m_e c^2/\alpha = 21$ MeV

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Multiple scattering (cont'd)

including energy loss $X_i \to X_f$, $E(X) = E(X_f) \times e^{\frac{X_f - X}{X_0}}$

$$X' = X_f - X,$$

$$\theta^2(i \to f) = \int d\theta^2 = \frac{E_s^2}{E_f^2} \int_0^{X_i - X_f} e^{-2X'/X_0} \frac{dX'}{X_0}, \quad (E_f > E_C)$$

and lateral displacement

$$D^2(i \to f) = \int X'^2 d\theta^2 \Rightarrow D = \frac{10 \text{ MeV}}{E} X_0 \quad (E > E_C)$$

i.e., 40 m at 100 MeV at sea level
$\to$ multiple scattering longitudinal lag 3 m, also $\searrow$ with $\nearrow$ energy

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Lateral distribution

flux of electrons given by NKG formula (Gaisser p 226, Stanev p 179)

$$n_e(r, X) = N_e(X) \frac{C}{r\, r_1} \left(\frac{r}{r_1}\right)^{s-1} \left(1 + \frac{r}{r_1}\right)^{s-9/2},$$

with

$$r_1 = \frac{E_s}{E_C} \frac{X_0}{\rho_{\mathrm{air}}}$$

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From showers to electric fields

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A first exercise

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A detour: Cerenkov light in air

consider Cerenkov radiation of a charge particle ($q = Z\, e$)
energy spectrum per unit length

$$\frac{d^2 E_C}{dL d\omega} = \alpha Z^2 \sin^2 \theta_C \frac{\omega}{\hbar c}$$

- Cerenkov in air $\theta_C \ll 1$
- vertical downward moving particle
- trajectory bit of length Δz around z_0 shines on a ring of mean radius $z_0\, \theta_C$ and width $\Delta z\, \theta_C$

$$\frac{\Delta E_C}{2\pi z_0 \Delta z \theta_C^2} = \alpha Z^2 \frac{\omega \Delta \omega}{h c z_0}$$

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Cerenkov radio

$\Rightarrow$ $dE_C/dS \sim 10^2$ MeV/m^2 using $hc = 1.24$ eV μm, $Z = 1$, $z_0 = 4$ km, $N_e = 5\,10^7$, $\lambda = 0.6$ μm and $\Delta\lambda = 0.4$ μm

radio (decametric)

- divide ω by $\sim 10^7$ and $\Delta\omega$ by $\sim 10^7$
- take $A_e \sim 10$ m^2

$\Rightarrow$ $\Delta E_C = 10^{-5}$ eV

- much too small since galactic noise gives

$$k_B T \times \Delta\nu \times T \to 2.5 \text{ eV} \times 40 \text{ MHz} \times 10 \text{ ns} = 1 \text{ eV}$$

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Coherence: a must in radio

solution: replace $N_e \to N_e^2$; incoherent $\to$ coherent

- at first: $N_- = N_+ \Rightarrow$ no field at all
- but *systematic* charge separation by
  1. earth magnetic field (and E field in thunderstorms)
  2. elementary processes $\to$ negative charge excess

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Different approaches

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Overview

- $\sum_{k=1}^{N} \vec{E}(t, A)$ with $\vec{E}$ single-charge electric field taken from textbook $\to$ Monte-Carlo based approach
- $\vec{E}[\rho, \vec{\jmath}]$
- Feynman formula for relativistic charges:

$$\vec{E} = \frac{-q}{4\pi\epsilon_0 c^2} \vec{e}_{r'}''$$

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From individual charges

More thorough study to date: T Huege, H Falcke, Astronomy & Astrophysics 412, 19 (2003); Astronomy & Astrophysics 430, 779 (2005); Astropart. Phys. 24, 116 (2005); T Huege et al, Astropart. Phys. 27, 392 (2007)
Building block:

$$\vec{E}_q(t, A) = \frac{q}{4\pi\epsilon c^2} \frac{\vec{R} \wedge \left[(\vec{R} - R\vec{v}/c) \wedge \vec{a}\right]}{||R - \vec{R} \cdot \vec{v}/c||^3}$$

with

$$\vec{a} = \frac{q\vec{v} \wedge \vec{B}}{\gamma m_e}$$

From charges and currents

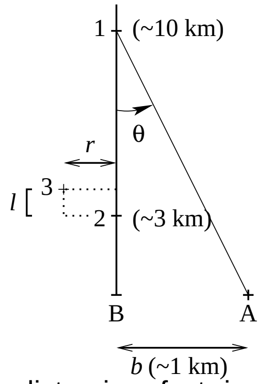
shower electromagnetic field as a standard electromagnetism exercise

$$(\vec{E}, \vec{B}) = F[\rho, \vec{j}]$$

how to carry out such a program ?

- Kahn and Lerche approach
  - ringlike geometry
  - no shower evolution (contribution around $N_{\max}$) + estimate for shower decay
  - formulation in Fourier space
  - geomagnetic contribution $>$ Askaryan effect (charge excess)
- more realistic model, numerical implementation. . . not (yet) followed

Time scales



- particle Q moves at $\approx c$
- Q =B at $t = 0$
- $ct_i = \sqrt{d_{iB}^2 + b^2} - d_{iB}$
- $\Delta t_{12} \approx 0.4\, b^2$ (small θ)
- $\Delta t_{2B} = 3.3\, b$

(time in μs and distance in km)

- Doppler distorsion: fast rise and slow decay
- $v \approx c$ valid at $\theta \gg |c - v|$
- $\Delta t_{32} \approx l/c + b\, r/(cd_{2B})$, both terms < 30 ns

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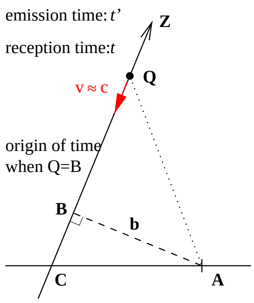
Large impact parameters

$\Rightarrow$ at large $b \longrightarrow$ (distorted) image of $N(X)$

- pointlike approximation: all timescales but obliquity set to 0



$$-c\,t' \gg b^2/2 \gg c\,t$$

$$ct\, ct' \approx -b^2/2,$$

$$E(t, A) = \frac{e\, N_{ee}(t')\, a_T}{4\pi\epsilon c^2} \frac{b^2}{2\,(ct)^3}.$$

$$a_T = \frac{e\, c\, B\, \sin\alpha}{\gamma m_e}$$

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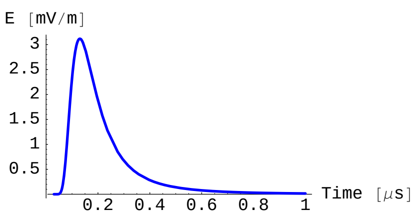
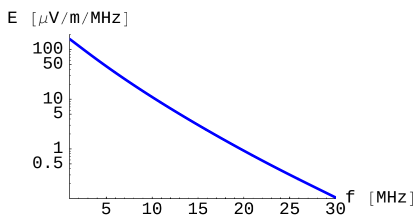
Large impact parameters (cont'd)

(vertical, 10^{19} eV, 700 m)





makes it possible problem inversion and discussion of antenna spacing for a giant array

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