The Basics of Radio Interferometry

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The Basics of Radio Interferometry

The role of interferometry in astronomy = role of venetian blinds in *Film Noir*

The Basics of Radio Interferometry

References

- **Optics/interferences**
  - Michelson, A., A., *"Studies in Optics"*, Dover publication
  - Hecht, *"Optics"*, Addison-Wesley
- **Fourier transform**
  - Bacewell, R. *"The Fourier Transform and its Applications"*, McGraw-Hill
- **Radio astronomy**
  - Kraus, *"Radio Astronomy"*, Cygnus Quasar Books
  - Rohlfs, K., Wilson, T., *"Tools of Radio Astronomy"*, Springer
- **Radio interferometry**
  - Thompson, Moran, Swenson, *"Interferometry and Synthesis in Radio Astronomy"*, Wiley-interscience publication
  - NRAO and IRAM interferometry summer schools

The Basics of Radio Interferometry

I. The concepts

a) Is it difficult to understand?
b) From fringes to “visibilities”
c) From visibilities to images
d) Resolution and artifacts
e) Interferometer array design

II. In practice

a) How does an interferometer array work?
b) Examples of working instruments
c) Aperture synthesis
d) Calibration and “Deconvolution”
e) Examples of data cubes

The concepts

Difficult to understand?

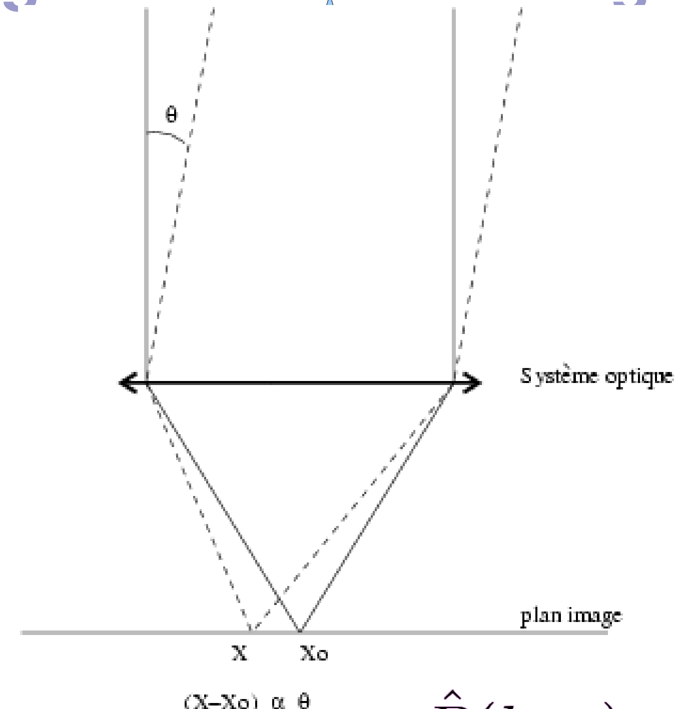
- **Not a natural technique**
  - no animals equipped with interferometers
- **Terminology can be confusing**
  - Radio astronomers have their own language (*dish*, *beam*, *sidelobes, baseline, ...)*
  - *Interference* is a phenomenon not a measure, what is actually measured with an “interferometer”?
  - *Visibility* can have several meanings
  - *Deconvolution* is not really what is meant when building images from interferometry measurements

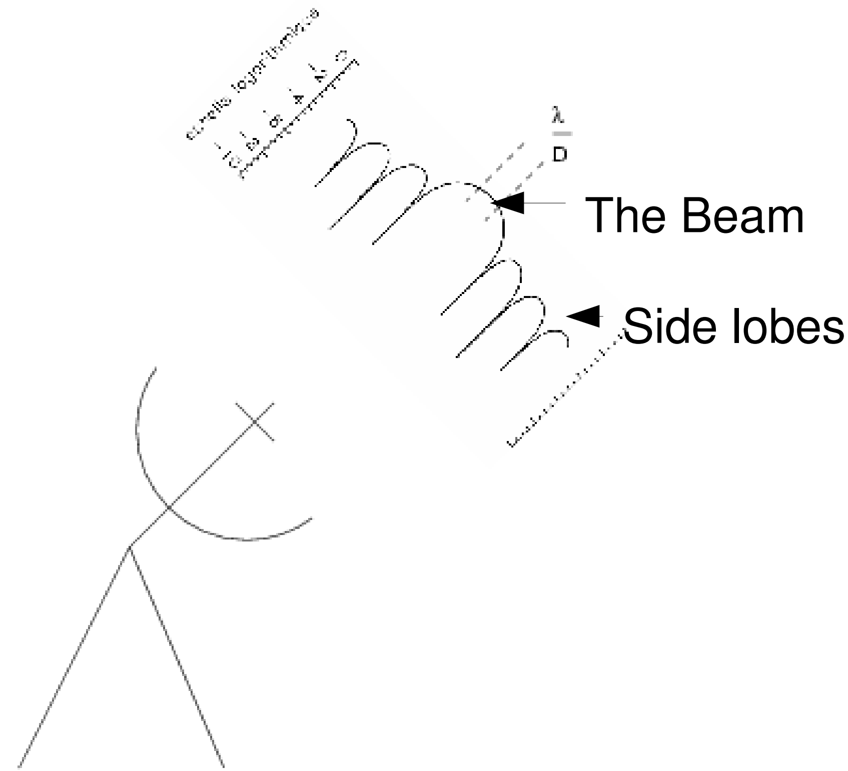
The concepts

Difficult to understand?

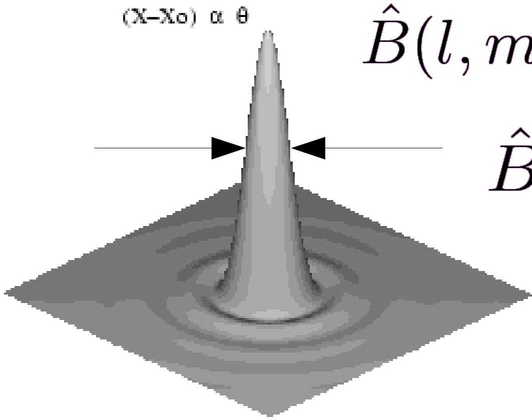
- **Not a direct technique**
  - To get an image the astronomical signal must be
    - processed by the electronics (the backends)
    - processed numerically by deconvolution software
- **The maths can be confusing**
  - Fourier transform is a mathematical concept not a real property of the source, how do the measurements relate to the source properties?
  - Imaging involves solving an “ill posed” inverse problem -> not trivial

Digression: direct imaging





$$\hat{B}(l,m) = \iint B(l',m')\,\mathrm{PSF}(l-l',m-m')\,dl'\,dm'$$



$$\hat{B} = B * \mathrm{PSF}$$

$$\delta\theta \simeq \frac{\lambda}{D} \quad \Longleftrightarrow$$

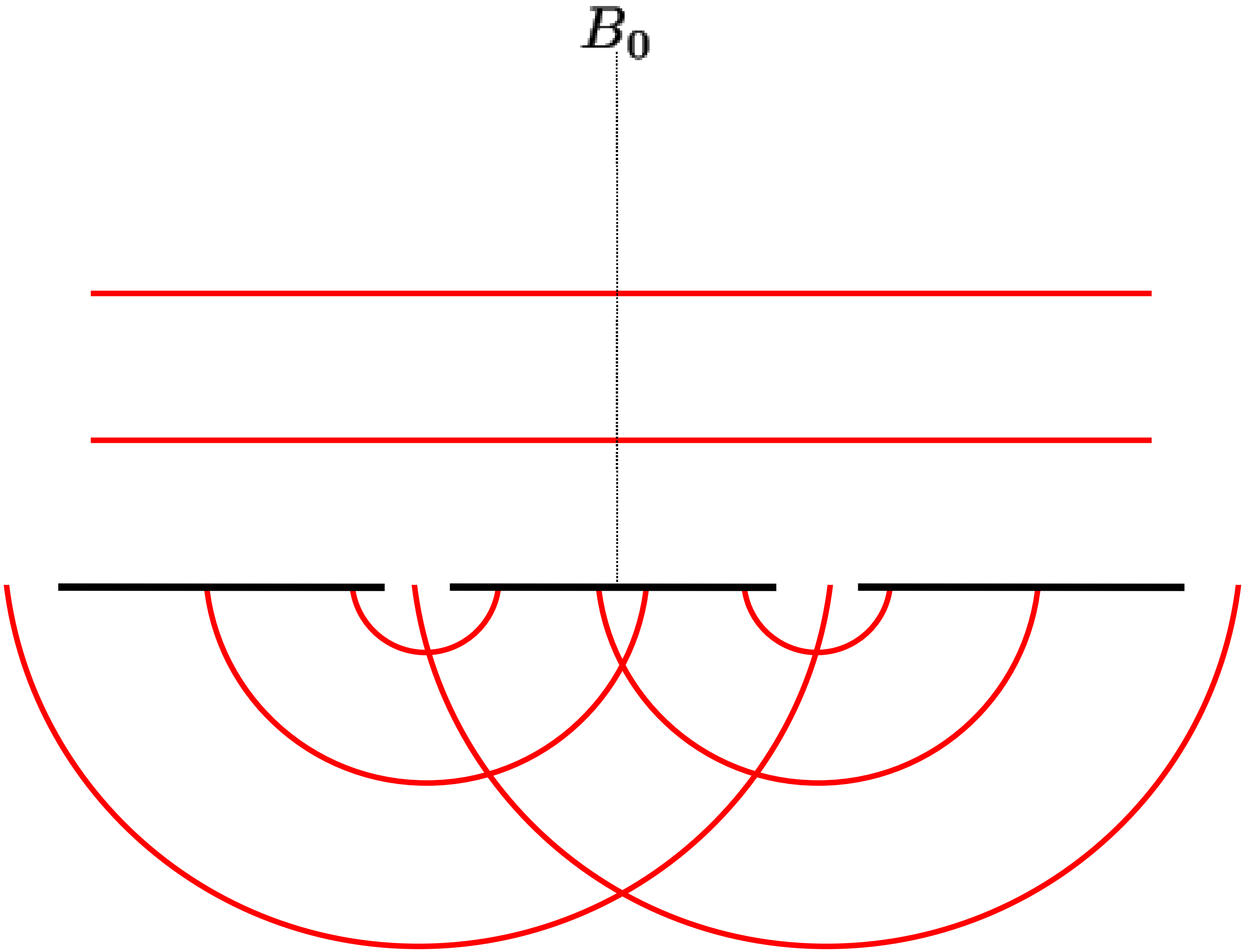
Field of view of a radio-telescope = primary beam

The concepts

From fringes to visibilities, a heuristic introduction

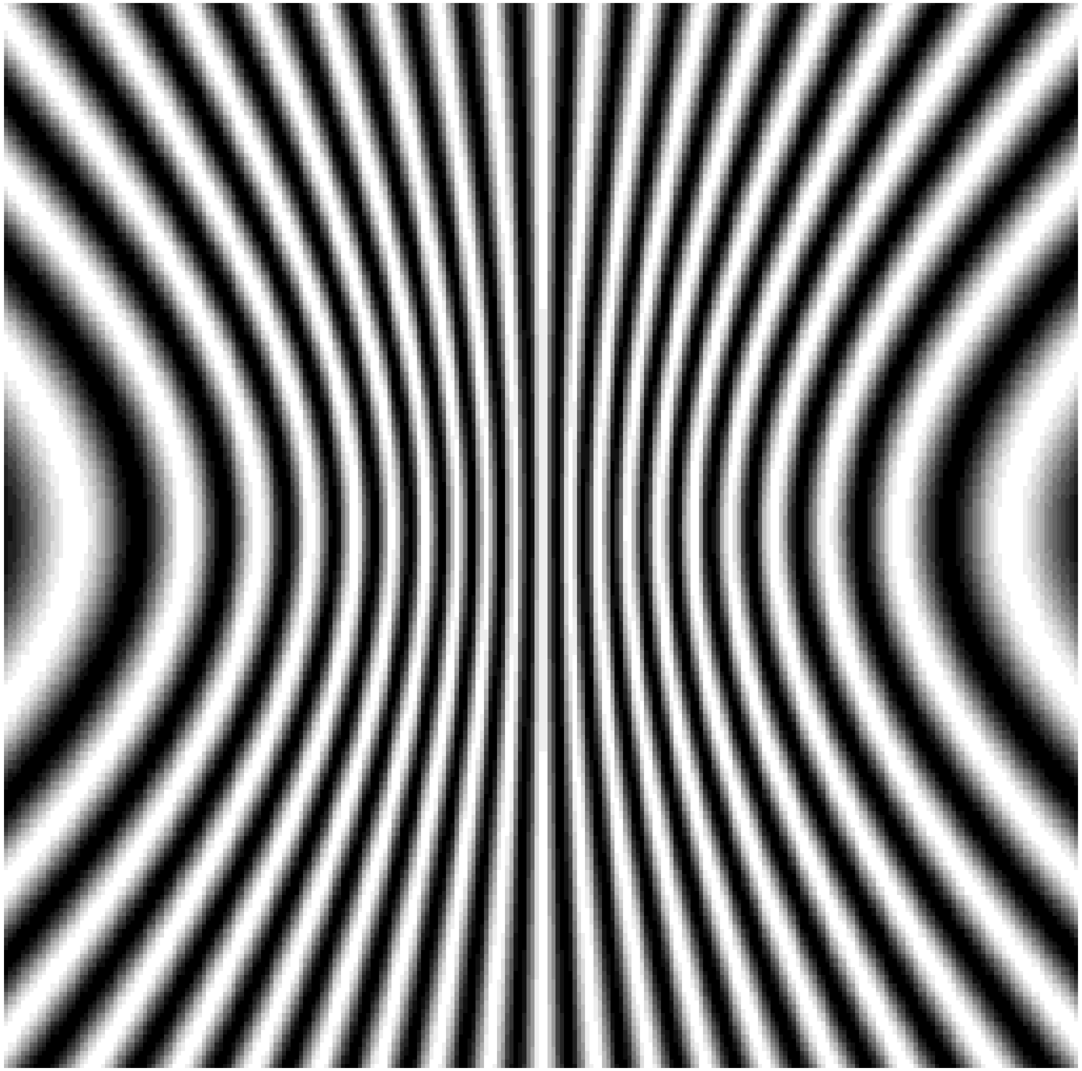
- The Young slits experiment
- What is a *visibility* (in radioastronomy)?
- How to measure a visibility?
- What kind of information are contained in a visibility?

Young's experiment

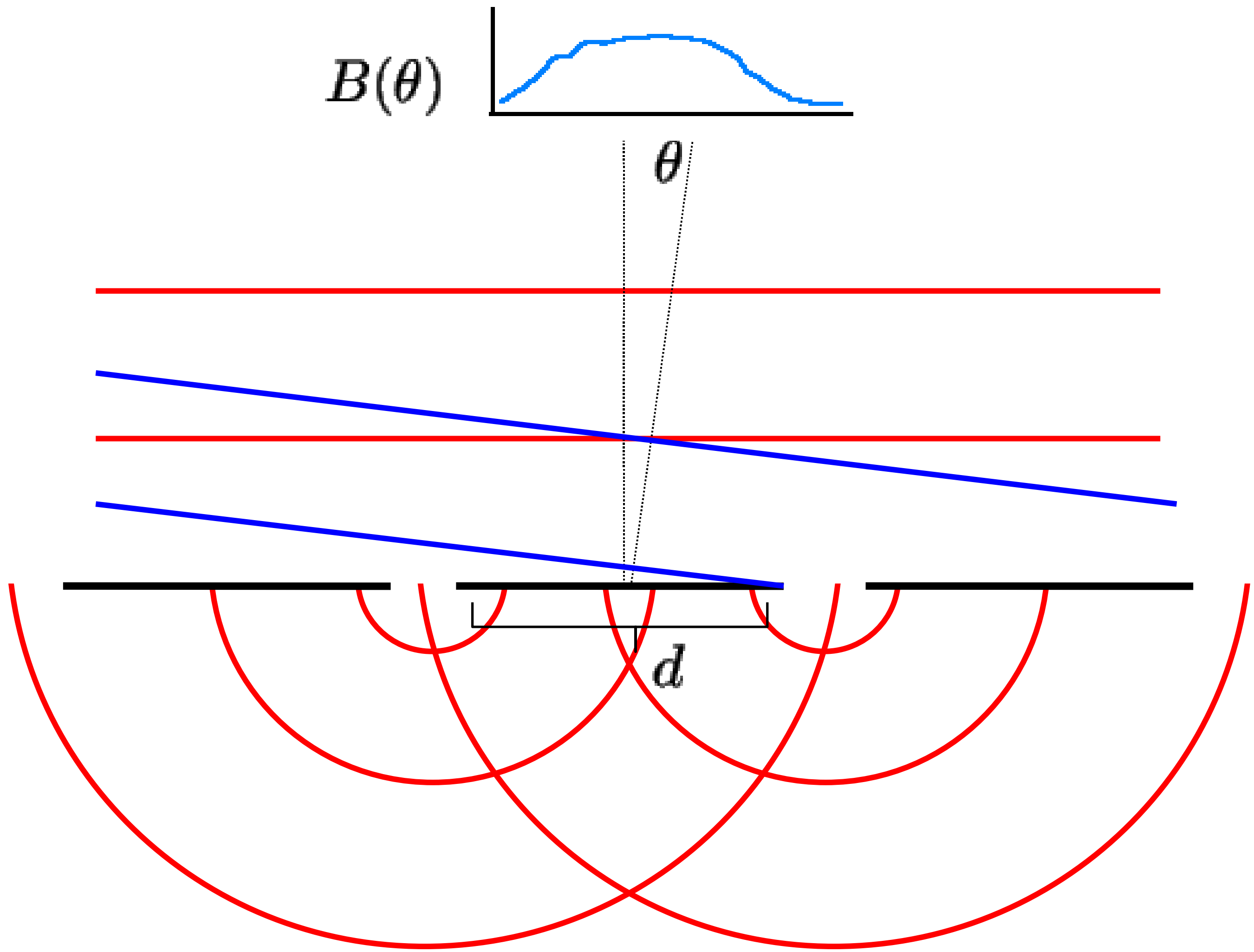


$$I = B_0(1 + \cos[2\pi\nu\Delta t])$$

Young's experiment



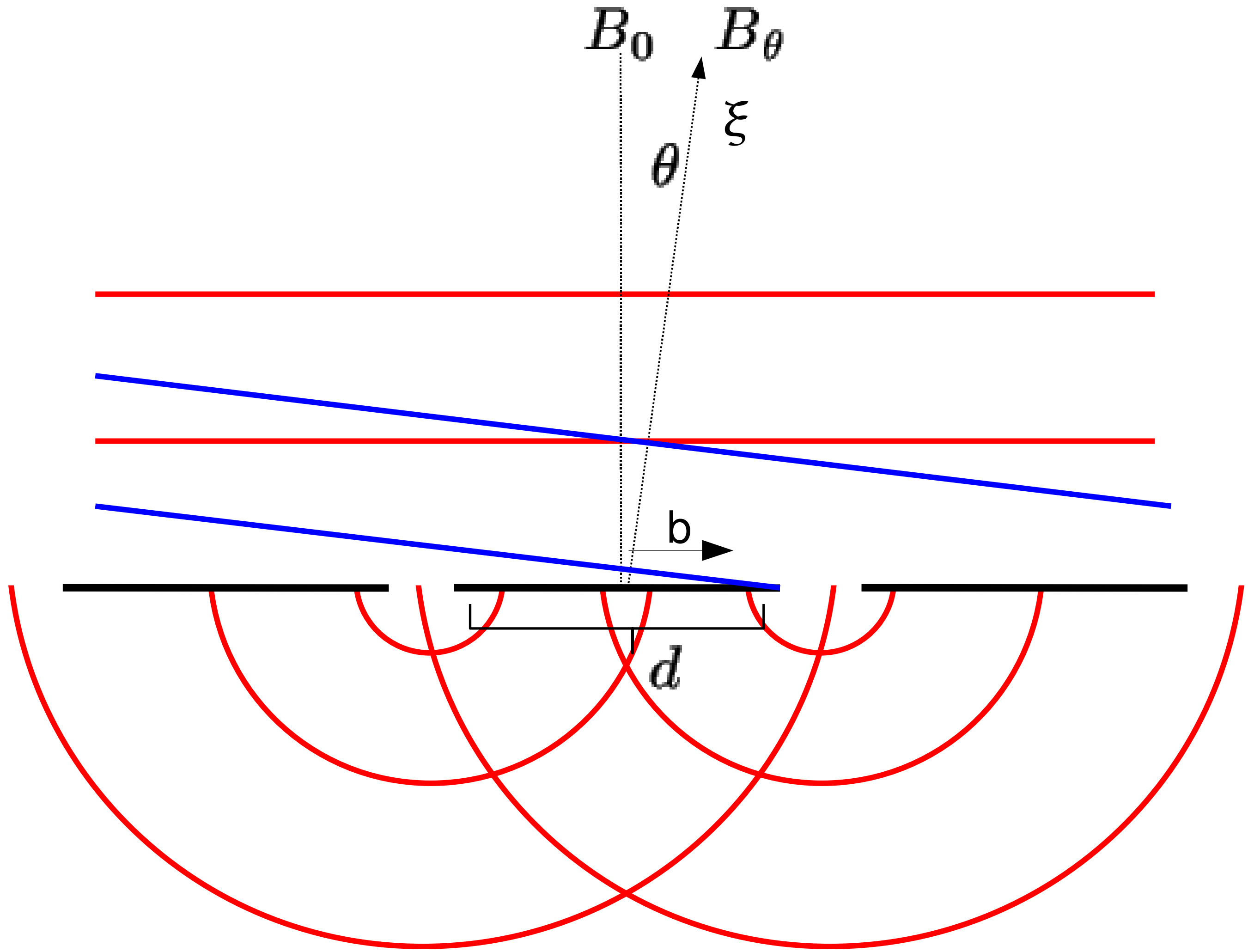
Fringes produced on a screen behind the apertures



$$I = B_0(1 + \cos[2\pi\nu\Delta t])$$

$$I = B_\theta(1 + \cos[\underbrace{2\pi\nu\Delta t} + 2\pi\frac{d}{\lambda}\theta])$$

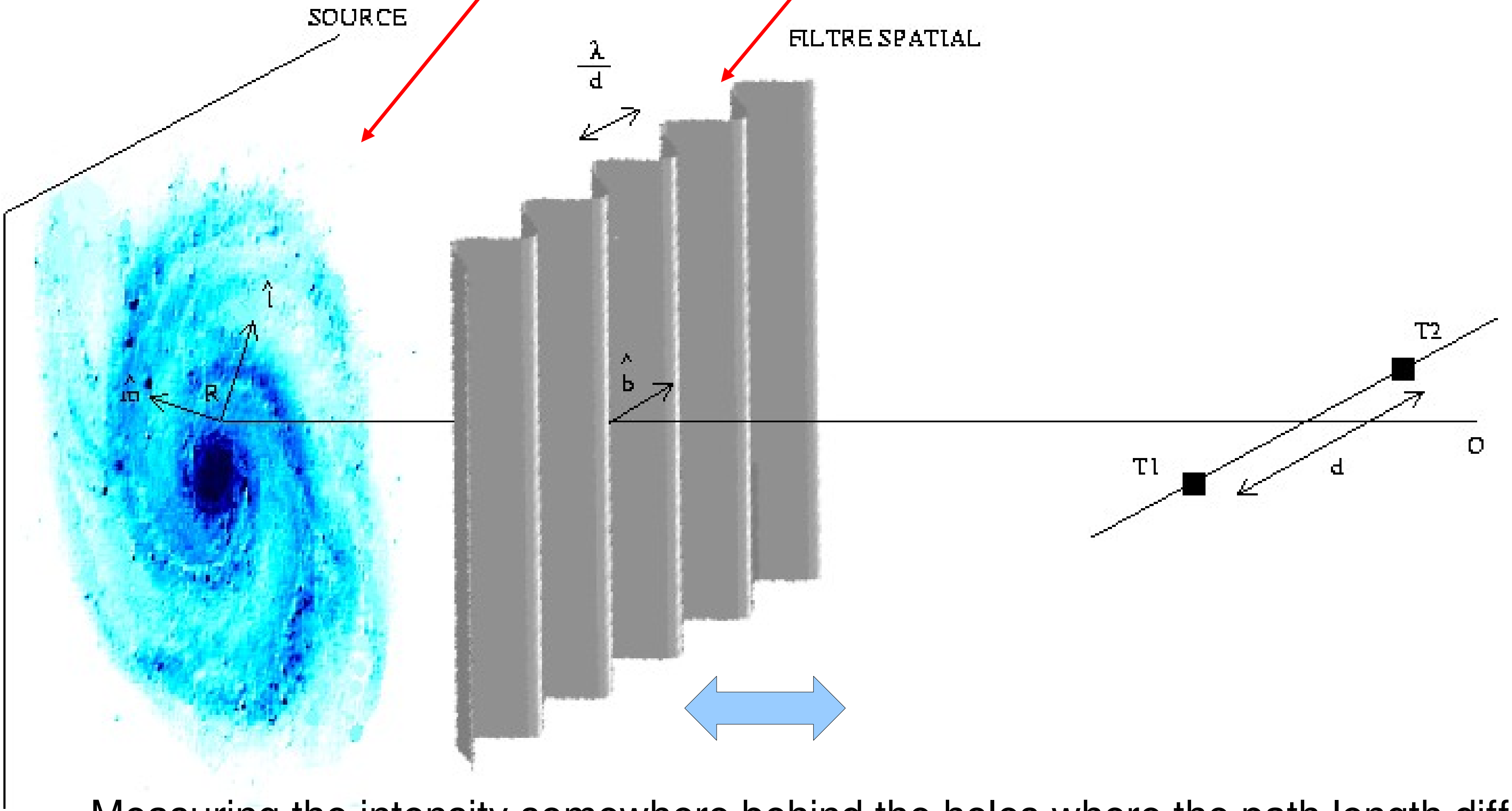
$$I = B_T + \int_{\Delta\theta} B(\theta)\cos[\varphi + 2\pi\frac{d}{\lambda}\theta]\ d\theta$$



$$I = B_T + \int_{\Delta\theta} B(\theta)\cos[\varphi + 2\pi \frac{d}{\lambda}\theta]\ d\theta$$

$$I = B_T + \int_{\Delta\Omega} B(\boldsymbol{\xi})\cos\left[\varphi + 2\pi \frac{d}{\lambda}\boldsymbol{\xi}\cdot\hat{\boldsymbol{b}}\right]\ d\theta$$

$$I = B_T + \int_{\Delta\Omega} B(\boldsymbol{\xi}) \cos\left[\varphi + 2\pi \frac{d}{\lambda} \boldsymbol{\xi} \cdot \hat{\boldsymbol{b}}\right] d\theta$$



Measuring the intensity somewhere behind the holes where the path length difference is Δt is **equivalent** to measuring the total intensity transmitted by a sine filter (~ a venetian blind) with an angular frequency d/λ oriented parallel to the vector defined by the holes and offset by $\varphi = 2\pi\nu\Delta t$ => spatial information

What is a visibility?

- The intensity measured at any location behind the holes is

$$
\begin{aligned}
I &= B_T + \int_\Omega B(\boldsymbol{\xi}) \cos\left[2\pi \frac{d}{\lambda} \boldsymbol{\xi} \cdot \hat{\boldsymbol{b}} - \varphi\right] \, d\Omega \\
&= B_T + \int_\Omega B(\boldsymbol{\xi}) \left\{ \cos\left[2\pi \frac{d}{\lambda} \boldsymbol{\xi} \cdot \hat{\boldsymbol{b}}\right] \cos\varphi + \sin\left[2\pi \frac{d}{\lambda} \boldsymbol{\xi} \cdot \hat{\boldsymbol{b}}\right] \sin\varphi \right\} \, d\Omega \\
&= \cos\varphi \, I_1 + \sin\varphi \, I_2 + B_T
\end{aligned}
$$

- φ depends on the position of the measure w.r.t. the two holes -> not related to the source

- <u>All the information</u> about the source is contained in

$$
\begin{aligned}
I_1 &= \int_\Omega B(\boldsymbol{\xi}) \cos\left[2\pi \frac{d}{\lambda} \boldsymbol{\xi} \cdot \hat{\boldsymbol{b}}\right] \, d\Omega \\
I_2 &= \int_\Omega B(\boldsymbol{\xi}) \sin\left[2\pi \frac{d}{\lambda} \boldsymbol{\xi} \cdot \hat{\boldsymbol{b}}\right] \, d\Omega
\end{aligned}
\quad \Rightarrow \quad \text{Two observables}
$$

- We <u>define</u> the visibility as

$$
\boxed{\mathcal{V} = I_1 + iI_2 = |\mathcal{V}| e^{i\phi}}
$$

How to measure a visibility?

Measure I_1 and I_2

$$I = B_T + \int_{\Delta\Omega} B(\boldsymbol{\xi}) \cos\left[\varphi + 2\pi \frac{d}{\lambda} \boldsymbol{\xi} \cdot \hat{\boldsymbol{b}}\right] \; d\theta$$

With $\varphi = 2\pi\nu\Delta t$

$$I_1 \quad = \quad \int_{\Omega} B(\boldsymbol{\xi}) \cos\left[2\pi \frac{d}{\lambda} \boldsymbol{\xi} \cdot \hat{\boldsymbol{b}}\right] \; d\Omega \quad \Rightarrow \quad \varphi = 0 \qquad \Delta t = 0$$

$$I_2 \quad = \quad \int_{\Omega} B(\boldsymbol{\xi}) \sin\left[2\pi \frac{d}{\lambda} \boldsymbol{\xi} \cdot \hat{\boldsymbol{b}}\right] \; d\Omega \quad \Rightarrow \quad \varphi = \pi/2 \qquad \Delta t = \lambda \; / \; 4c$$

To measure a visibility one can measure the intensity in the fringes at a point where the optical paths have the same length (in the median plan) and at a point where the difference in length is equal to one quarter of a wavelength.

Note: When I_1 and I_2 are measured the visibility $\mathcal{V}$ is known as well as its complex conjugate $\mathcal{V}^* = I_1 - i\, I_2$, i.e. the visibility that would be measured by inverting the role of the two apertures.

What kind of information are in a visibility?

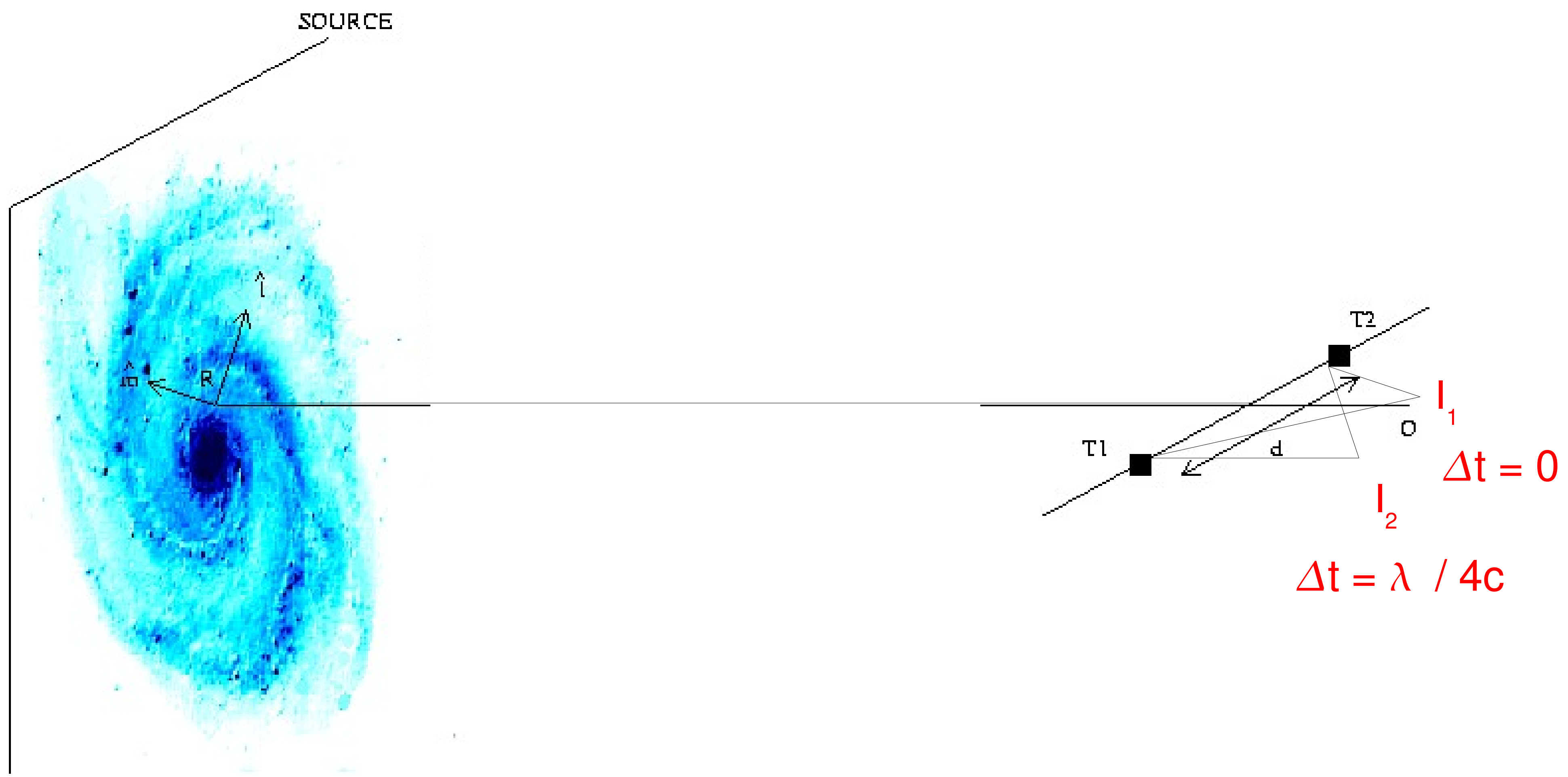
What is the meaning of $|\mathcal{V}|$ and ϕ?

$$\mathcal{V} = I_1 + iI_2 = |\mathcal{V}|e^{i\phi} \qquad (I_1, I_2) \Leftrightarrow (|\mathcal{V}|, \phi)$$

$$\begin{aligned} I &= \cos\varphi\, I_1 + \sin\varphi\, I_2 + B_T \\ &= |\mathcal{V}|\cos\varphi\cos\phi + |\mathcal{V}|\sin\varphi\sin\phi + B_T = |\mathcal{V}|\cos(\varphi - \phi) + B_T \end{aligned}$$

- **Take a sine filter with spatial frequency d/λ and with undulations oriented along the vector b.**
  - The phase, ϕ, of the visibility, corresponds to the offset of the sine filter that maximizes the total transmitted intensity
  - the amplitude of the visibility, $|\mathcal{V}|$, is the value of this maximal transmitted intensity.

NOTE: a single visibility contains information on the distribution of the emission in the source as a whole, not just at a given coordinate or within a given subregion.


SOURCE
R
T2
T1
d
O
I_1
$\Delta t = 0$
I_2
$\Delta t = \lambda / 4c$

From fringes to visibilities, Summary

- **Fringes**
  - Measuring the intensity in the fringes behind two apertures separated by *d* is equivalent to measuring the intensity transmitted by a sine filter with an angular wavelength λ/d and oriented in the direction defined by the two apertures (***b***).
- **Fringes contain spatial information about the source**
  - All this information is described by 2 numbers, that can be expressed as a complex number. By definition this complex number is called visibility.
  - These 2 numbers can be measured, e.g. by measuring the intensity:
    - at a point where there is no difference in the optical path lengths
    - at a point where the difference is equal to a quarter of a wavelength (λ/4).
- **The visibility corresponds to a spatial property of the source**
  - The phase of the visibility corresponds to the offset of the imaginary sine filter that would maximize the transmitted intensity
  - The amplitude of the visibility is the value of this maximum intensity

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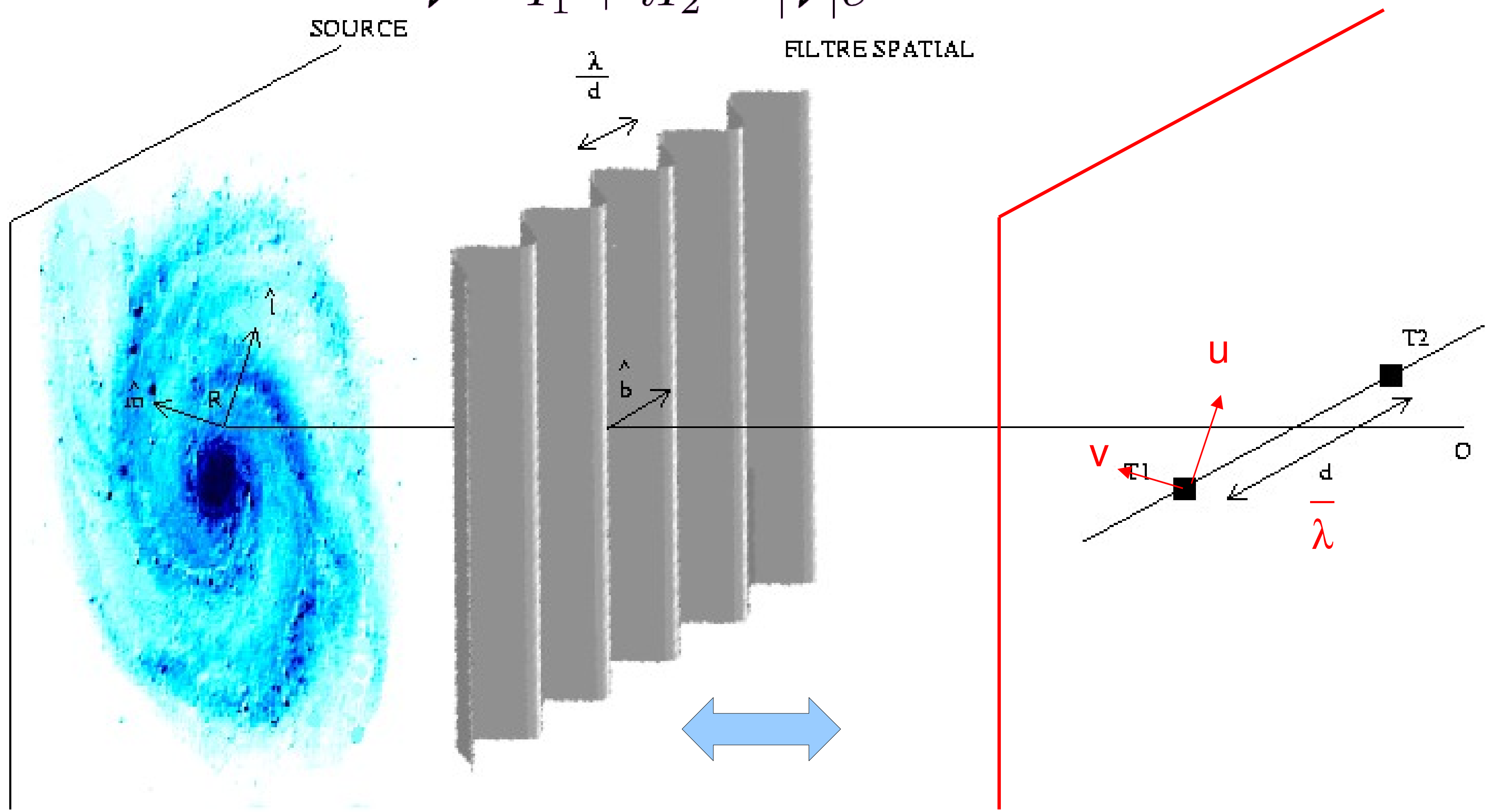
$$\mathcal{V} = I_1 + iI_2 = |\mathcal{V}|e^{i\phi}$$



To get as much information as possible on the source it is necessary to observe it through as many different sine filters as possible, i.e. to change the spacing between the apertures and change the orientation of the baseline vector, ***b***, they subtend.

The concepts

From visibilities to images

- **Is it feasible to reconstruct an image from a limited number of visibility measurements?**
- **How many measurements are required?**
- **How to proceed?**

Feasibility

Fourier formalism

- **Definition of the Fourier transform**

$$F(\boldsymbol{u}) \equiv \mathcal{F}\{f(\boldsymbol{x})\} = \iint f(\boldsymbol{x}) e^{2i\pi \boldsymbol{u}\cdot\boldsymbol{x}} dx_1 dx_2$$

- **Definition of the visibility function** $\mathcal{V} = I_1 + iI_2 = |\mathcal{V}|e^{i\phi}$

$$\begin{aligned} \mathcal{V}(\boldsymbol{b}) &= \int_\Omega B(\boldsymbol{\xi}) \cos\left[2\pi\boldsymbol{\xi}\cdot\boldsymbol{b}\right] d\Omega + i\int_\Omega B(\boldsymbol{\xi}) \sin\left[2\pi\boldsymbol{\xi}\cdot\boldsymbol{b}\right] d\Omega \\ &= \int_\Omega B(\boldsymbol{\xi}) e^{2i\pi\boldsymbol{\xi}\cdot\boldsymbol{b}} d\Omega \end{aligned}$$

$$\boldsymbol{b} = \frac{d}{\lambda}\hat{\boldsymbol{b}} = (u, v) \qquad \boldsymbol{\xi} = (l, m)$$

$$\mathcal{V}(u,v) = \iint B(l,m) e^{2i\pi(ul+vm)}\, dl\, dm$$

Visibility, as a function of the baseline coordinates (u, v), is the Fourier transform of the source brightness distribution as a function of the sky coordinates.
The (u, v) plane is called the Fourier plane.

Feasibility

The problem

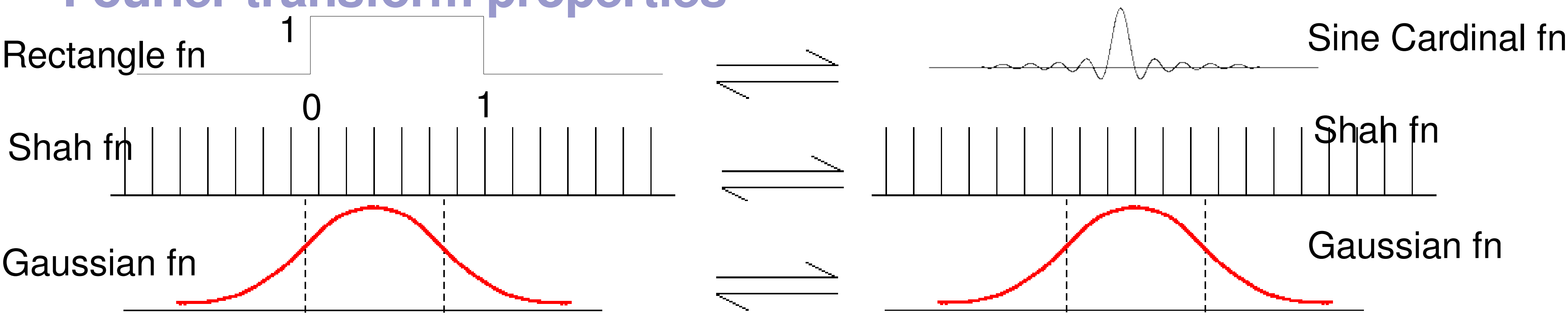
- **A visibility measurement is a sample of the visibility function at (*u, v*) and (*-u, -v*).**
- **Is it possible to estimate $\mathcal{V}(u,v)$ with only a limited number of samples?**

The answer

- **Yes if**
  - The size of the source is limited.
    This is always the case because of the limited field of view.
  - The image of the source has a limited resolution.
    All imaging techniques are limited in resolution anyway (the PSF), the problem is to get the highest possible resolution.

Demonstration

Fourier transform properties



$$\mathcal{F}\{\Box(l,m)\} = \frac{\sin(\pi u)}{\pi u}\frac{\sin(\pi v)}{\pi v} = \mathrm{sinc2D}(u,v)$$

$$\mathcal{F}\{\sqcup\!\sqcup(l,m)\} = \sqcup\!\sqcup(u,v)$$

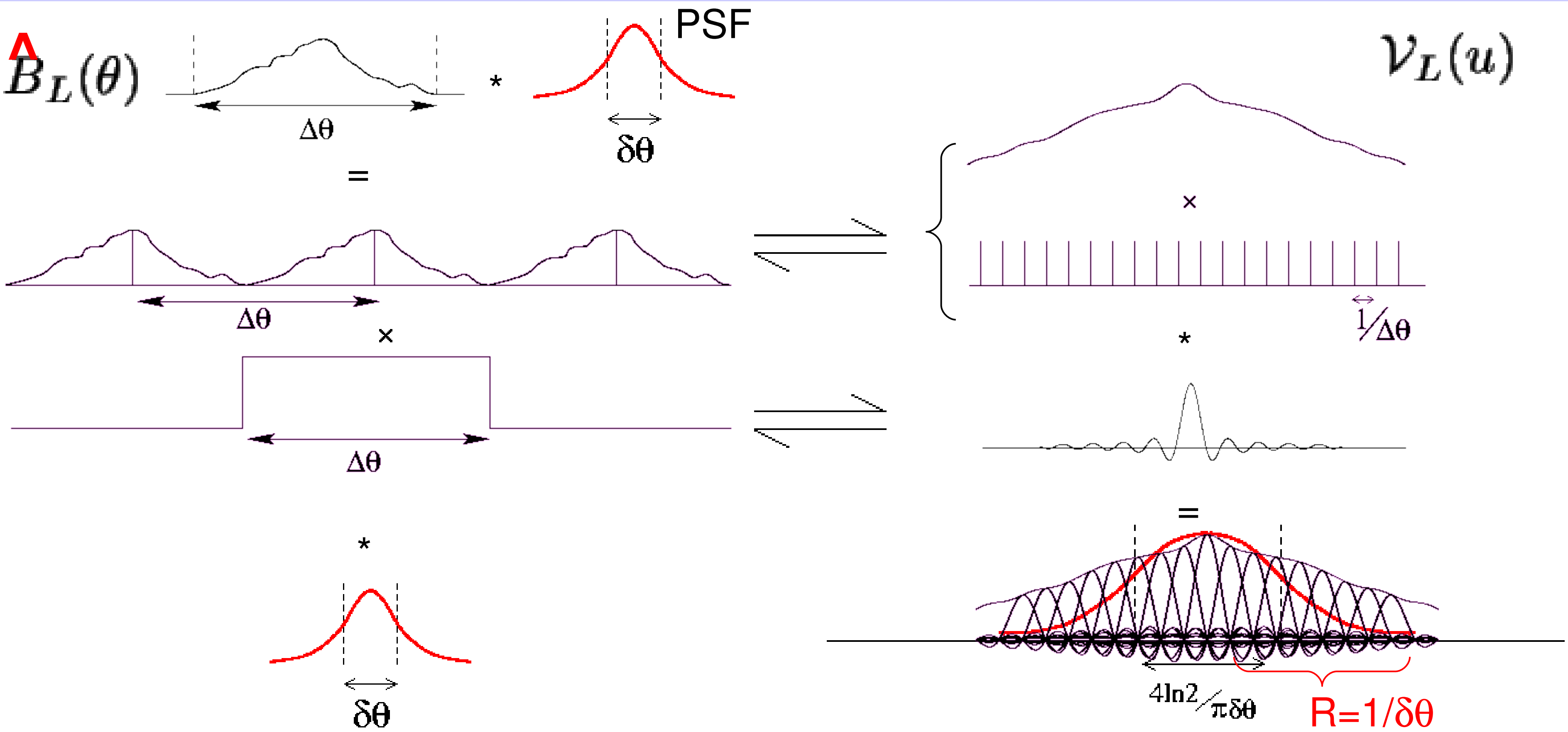
$$\mathcal{F}\{G_{\delta\theta}(l,m)\} = 2\pi\, a^2\, \delta\theta^2\, G_{\delta b}(u,v)$$

$$\text{with } \delta b = \frac{1}{2\pi a^2 \delta\theta} = \frac{4\ln 2}{\pi\, \delta\theta}$$

$$\mathcal{F}\{f(Cx)\} = \frac{1}{|C|} F\left(\frac{u}{C}\right)$$

Bracewell, R. The Fourier Transform and Its Applications, 3rd ed. New York: McGraw-Hill, 1999.

Demonstration



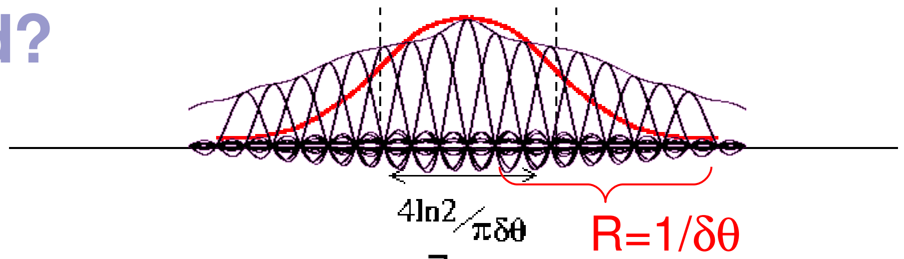
$$\hat{\mathcal{V}}_L(u,v) = \left(\sum_{i\in\mathbb{Z}}\sum_{j\in\mathbb{Z}} \mathcal{V}_L\left(\frac{i}{\Delta\theta},\frac{j}{\Delta\theta}\right) \mathrm{sinc2D}\left(u-\frac{i}{\Delta\theta}, v-\frac{j}{\Delta\theta}\right)\right)$$

V_{ij}

⇒ a limited number of sinc are involved, their amplitudes, V_{ij} (called "Fourier components"), can be estimated from a limited number of measurements.

Number of visibilities required

How many measurements required?



$$\hat{\mathcal{V}}_L(u,v) = C\left[\sum_{i\in\mathbb{Z}}\sum_{j\in\mathbb{Z}} V_{ij}\,\mathrm{sinc2D}\left(u-\frac{i}{\Delta\theta}, v-\frac{j}{\Delta\theta}\right)\right]\times G_{\Delta b}(u,v)$$

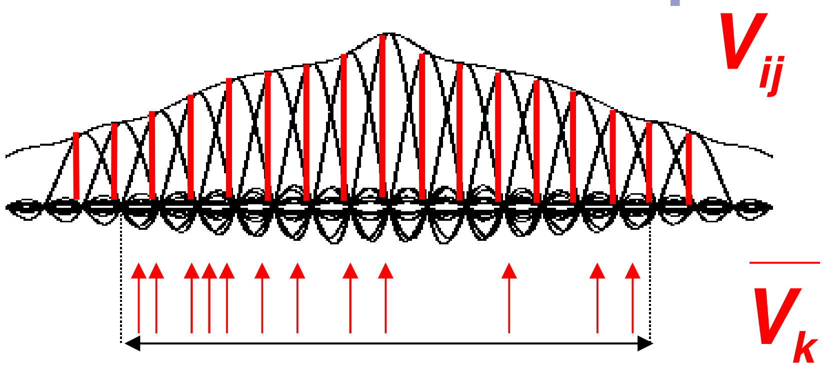
To fully determine $\mathcal{V}_L$ inside *R* it is necessary to estimate the value of the visibility function $\mathcal{V}$(u,v) at each node of the grid within *R*. The step of the grid is δu=1/$\Delta\theta$, the number of samples to estimate is therefore

$$N \simeq \left(\frac{R}{\delta u}\right)^2 = \left(\frac{\Delta\theta}{\delta\theta}\right)^2$$

$\Delta\theta/\delta\theta$ is the ratio of the source size by the resolution, also known as the spatial **dynamic range**. It represents the quantity of information embedded in the image. The smaller the source the less the number of measurements required.

How to proceed?

Estimate the *N* Fourier Components

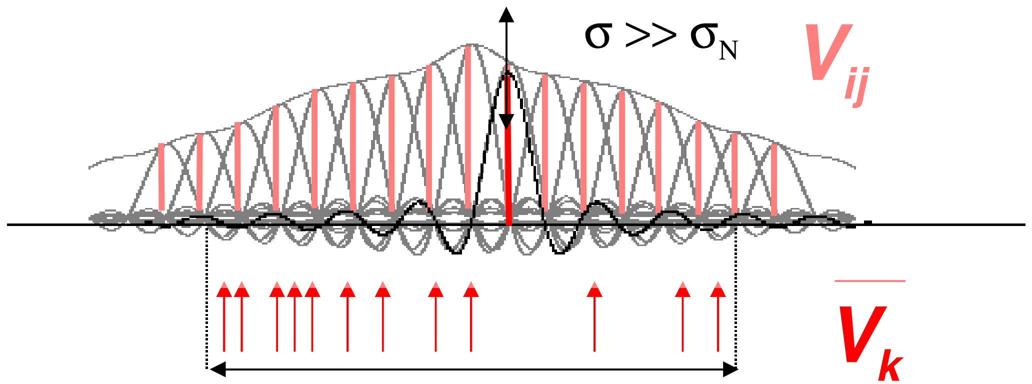


To estimate the N parameters V_{ij} at least N/2 visibilities V_k need to be measured. If N' is the number of visibilities actually measured then the V_{ij} are solution of the 2N' linear equations:

$$\begin{cases} \sum_i \sum_j \Re\{V_{ij}\}\, g_{ij}(u_k, v_k) + \Re\{V_{ij}\}\, g_{-i-j}(u_k, v_k) = \dfrac{\Re\{\overline{V}_k\}}{C} & I_1 \\ \sum_i \sum_j \Im\{V_{ij}\}\, g_{ij}(u_k, v_k) - \Im\{V_{ij}\}\, g_{-i-j}(u_k, v_k) = \dfrac{\Im\{\overline{V}_k\}}{C} & I_2 \end{cases}$$

Where g_{ij} are the values of the Fourier transform of the source support (e.g. sinc2D if the support is square) at each point of the grid

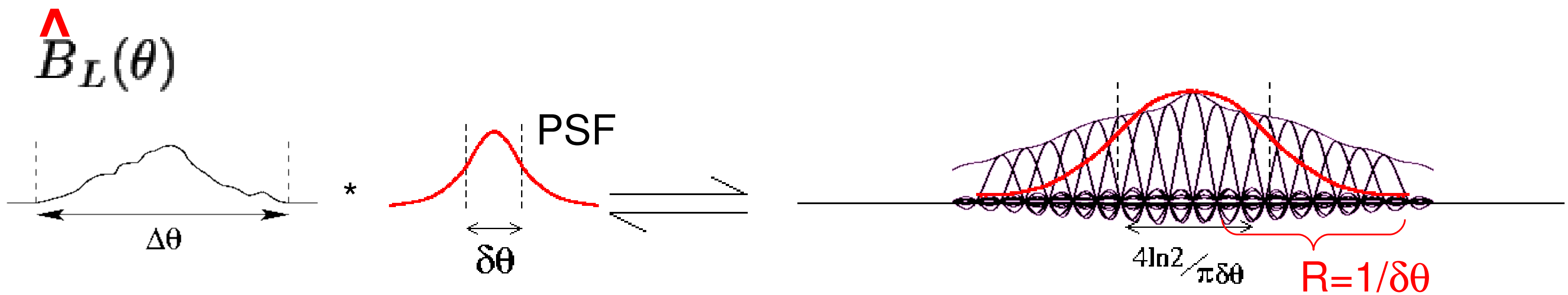
How to proceed?



- **The error on the estimate of a V_{ij} depends on**
  - The local density of measurements
  - The *a priori* knowledge of the source (the support via g_{ij})
- **Where should the visibilities be measured in the (*u, v*) plane?**
  - Without noise the coordinates of the measurements do not matter -> no sampling theorem like Shannon!
  - In practice there is always noise and the g_{ij} functions decrease rapidly -> to measure a given V_{ij} best to measure as close as possible to the center of the sinc, i.e. close to the *ij* grid node

How to proceed?

- **Affect weights to the V_{ij} corresponding to the Fourier transform of the wanted PSF ("clean beam")**
- **Fourier transform to get the image**



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Resolution

A high resolution imaging technique

- The radius, R, of the region sampled in the (u,v) plane fixes the resolution, $\delta\theta$, through $R=1/\delta\theta$
- The distance to the center in the (u,v) plane is the baseline length divided by the wavelength
- ⇒ The largest baseline, b_m, is related to the resolution by

$$\delta\theta \simeq \frac{\lambda}{b_m}$$

- ⇒ The largest baseline, b_m, plays the same role as the telescope diameter in direct imaging
- Monolithic telescopes are limited in size but there is no limit to the separation of the apertures of an interferometer. They can be thousands of kilometers apart (VLBI)!
- <u>High resolution imaging technique</u> = main motivation

Artifacts

A technique prone to artifacts

- Increasing the resolution requires to increase the number of samples in the *uv* plane

$$N \simeq \left(\frac{R}{\delta u}\right)^2 = \left(\frac{\Delta\theta}{\delta\theta}\right)^2$$

- When there are not enough samples to evaluate all the required V_{ij} then some information is missing to allow the image to be reconstructed properly (uv coverage)
- As the property measured by a visibility is related to the distribution of the emission in the source as a whole, missing or corrupted visibilities will affect the whole image.
- The separation between the telescope cannot be < 2xD --> impossible to sample uv-plane inside 2D/λ ("short spacing problem").
- Visibilities are affected by instrumental and atmospheric effects. The phase and the amplitude of the visibilities are affected by different things.

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Interferometer array design

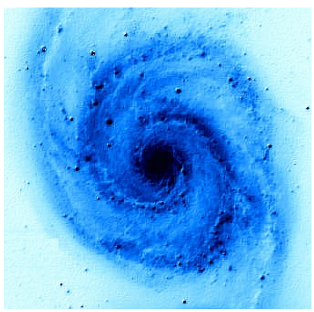
The configuration problem

- **We have N_a apertures (telescopes) and we can measure 2 visibilities with each pair of apertures, i.e. $N_a(N_a-1)$ visibilities**
- **Each visibility has (u, v) coordinates equal to the coordinates of the baseline vector, *b*, subtended by the apertures $b = (u,v)$ (for a source at zenith otherwise need to project on the sky plane).**
- **What is the optimal aperture configuration?**
  i.e. Which aperture configuration maximizes the quality of the reconstructed image?

Interferometer array design

Two approaches to the configuration problem

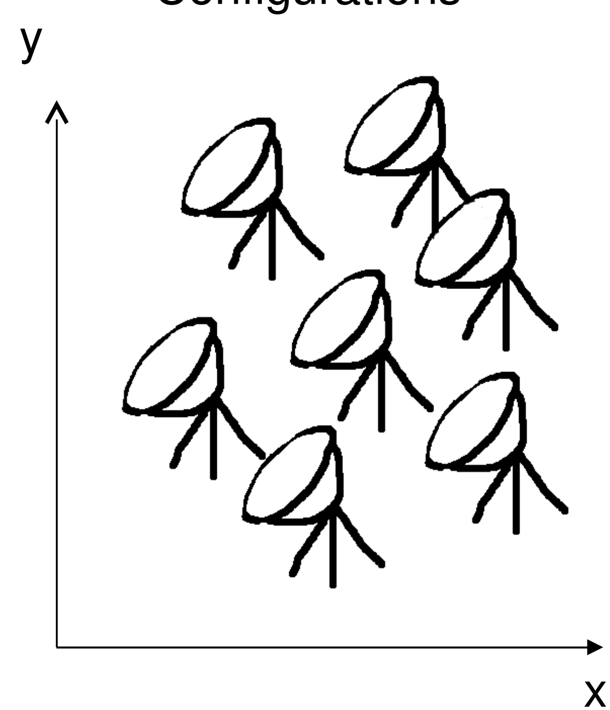
Specifications on the image quality



Direct (trial-error)

inverse

Configurations



Interferometer array design

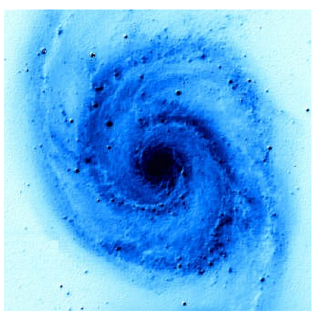
Two approaches to the configuration problem

- **Direct**
  - Need to try many different geometrical shapes (e.g. Y-shape, circle, triangle...)
  - Not trivial how to improve a given configuration shape in the trial-error process
  - No guarantee that the best configuration within the configurations tried is indeed the optimal one
- **Inverse**
  - Need to develop a method/algorithm
  - Ill-posed problem
  - In principle the solution is really the optimal one
  - Can be adapted to complex situations (e.g. Multiconfiguration)

Interferometer array design

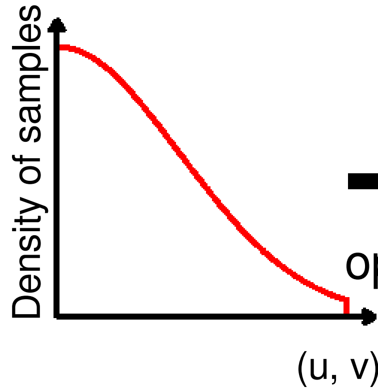
The inverse approach

Specifications on the image quality i.e. The PSF



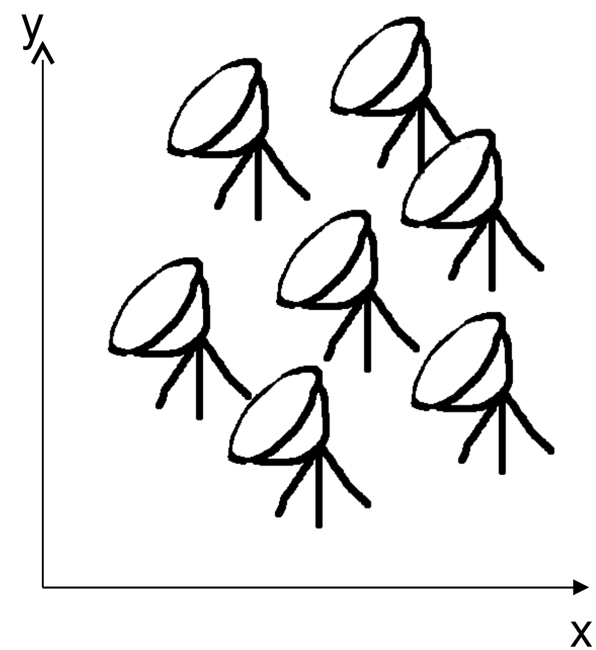
analysis

Specifications on the distribution of visibilities in Fourier plane



optimization

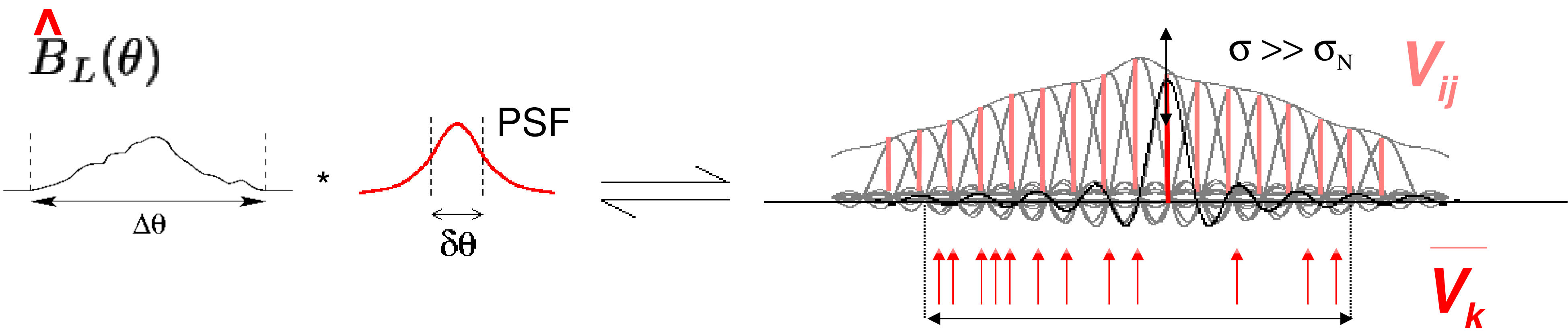
Configurations



Interferometer array design

Specifying the distribution of samples

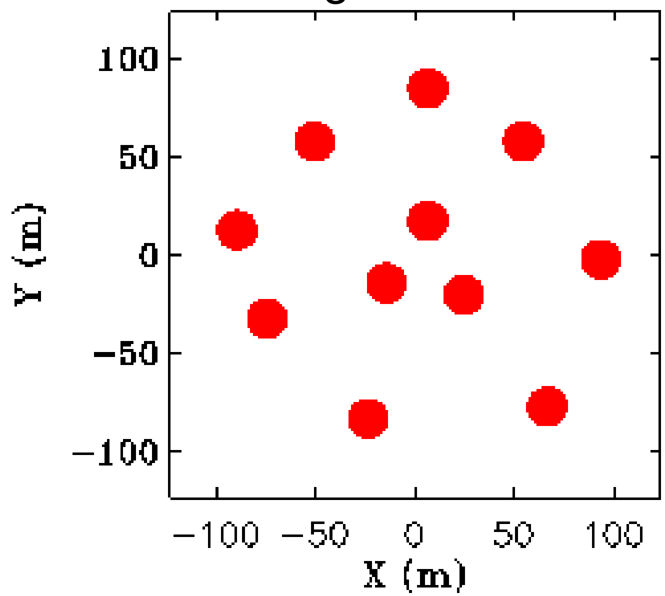
- No holes, when few samples => uniform distribution
- The Fourier transform of the PSF wanted gives the weights of the V_{ij}
- The higher the weight of a V_{ij} the higher the accuracy on its measurement should be <=> the higher the density of measurements around the *ij* node should be



Interferometer Array Design

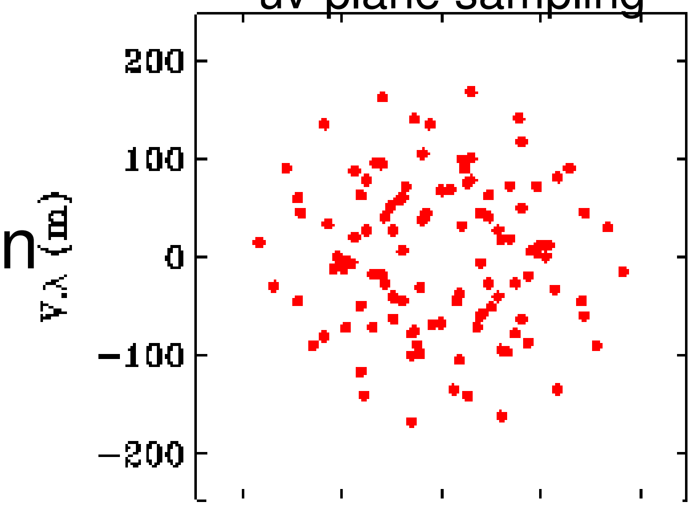
From the distribution of samples to the configuration

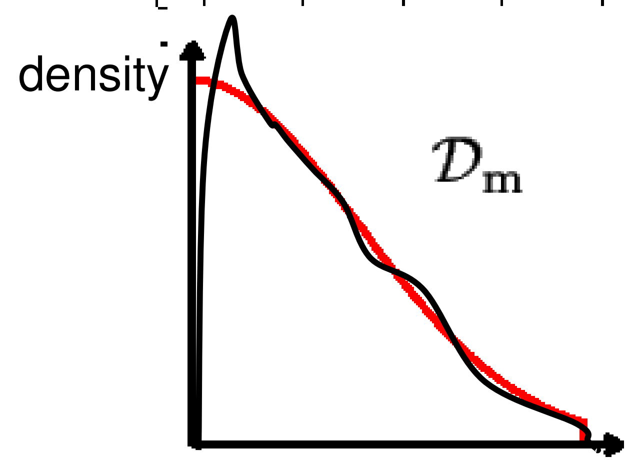
configuration



Autocorrelation

uv-plane sampling



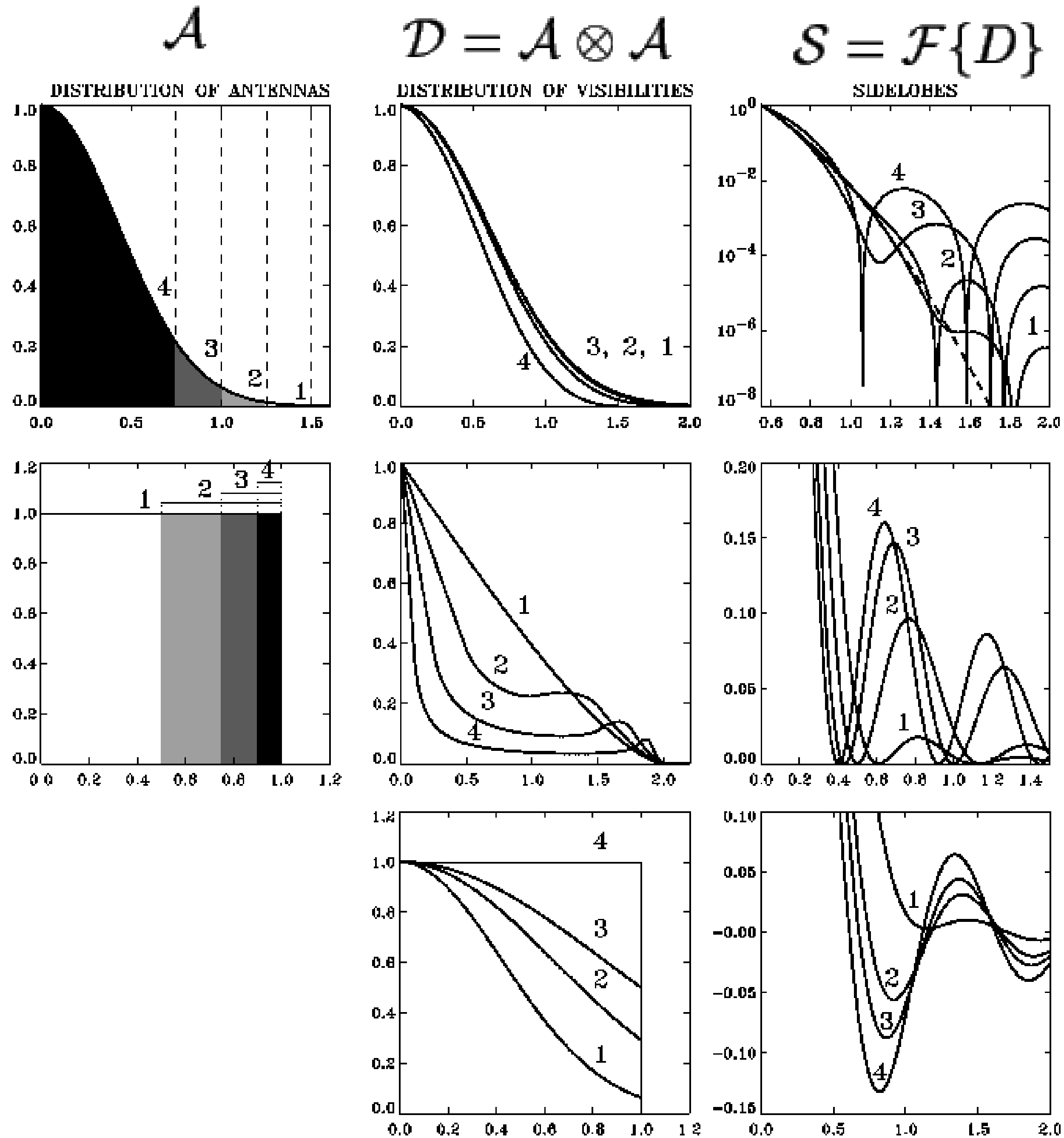


$$\mathcal{A}_{\rm m}, \quad \mathcal{D}_{\rm m} = \mathcal{A}_{\rm m} \otimes \mathcal{A}_{\rm m}$$

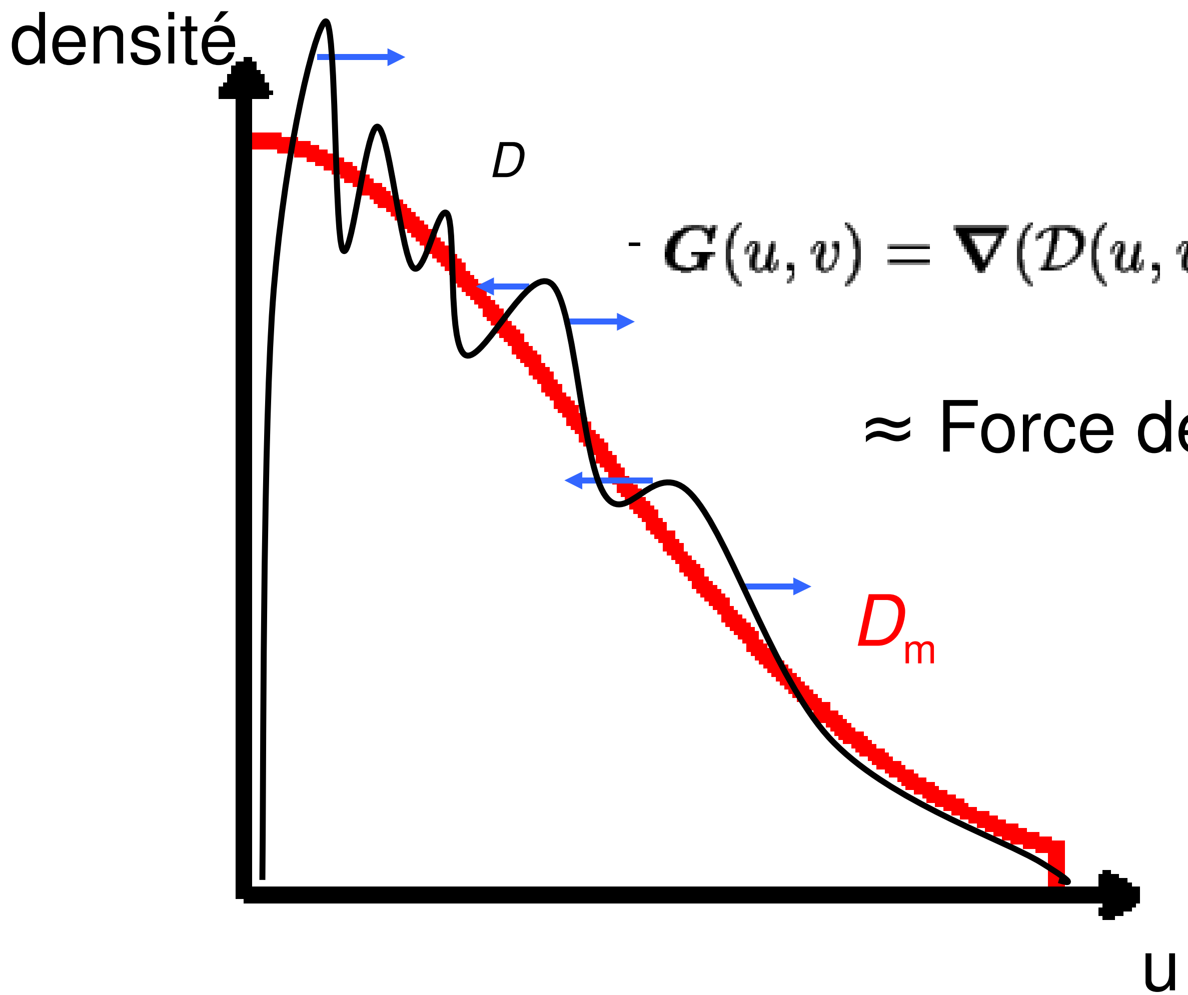
$$\mathcal{A}, \quad \mathcal{D} = \mathcal{A} \otimes \mathcal{A}$$

$$[\mathcal{D}, \mathcal{D}_{\rm m}] = \min$$

U

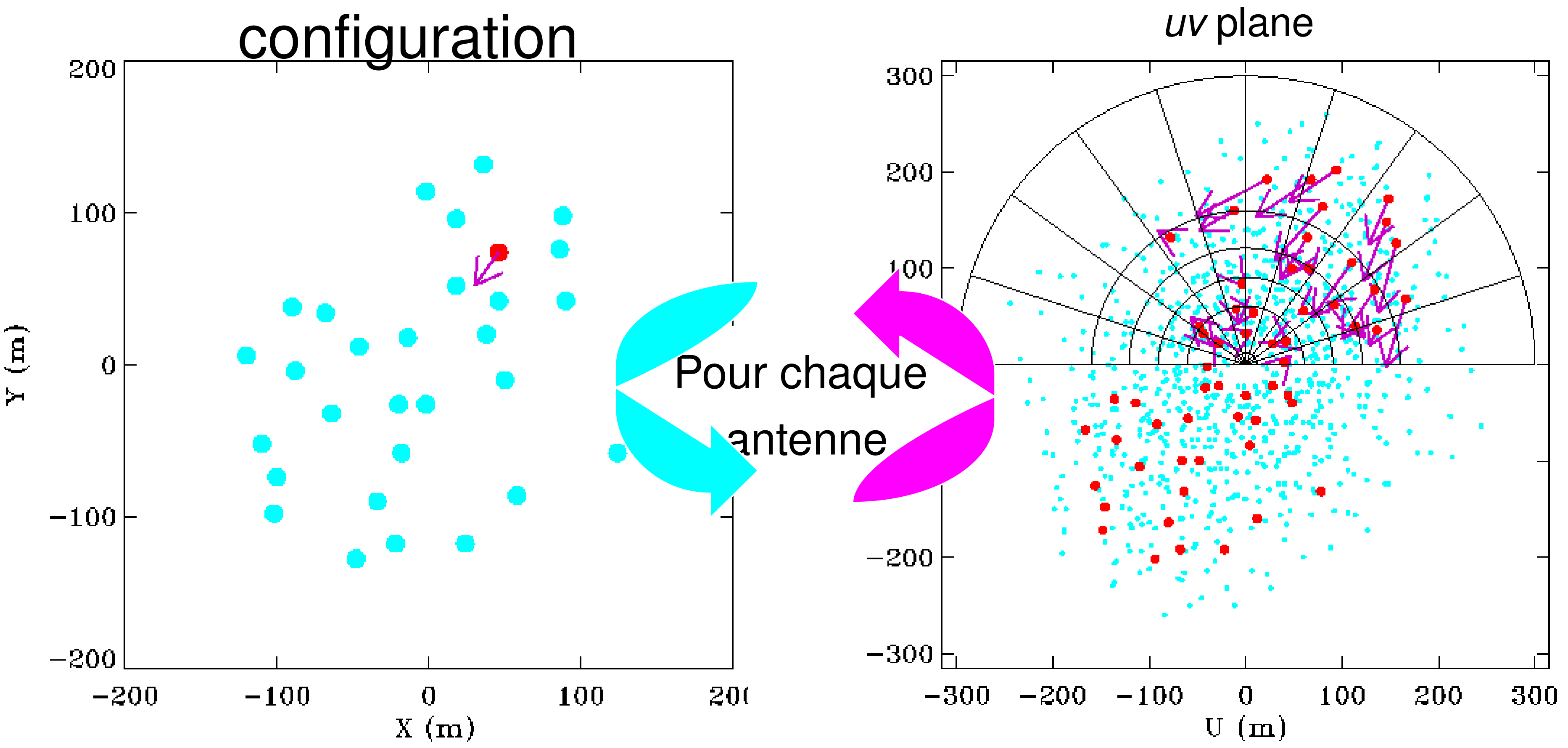

$\mathcal{A}$
$\mathcal{D} = \mathcal{A} \otimes \mathcal{A}$
$\mathcal{S} = \mathcal{F}\{D\}$
DISTRIBUTION OF ANTENNAS
DISTRIBUTION OF VISIBILITIES
SIDELOBES
3, 2, 1

Interferometer array design

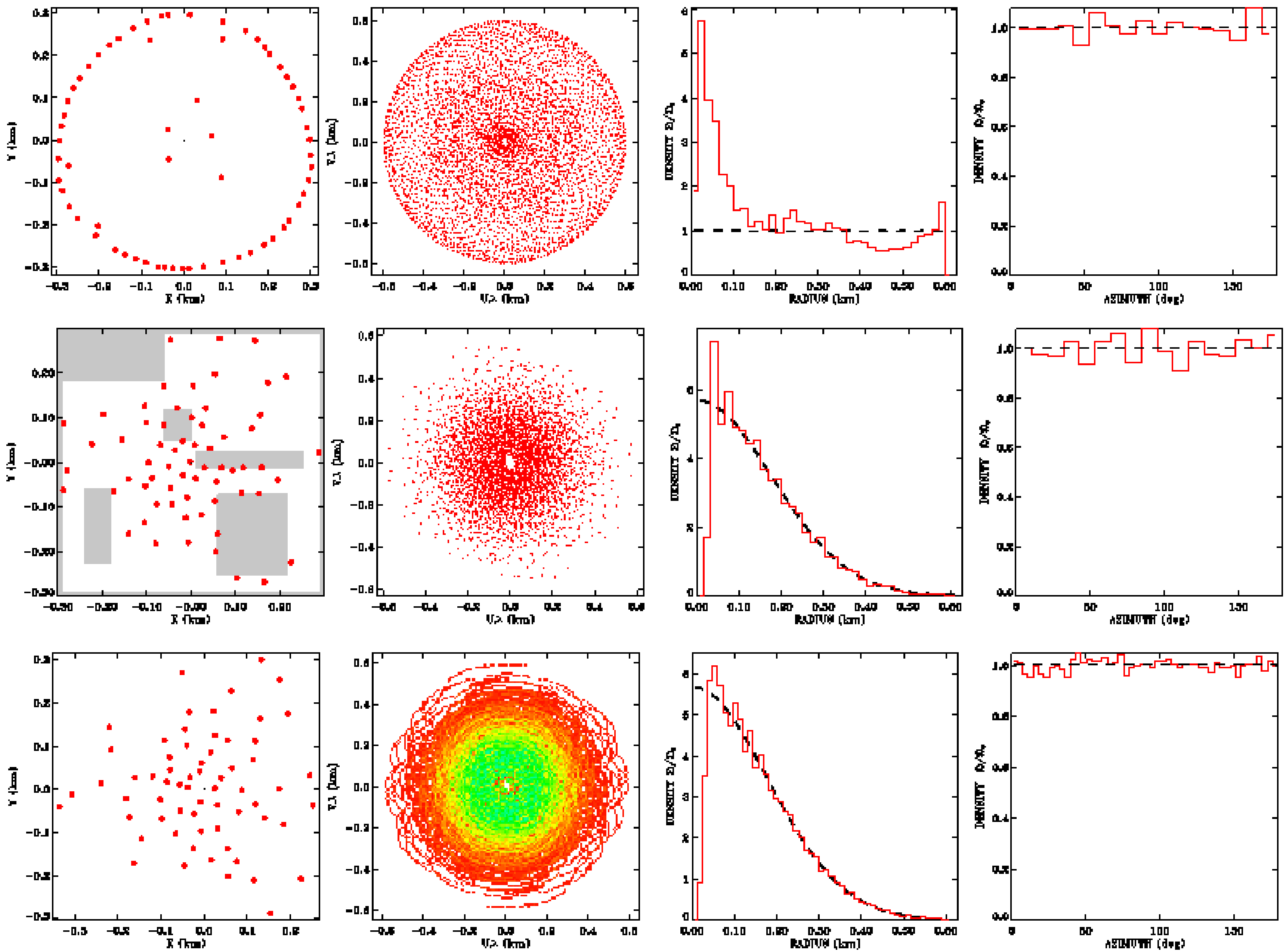


$$-\boldsymbol{G}(u,v) = \boldsymbol{\nabla}(\mathcal{D}(u,v) - \mathcal{D}_{\mathrm{m}}(u,v))$$

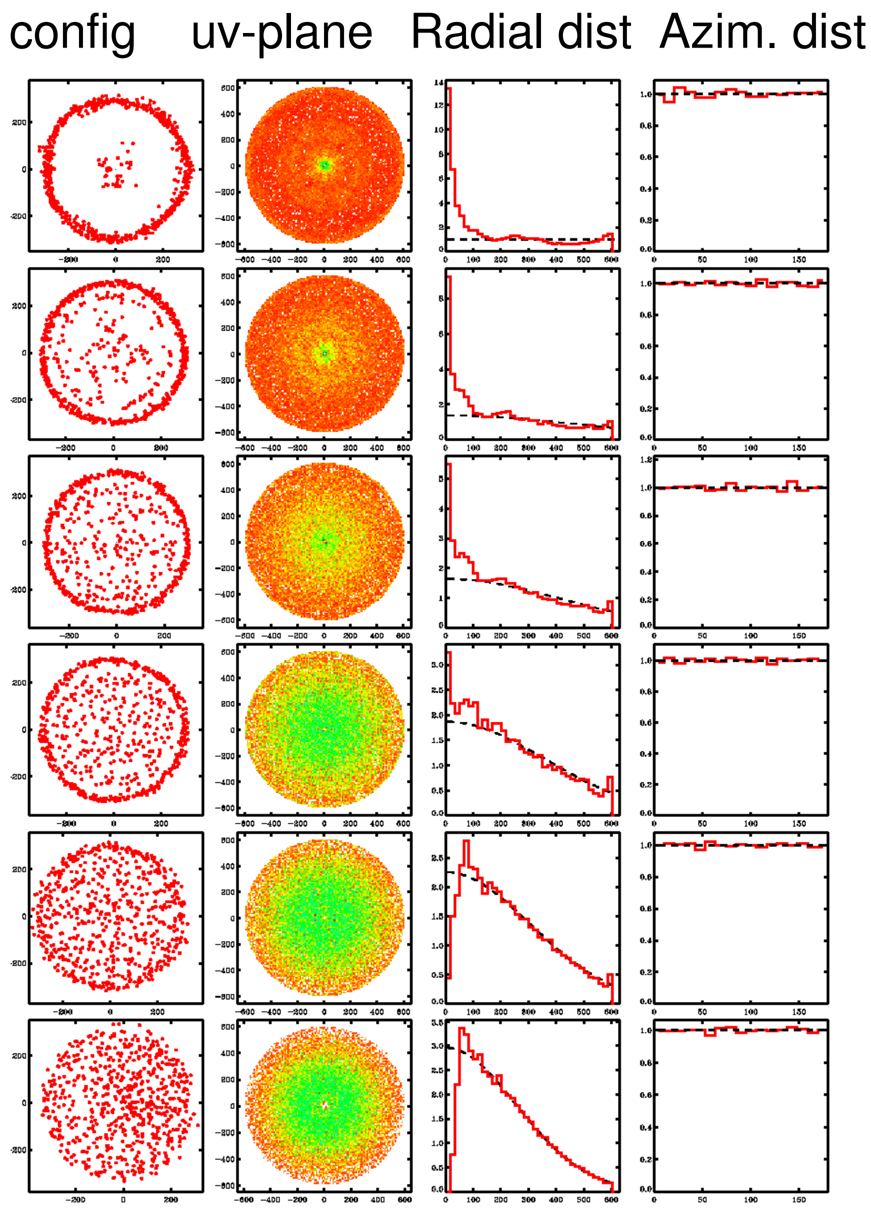
≈ Force de pression



$$D = g \sum_{i=1}^{n_a - 1} MG(u_i, v_i)$$



Interferometer array design



The Basics of Radio Interferometry

I. The concepts

a) Is it difficult to understand?
b) From fringes to "visibilities"
c) From visibilities to images
d) Resolution and artifacts
e) Interferometer array design

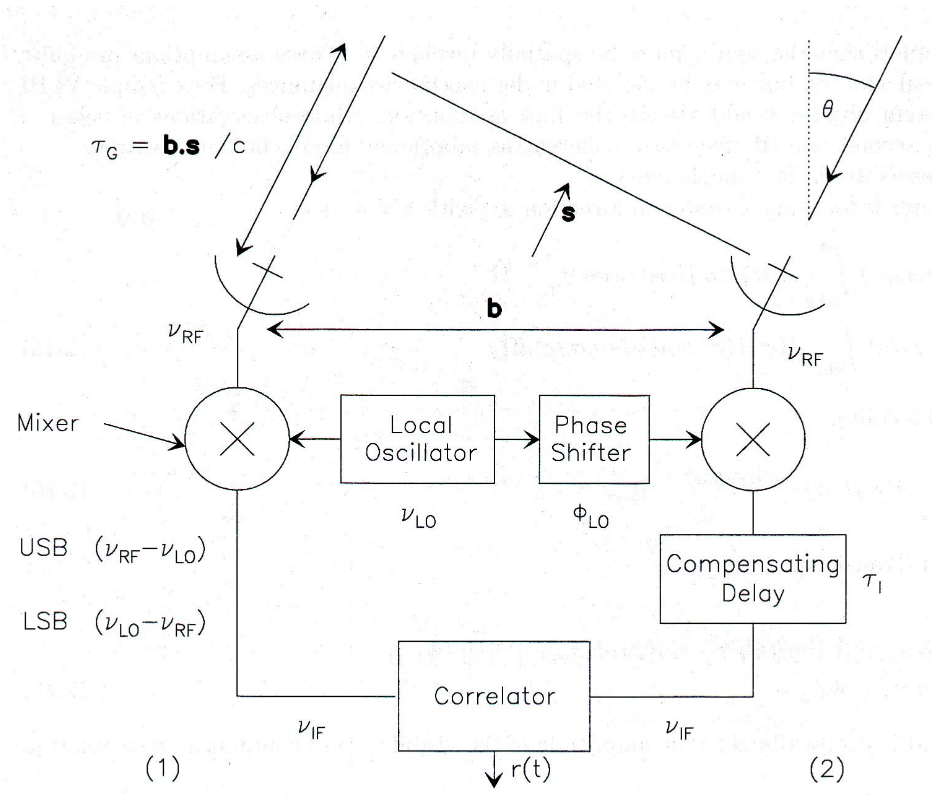
II. In practice

a) How does an interferometer array work?
b) Examples of working instruments
c) Aperture synthesis
d) Calibration and "Deconvolution"
e) Examples of data cubes

In Practice

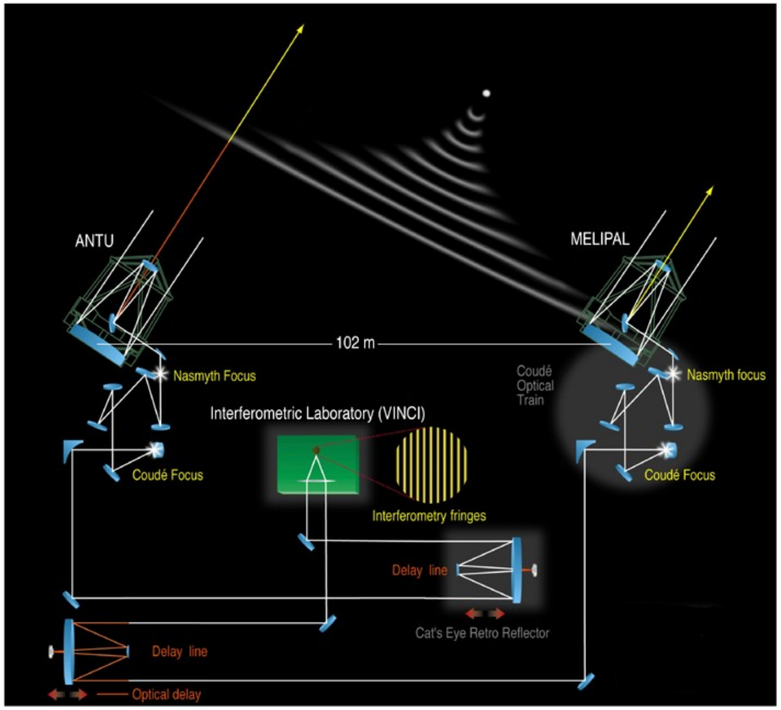
Radio interferometry

- **Receiver at focus of each telescope**
  - Converts electrom. wave into an electronic signal (amplitude and phase conserved)
- **Delay compensator (cable or electronics)**
  - Compensate delay to have Δt=0 for the center of the source ("center of phase")
- **Correlator**
  - Computes I_1 and I_2



(from Guilloteau, IRAM summer school 2000)

In Practice

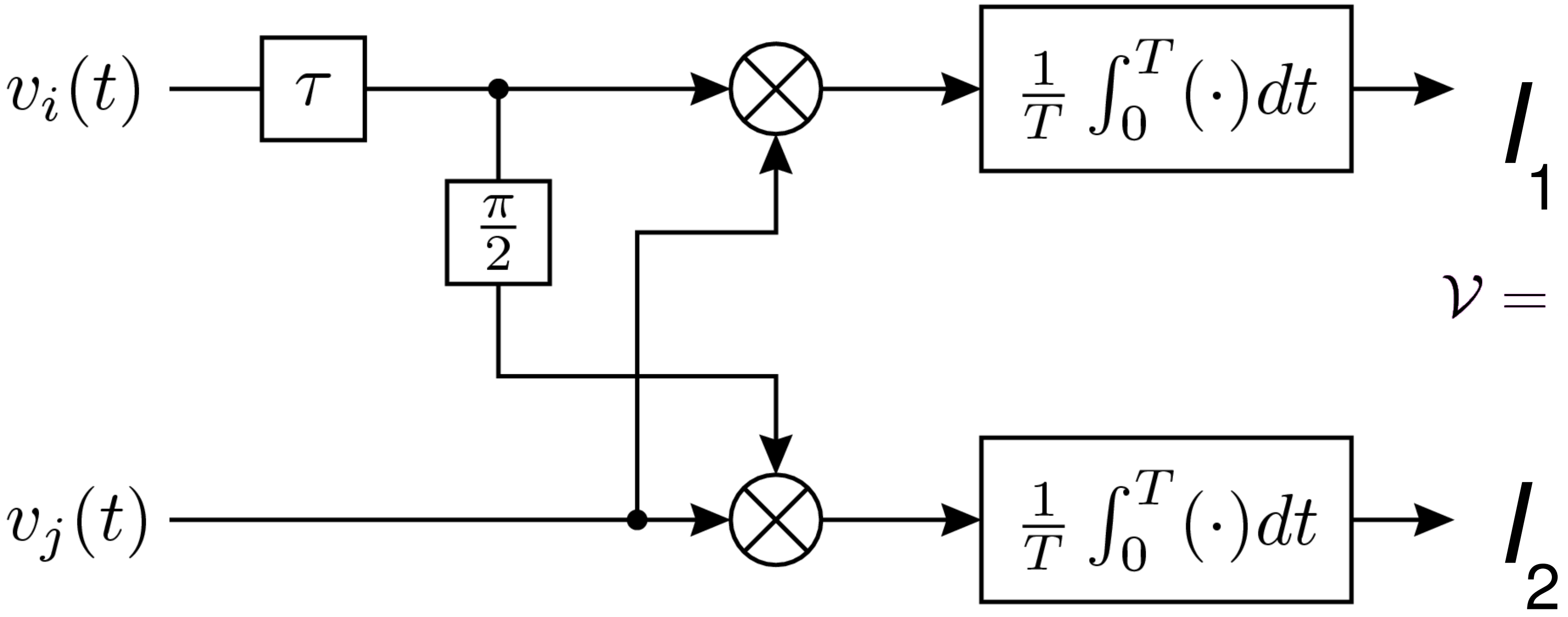


The VLT Interferometer with ANTU and MELIPAL

ESO PR Photo 30a/01 (5 November 2001)

In practice

The minimal correlator

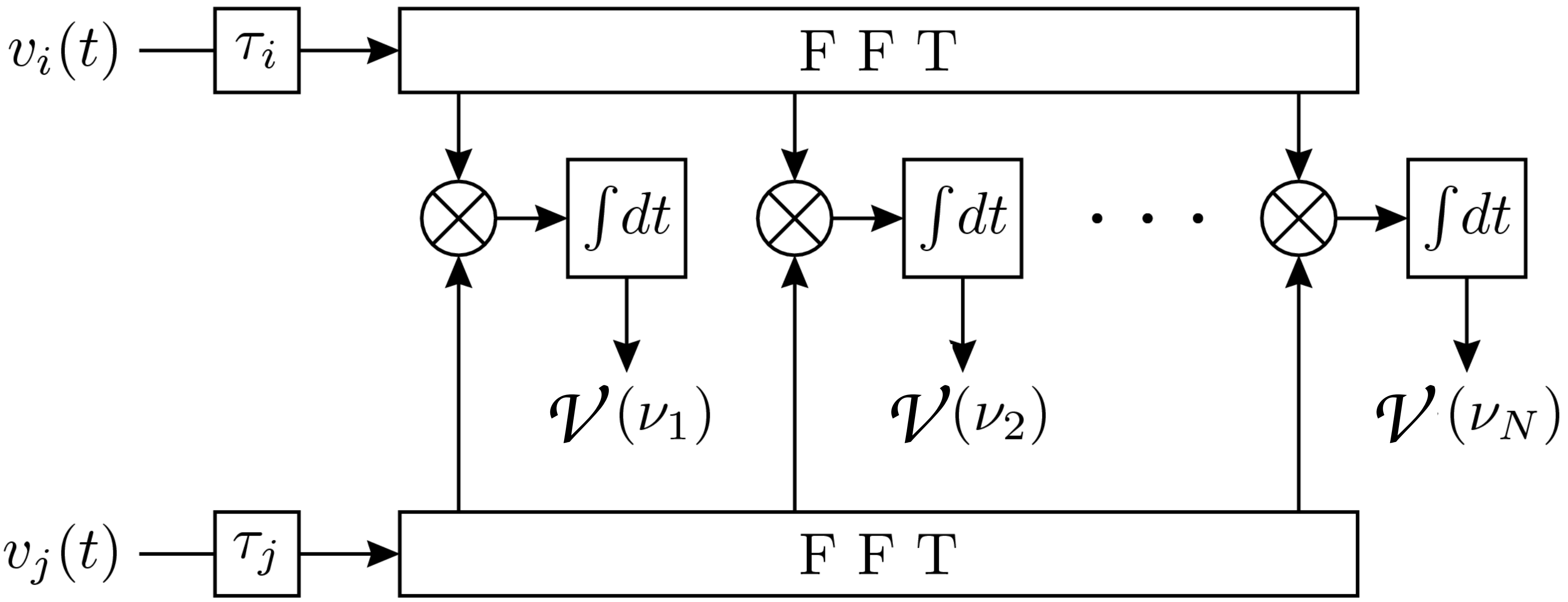


$$\mathcal{V} = I_1 + iI_2$$

(from W., Brisken, NRAO Summer School, 2006l)

In Practice

The correlator with FFT

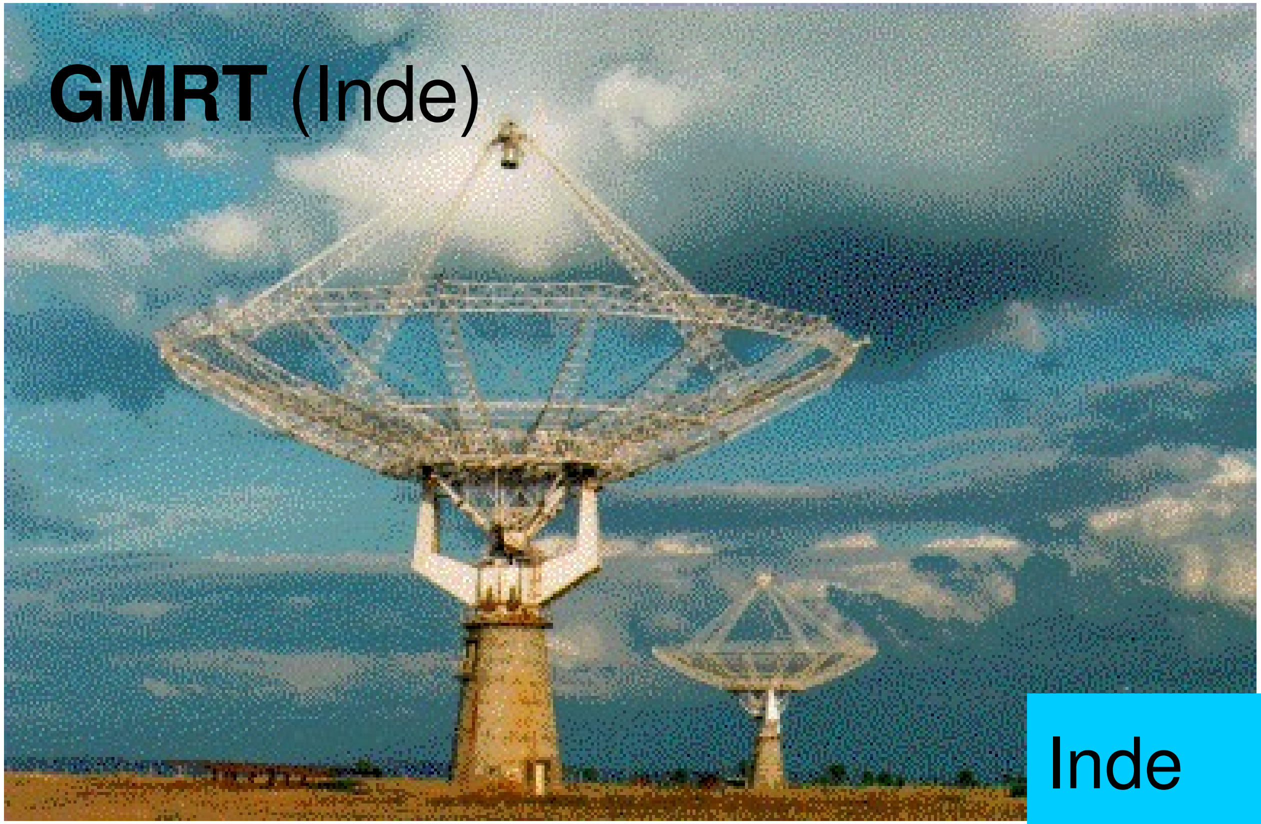


=> at each integration stamp the visibility is measured at N frequencies

=> an image can be reconstructed for each frequency

=> SPECTRO IMAGERY

Radio Interferometers



Inde

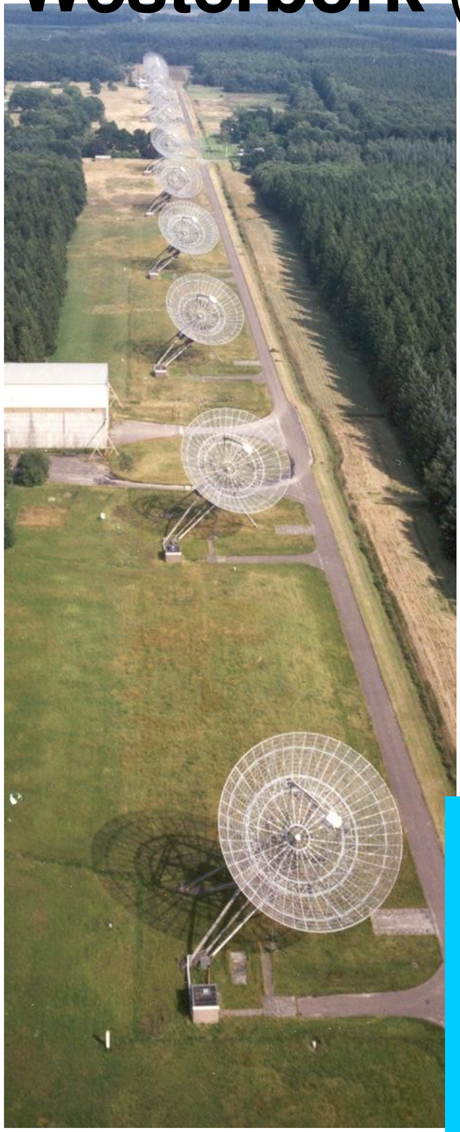
30 x 45m antennas

Baseline max: 25 km

$\lambda \sim 1$m

Radio Interferometers

Westerbork (ASTRON)



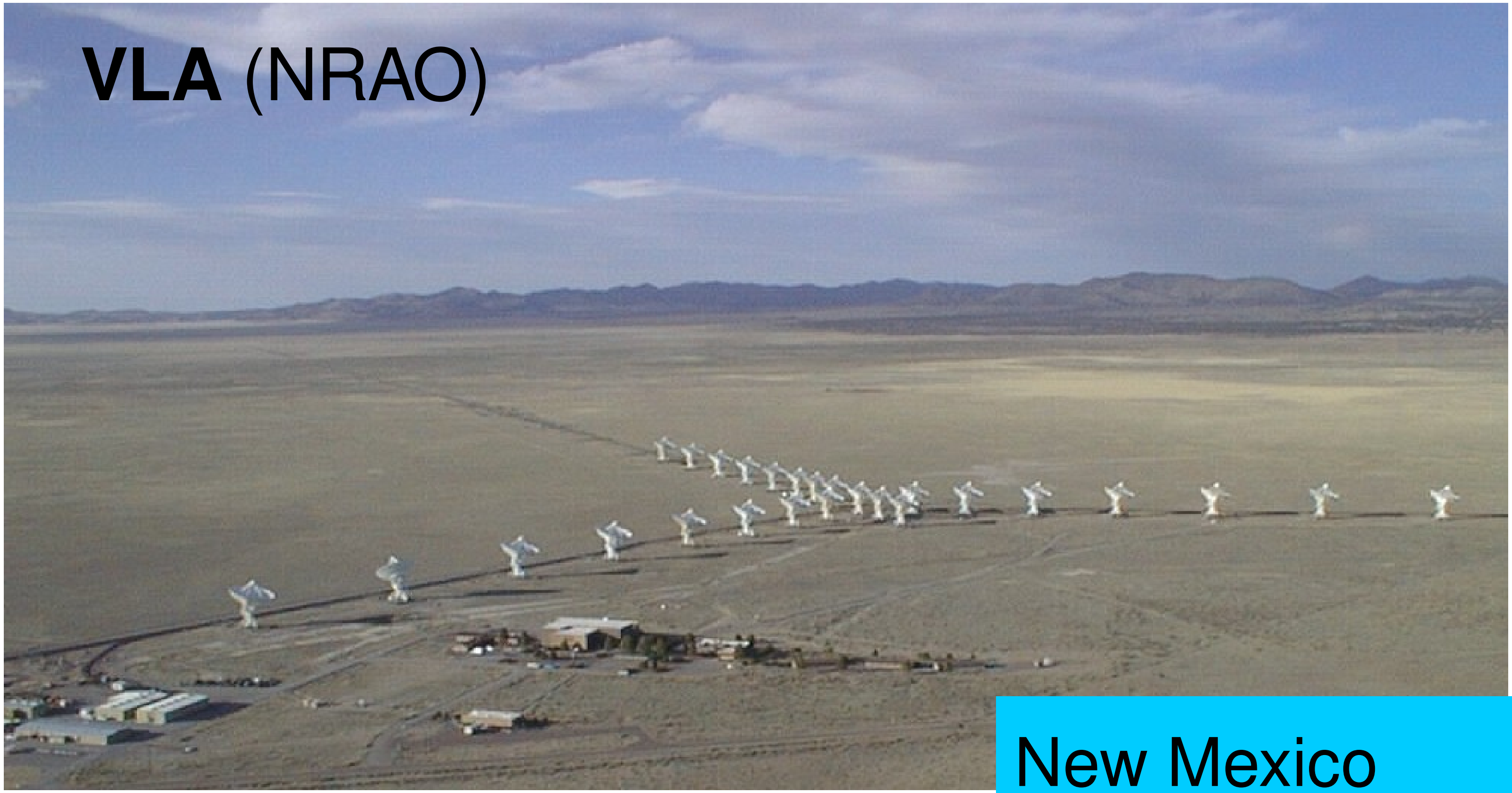
Netherlands

14 x 25m antennas

Baseline max: 2.7 km

$\lambda \sim$10cm – 1m

Radio Interferometers



New Mexico

27 x 25 m antennas

Baseline max: 36 km

$\lambda \sim$1cm – 1m

Radio Interferometers

France

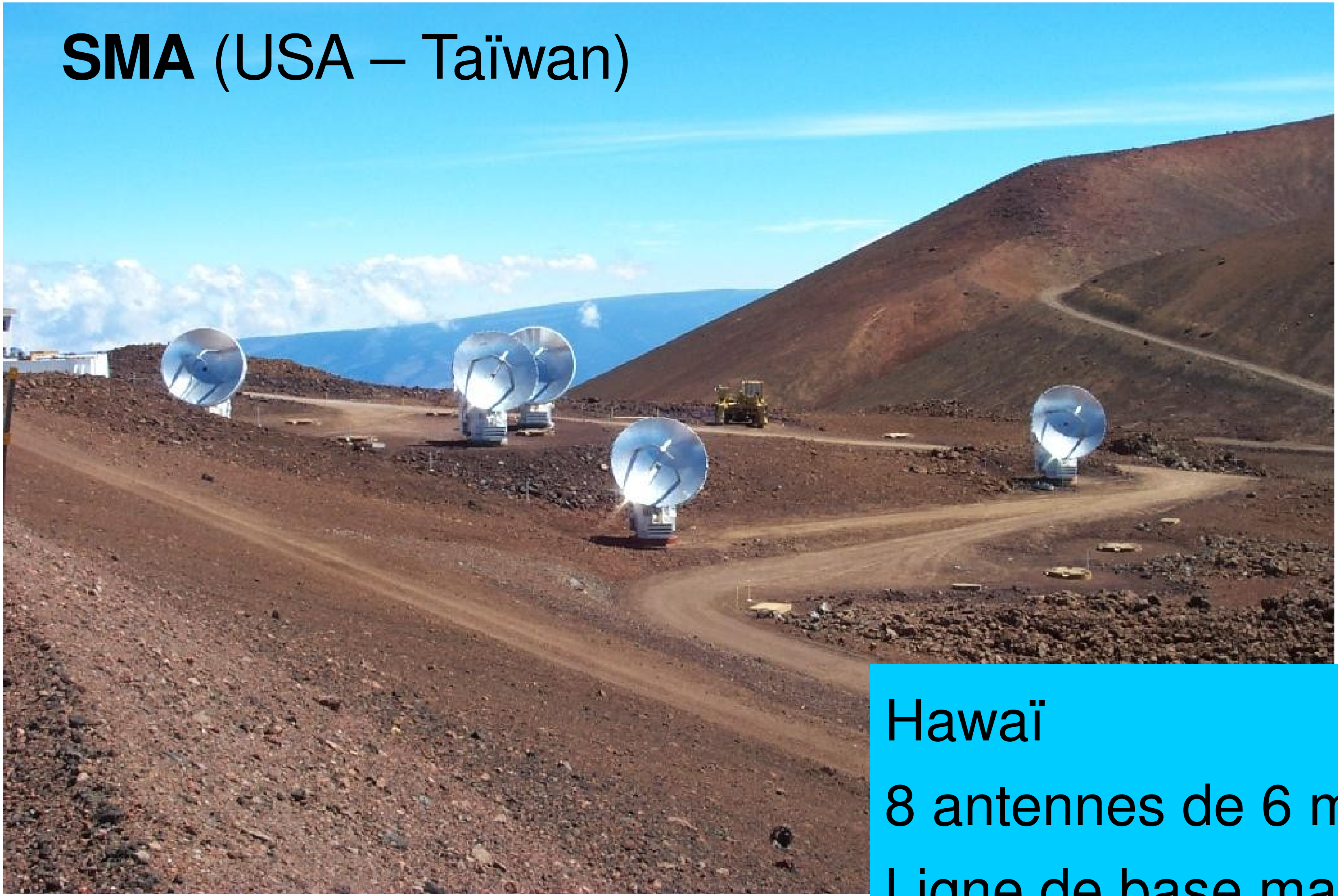
6 x 15m antennas

Baseline max: ~1 km

$\lambda \sim$1mm

Radio Interferometers

SMA (USA – Taïwan)



Hawaï

8 antennes de 6 m

Ligne de base max: 0.5 km

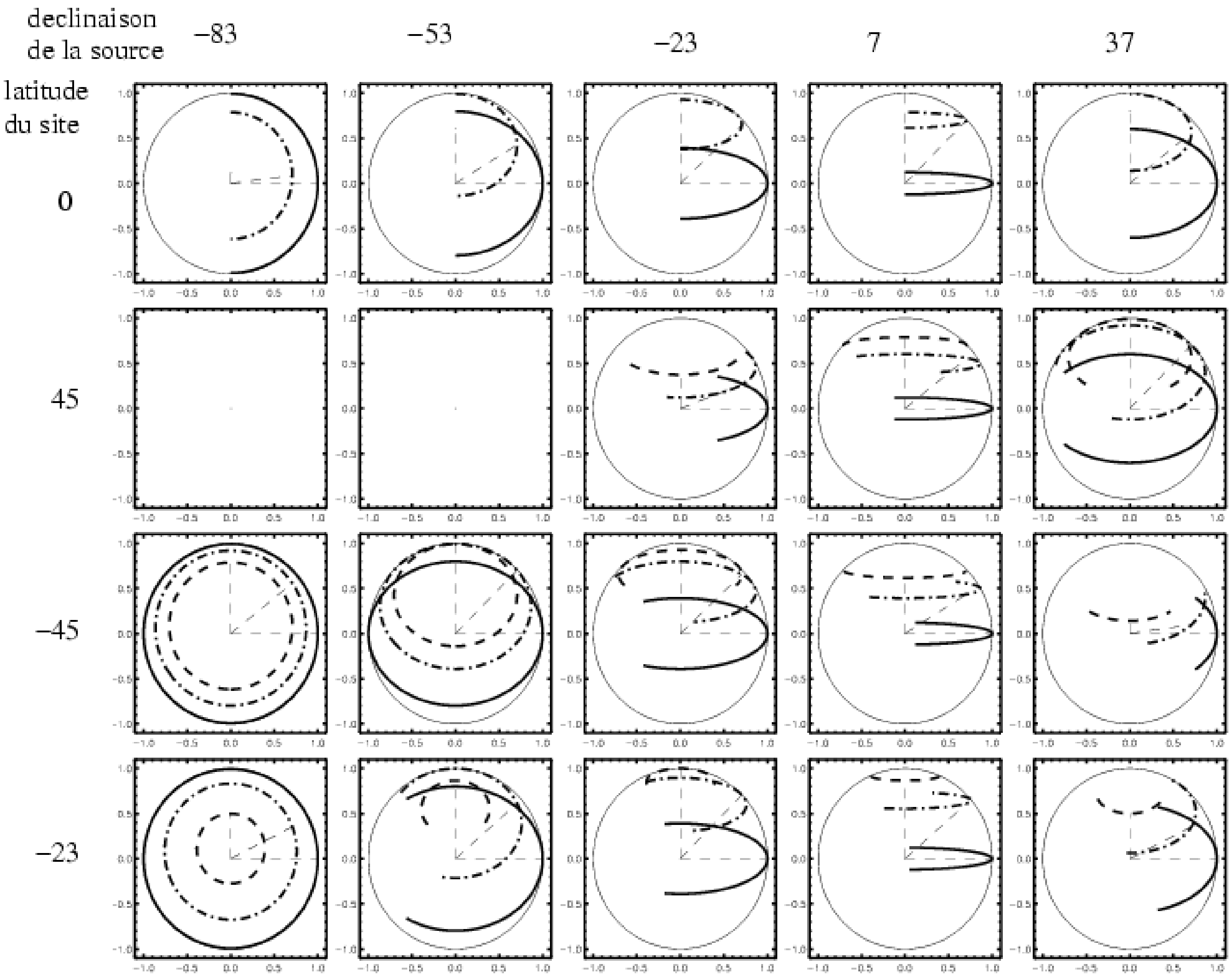
$\lambda \sim 0.5$mm

Aperture Synthesis

How to synthesize an aperture of diameter *d*?

- **Need to sample the uv-disk of diameter d/λ without holes**
  - The number of samples required $= (d/\lambda \times \text{source size})^2$
  - For a given number of antennas N_a the number of samples is $N_a (N_a - 1)$,
  - $\Rightarrow N_a \sim$ source size / resolution, but telescopes are expensive
- **For a given number of telescopes, maximize the size of the region sampled by**
  - Moving the telescopes on the ground (multiconfiguration observations)
  - Moving the telescope w.r.t. the source thanks to the Earth rotation ("supersynthesis").
  - Change the frequency (possible only when the spectral energy distribution of the source is known a priori), this changes the baseline lengths, d/λ.

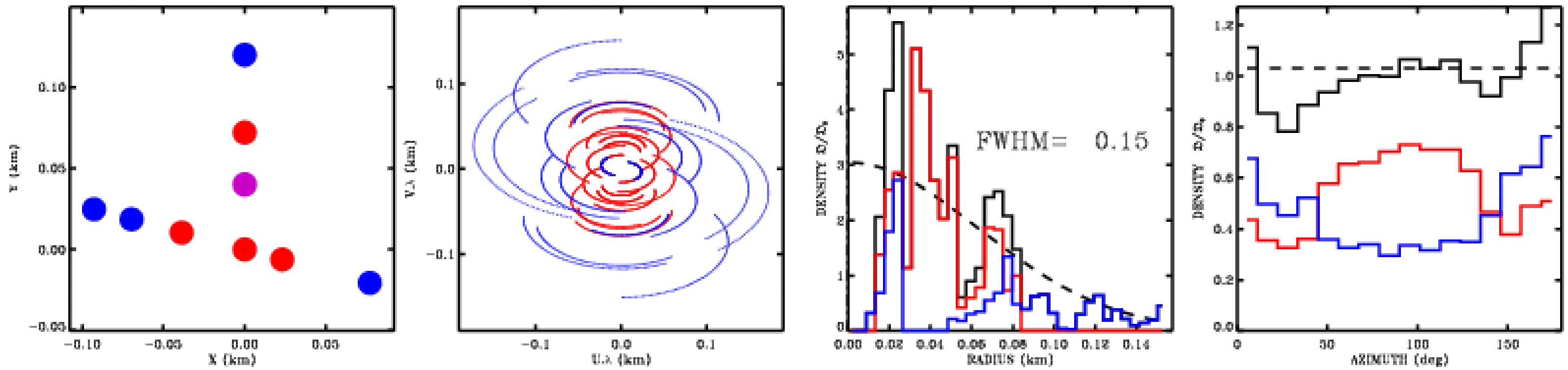
Aperture Synthesis



Ellipse arcs in uv-plane produced by 3 different baselines for different site latitudes and different source declinations.

Aperture Synthesis

configs uv-plane Radial dist Azim. dist

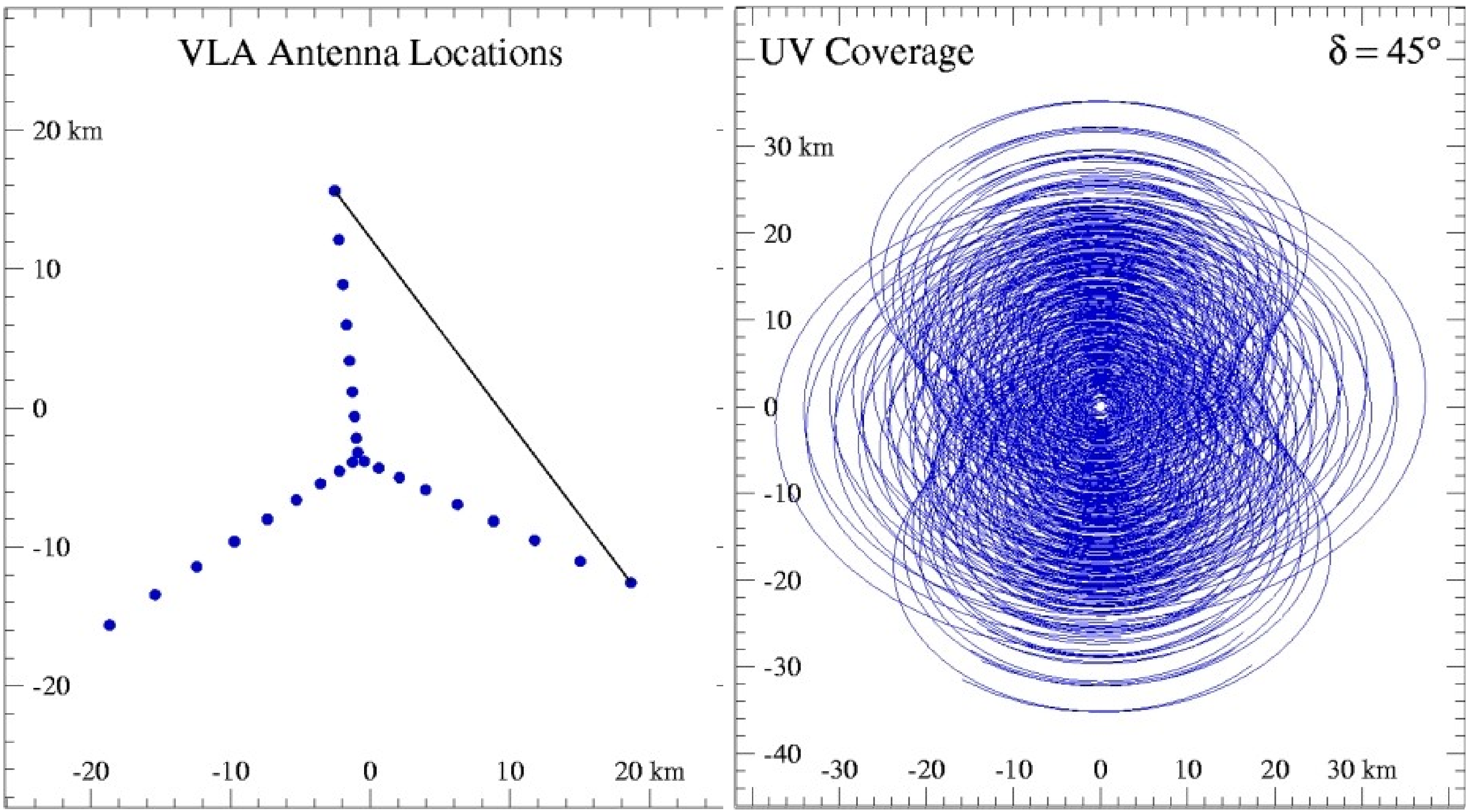


Plateau de Bure observations, supersynthesis + multiconfiguration

Aperture Synthesis

snapshot

12 hours of integration



(from A. Cohen, NRAO Summer Schoo, 2006l)

Calibration

Bandpass

- Observe a strong continuum source
- compute the gain of each frequency channel

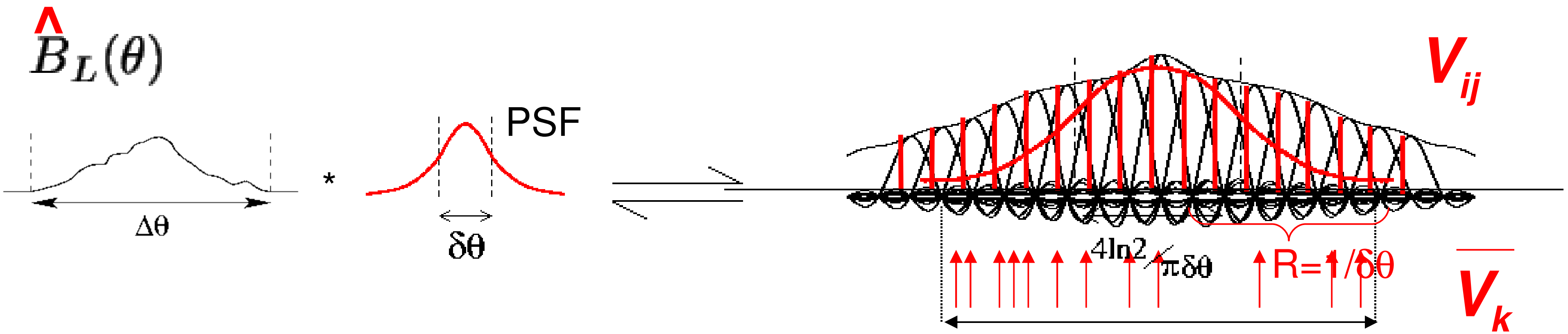
Phase/amplitude

- Observe a strong unresolved source (typically a quasar)
- Compute phase corrections such that the phases of all visibilities equal zero and amplitude corrections such that all amplitudes equal one.

Flux

- Observe a strong source of known flux (quasars are variable!), unresolved or with a know brightness distribution (a planet)
- Set the amplitude scale accordingly

“Deconvolution”



"Deconvolution"

How radio astronomer usually do

- Instead of estimating the Fourier components, V_{ij}, the radio astronomers directly Fourier transform the measurements
- The image obtained is called the "dirty map"

$$
\begin{aligned}
B_S(l,m) &= \iint \sum_k^N [\overline{V}_k \delta(u-u_k, v-v_k) + \overline{V}_k^* \delta(u+u_k, v+v_k)] e^{-2i\pi(ul+vm)}\, du\, dv \\
B_S(\boldsymbol{\xi}) &= \sum_k^N \left\{ |\overline{V}_k| \left(\cos\phi_k + i\sin\phi_k\right) \left(\cos[2\pi\boldsymbol{\xi}\cdot\boldsymbol{b}_k] - i\sin[2\pi\boldsymbol{\xi}\cdot\boldsymbol{b}_k]\right) \right\} + \\
&\qquad \left\{ |\overline{V}_k| \left(\cos\phi_k - i\sin\phi_k\right) \left(\cos[2\pi\boldsymbol{\xi}\cdot\boldsymbol{b}_k] + i\sin[2\pi\boldsymbol{\xi}\cdot\boldsymbol{b}_k]\right) \right\} \\
&= 2\sum_k^N |\overline{V}_k| \cos[2\pi\boldsymbol{\xi}\cdot\boldsymbol{b}_k - \phi_k]
\end{aligned}
$$

- This is equivalent to summing the sine filters (the venetian blinds) with the amplitudes and phase measured

“Deconvolution”

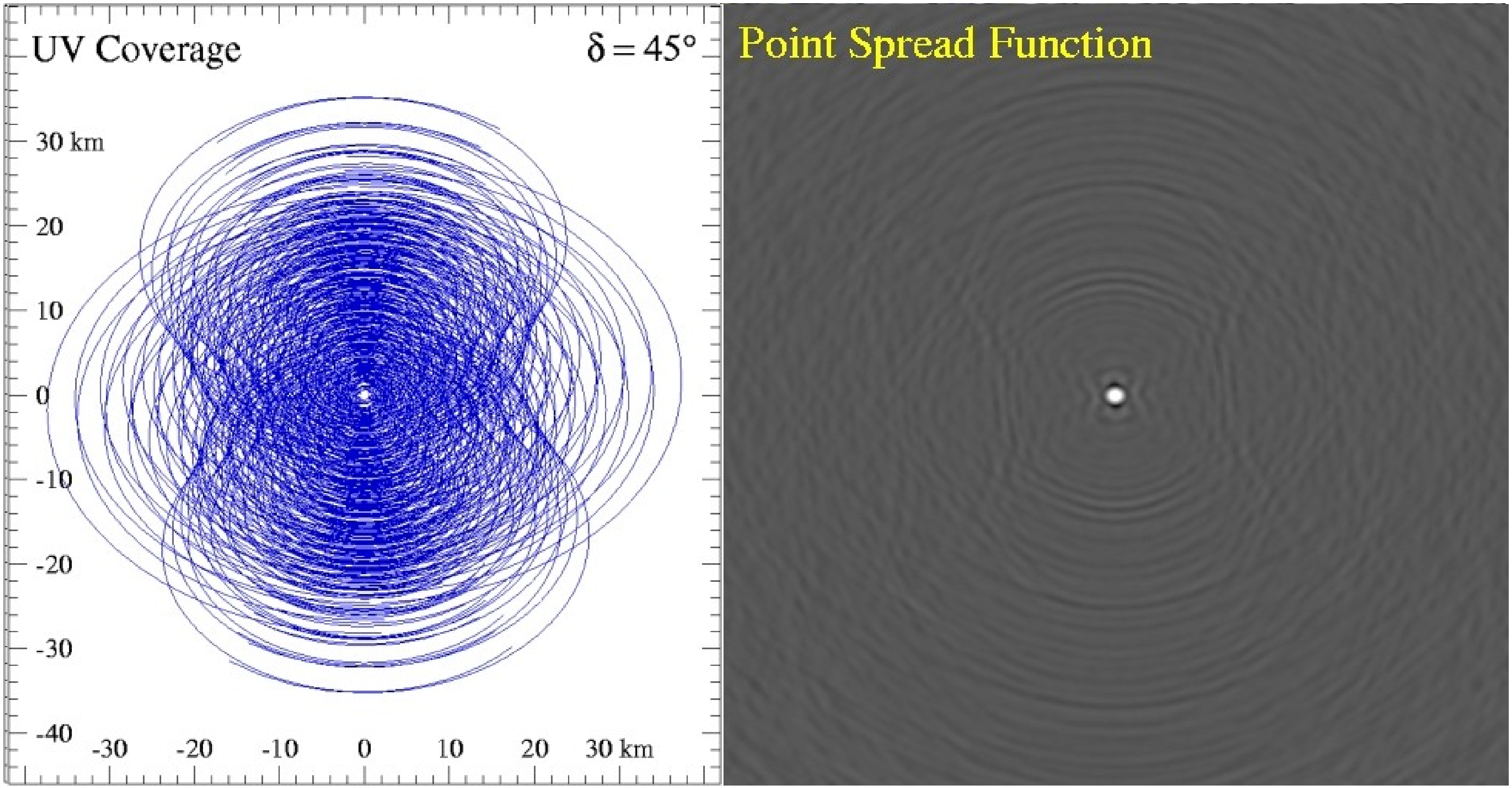
Imaging in practice

- Summing the sines is computationally expensive
- --> use FFT from one grid to another grid
- --> need to “grid” first.
- Interpolate the measurements at each node of a grid by convolving (not equivalent to computing the V_{ij})

$$B_s = F\{\underbrace{f}_{\text{Sampling function}} \times V_L\} = \underbrace{S}_{\text{Synthesized lobe}} * B_L$$

“Deconvolution”

snapshot
12 hours of integration

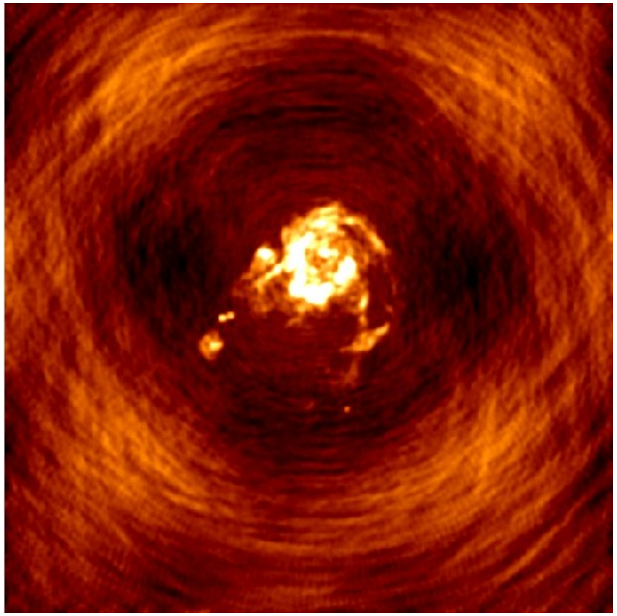


(from A. Cohen, NRAO Summer Schoo, 2006l)

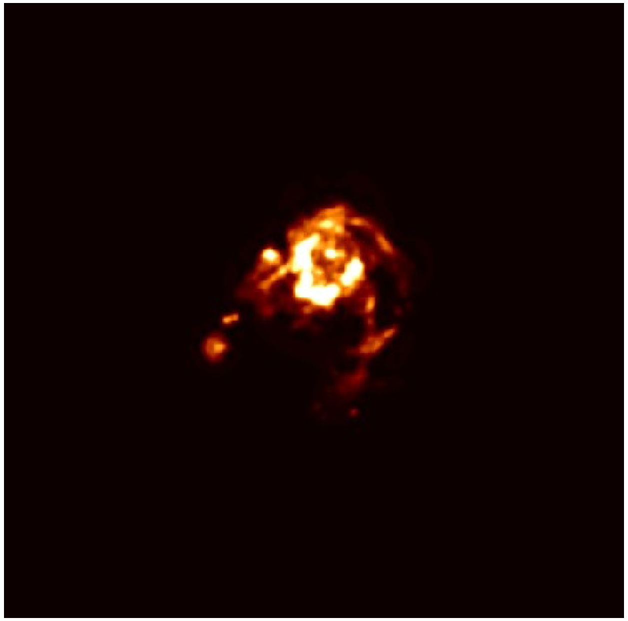
“Deconvolution”

Methods

- **CLEAN**
  - Assume source brightness distribution is a sum of point sources
  - Fit and subtract the synthesized beam iteratively
- **Maximum Entropy**
  - Maximize the “entropy” of the image (keep the pixel values in a range as small as possible)
- **There are methods working in Fourier Plane**
  - NNLS (Lawson & Hanson 1974, Briggs 1995)
  - WIPE (Lannes et al, 1994, 1996, 1997)

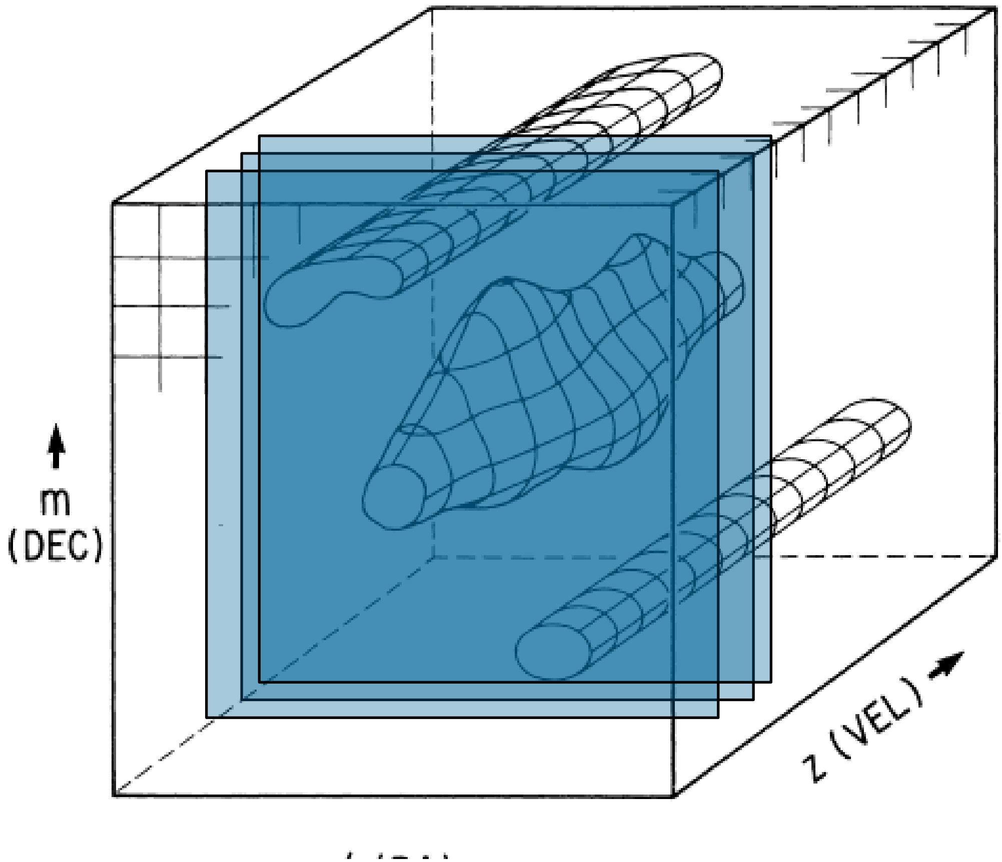


Dirty map



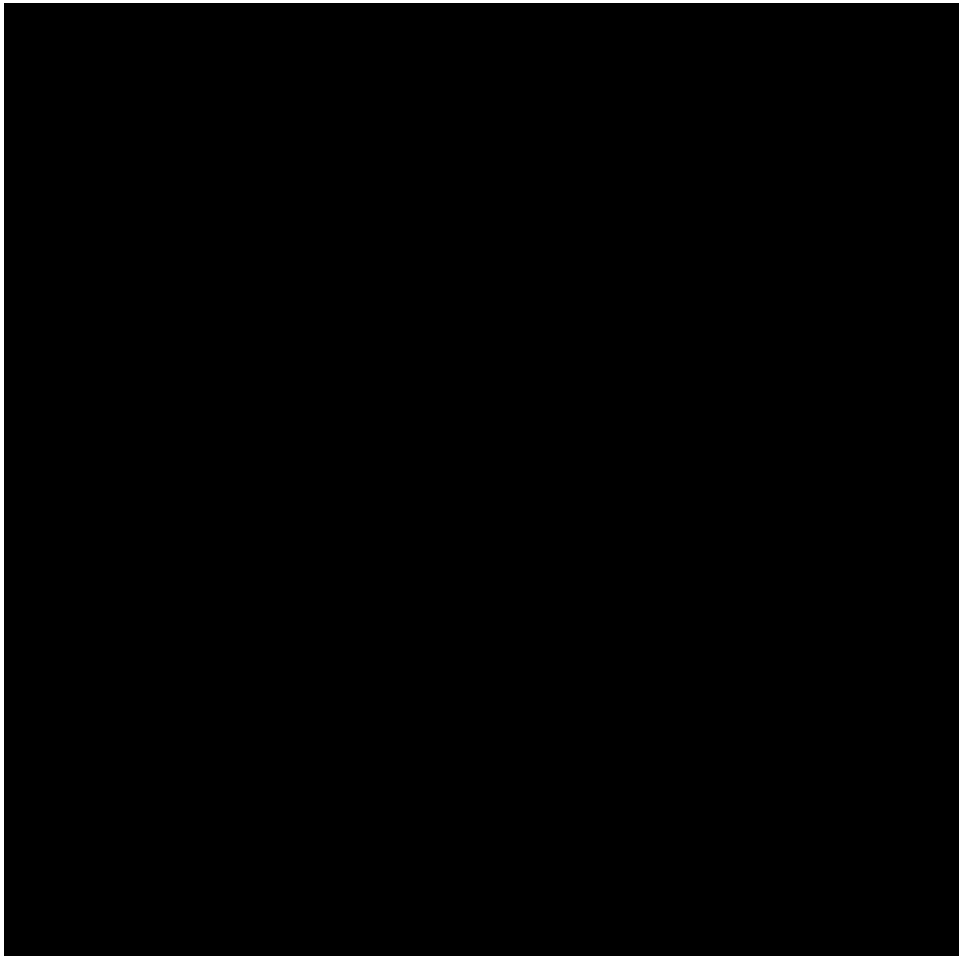
Clean map

Spectral Line Data



(From Mathews, NRAO summer school, 2006)

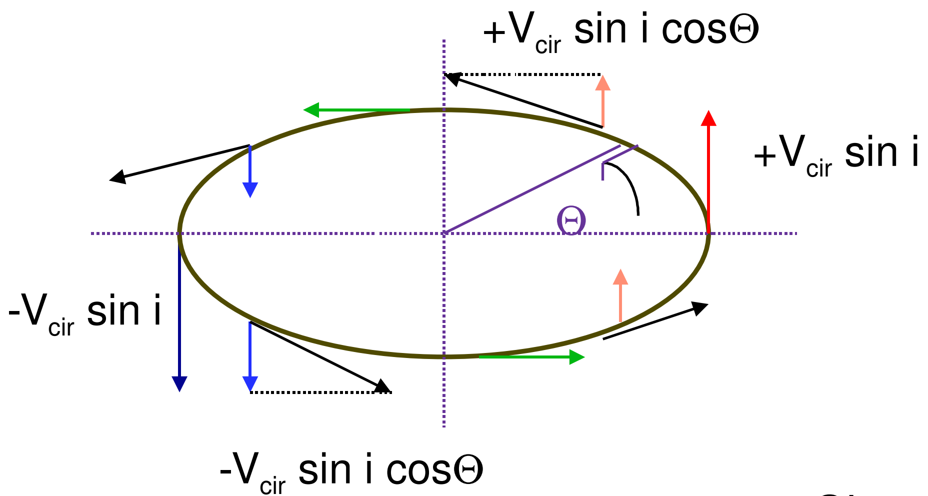
Spectral Line Data

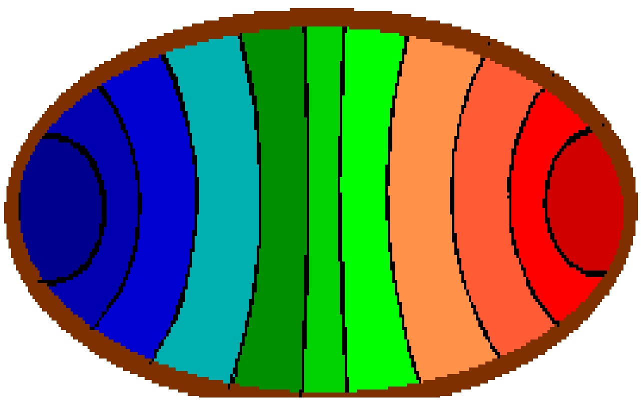


(From Mathews, NRAO summer school, 2006)

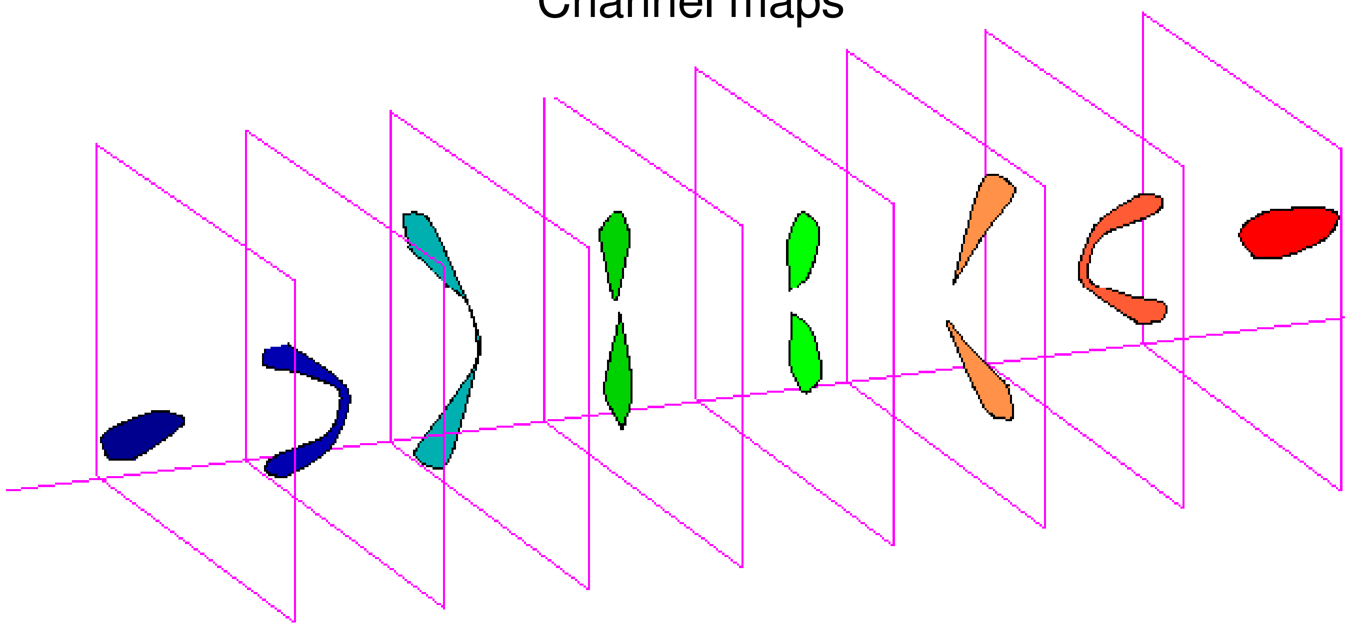
Spectral Line Data

Galactic disks





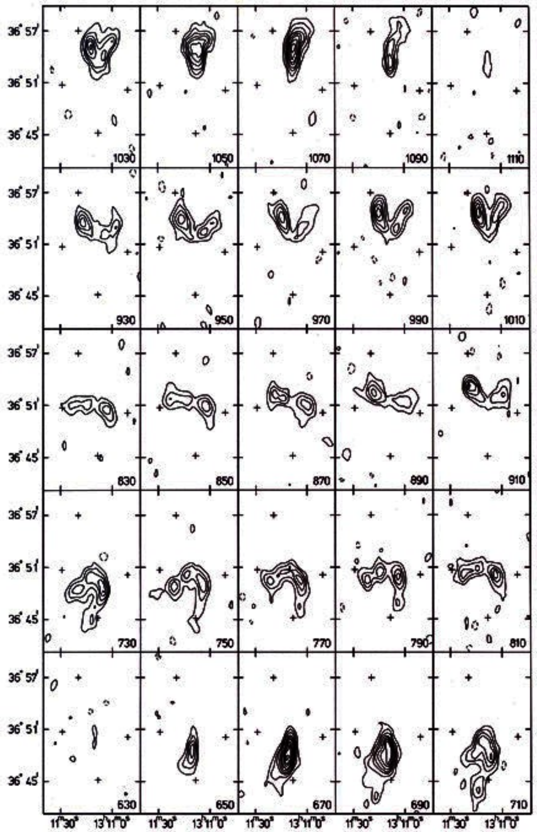
Channel maps



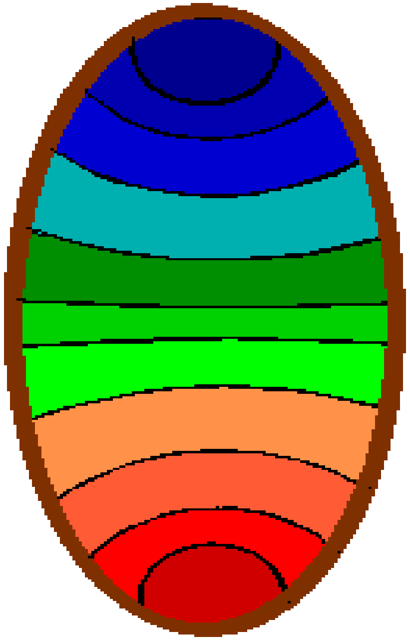
(From Mathews, NRAO summer school, 2006)

Spectral Line Data

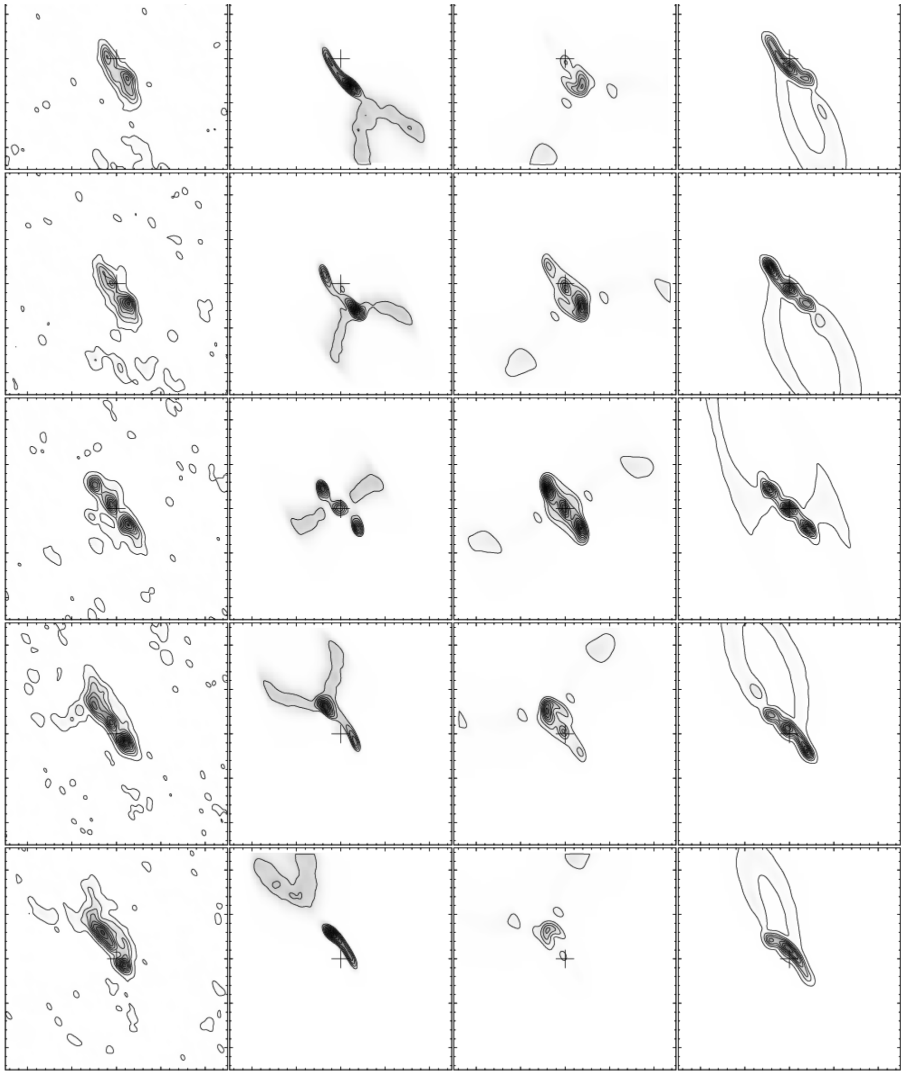
Galactic disks

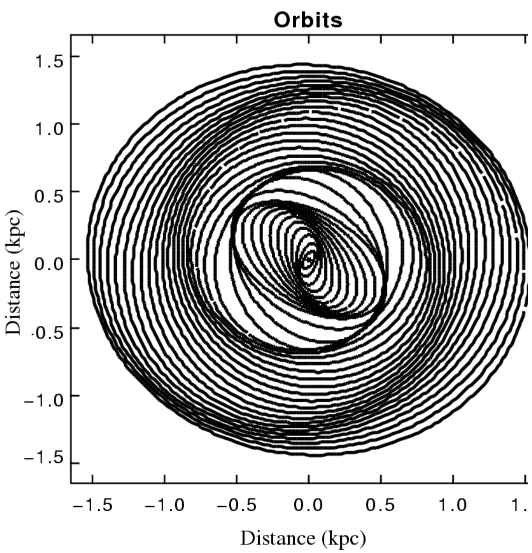


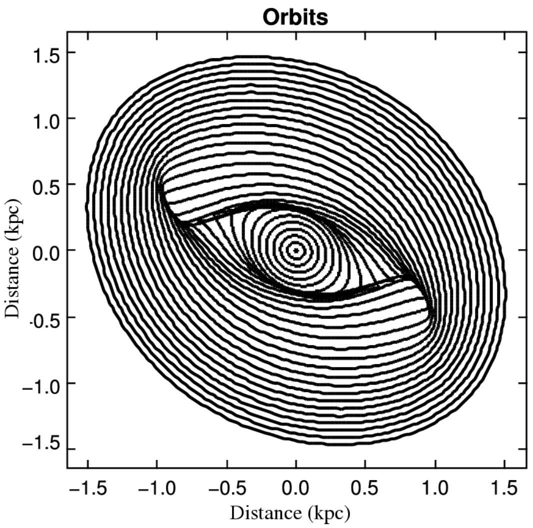
HI in the galaxy NGC 5033 (Bosma)



Spectral Line Data







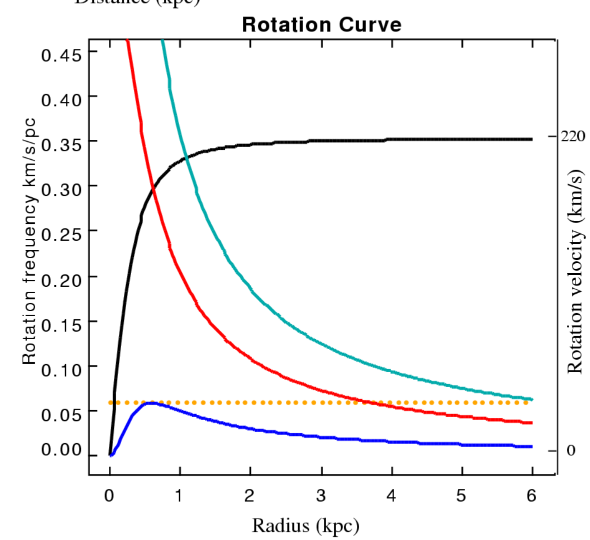


Fig. 6. Each row shows a 50″×50″ channel map of the observed data, the axisymmetric model, the ellipse orbit model and the barred potential model respectively (from left to right) at a given velocity. From top to bottom the velocities are -80, -50, 0, 50 and 80 km s^{-1}. We recall that, at 17 Mpc, 1″ corresponds to 82 pc along the major axis.

Conclusion

- **Interferometry is like looking through venetian blinds**
  - The separation between the apertures fixes the spatial frequency of the venetian blind
  - The orientation of vector subtended by the apertures fixes the orientation of the blind
  - Measuring visibilities is measuring
    - The phase of the venetian blind that maximizes the transmitted intensity
    - The value of this maximum intensity
  - With visibility measurements it is possible to reconstruct an image of the source
- **It is worth the trouble**
  - High Resolution
  - Spectroimagery (data cubes)
- **The future for high resolution at all wavelengths**