Imagerie & Interferometrie #2

James M Anderson

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Acknowledgments

- JIVE
- RadioNet
- ALBUS

- Fundamentals of Interferometry – Rick Perley
- Wide-Field Imaging -
- Sensitivity
- Spectro-imagery
- VLBI -
- (u,v) plane analysis-

Personal Background

- Software Scientist at JIVE
- Member of Advanced Long Baseline User Software (ALBUS) project
  - Ionosphere
  - Wide-field imaging
  - ParselTongue
    - Python-based scripting language for AIPS
- Research interests
  - Ionospheric calibration
  - LOFAR calibration and long-baseline development
  - Low-luminosity AGNs
- User of EVN, VLA, VLBA, Arecibo

http://www.radionet-eu.org/rnwiki/ParselTongue

Outline

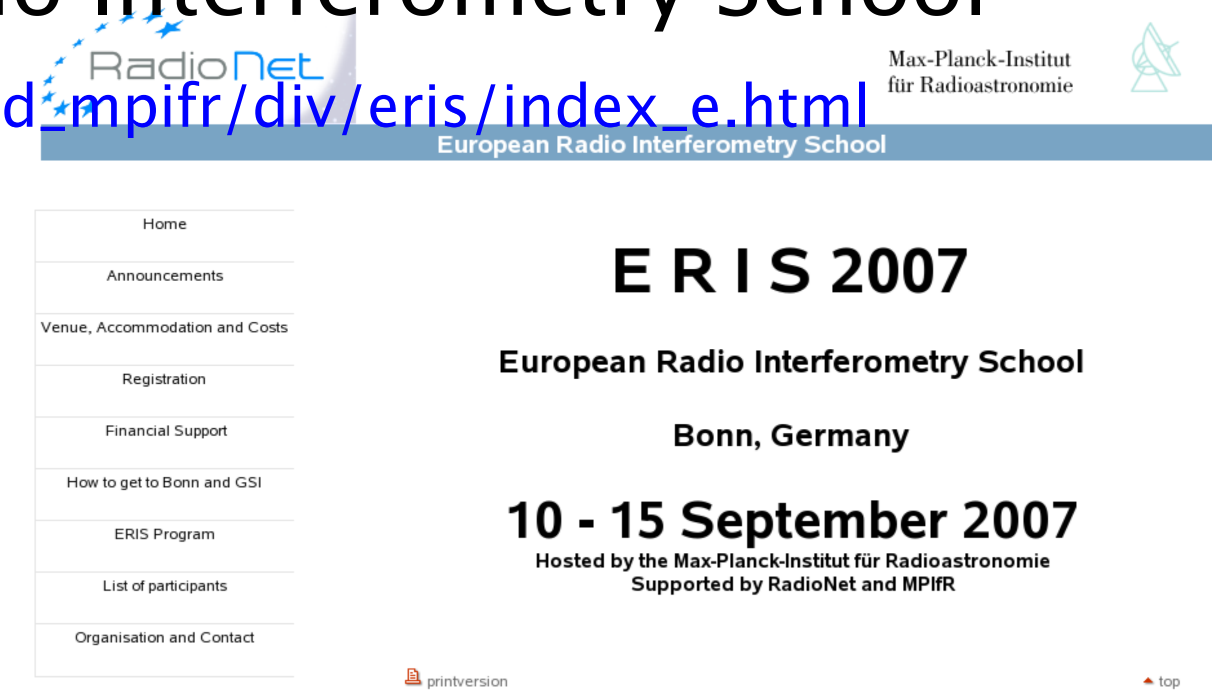
- Interferometry fundamentals
- VLBI
- lunch
- More VLBI
- Ionospheric calibration
- Wide fields of view

Books

- Interferometry and Synthesis in Radio Astronomy (Thompson, Moran, & Swenson 2001)
  - Leans toward physics/engineering side
- Synthesis Imaging in Radio Astronomy II (Taylor, Carilli, & Perley, eds. 1999)
  - Leans more toward astronomical user

Other Sources

- Lectures from the NRAO Synthesis Imaging Summer School
  - http://www.aoc.nrao.edu/events/synthesis/2006/lectures/
  - This is where I have stolen many of the slides in this presentation ...
- The European Radio Interferometry School
  - http://www.mpifr.de/old_mpifr/div/eris/index_e.html



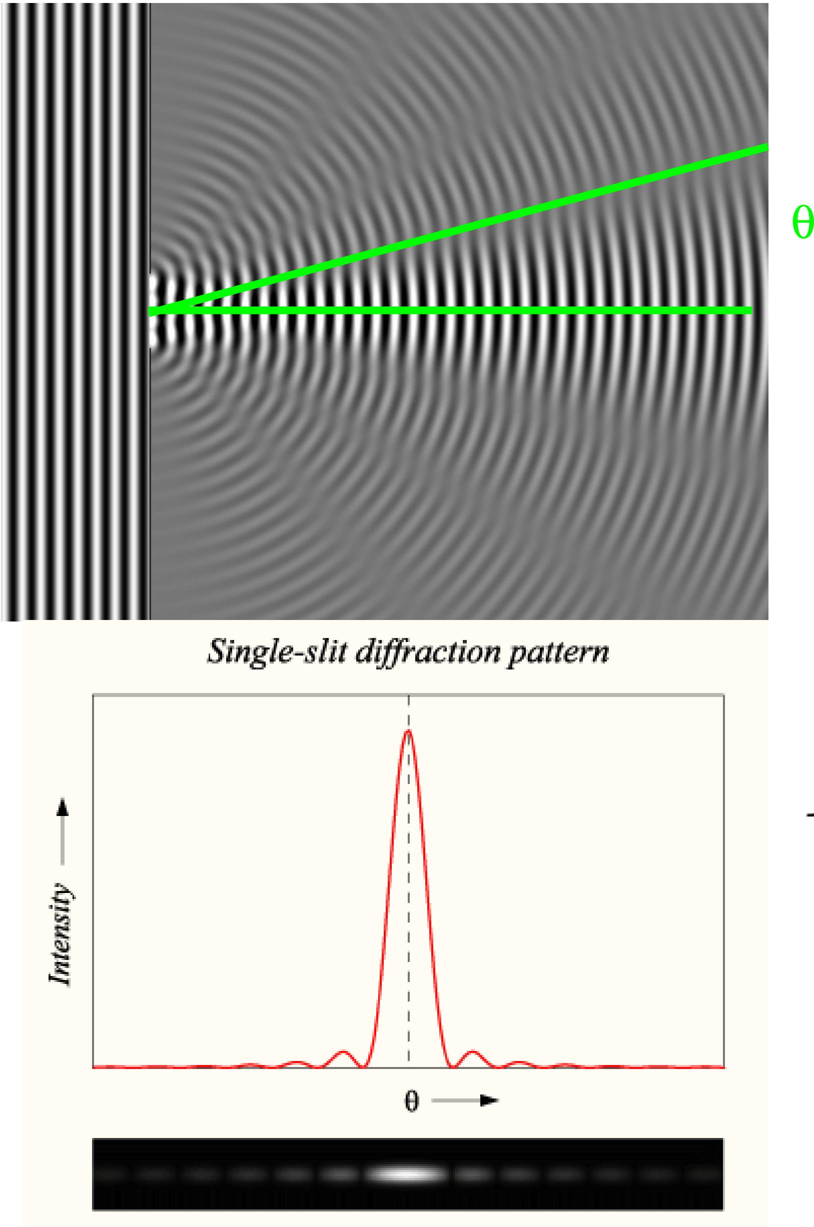
Interferometry Fundamentals

- Review of concepts from previous lectures by F. Boone
- Delay
- Basic physics
- Visibility
- Fourier Transform of the sky

Why You Should Pay Attention to the Details

- Interferometry and Fourier Transforms are probably not what you normally think about
- Interferometry is challenging, but has some tremendous benefits
- Major new telescopes such as LOFAR and the SKA will break simplifying assumptions often used in interferometry
- If you are not here to learn, why are you here?

Single-slit Diffraction



Wikipedia

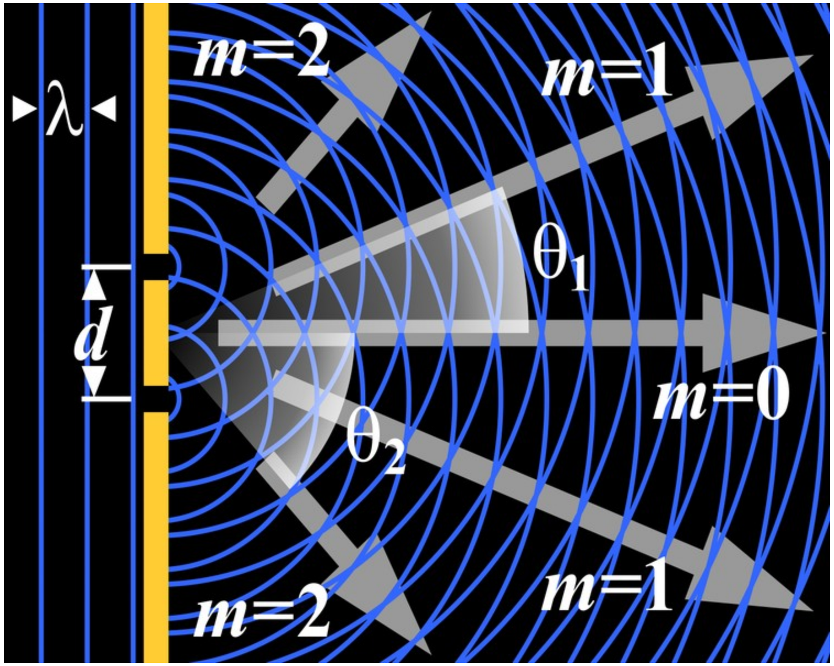
- Introductory university physics
- One-dimensional slit of size a
- Aperture corresponds to radio antenna size

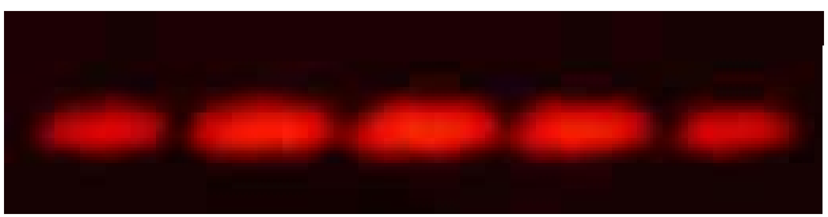
$$E = E_0 \operatorname{sinc}\left(\frac{a\nu}{c}\sin\theta\right)\cos(2\pi\nu t)$$

$$I = I_0\left[\operatorname{sinc}\left(\frac{a\nu}{c}\sin\theta\right)\right]^2$$

$$\operatorname{sinc}(x) \equiv \frac{\sin(\pi x)}{\pi x}$$

Two-slit Diffraction



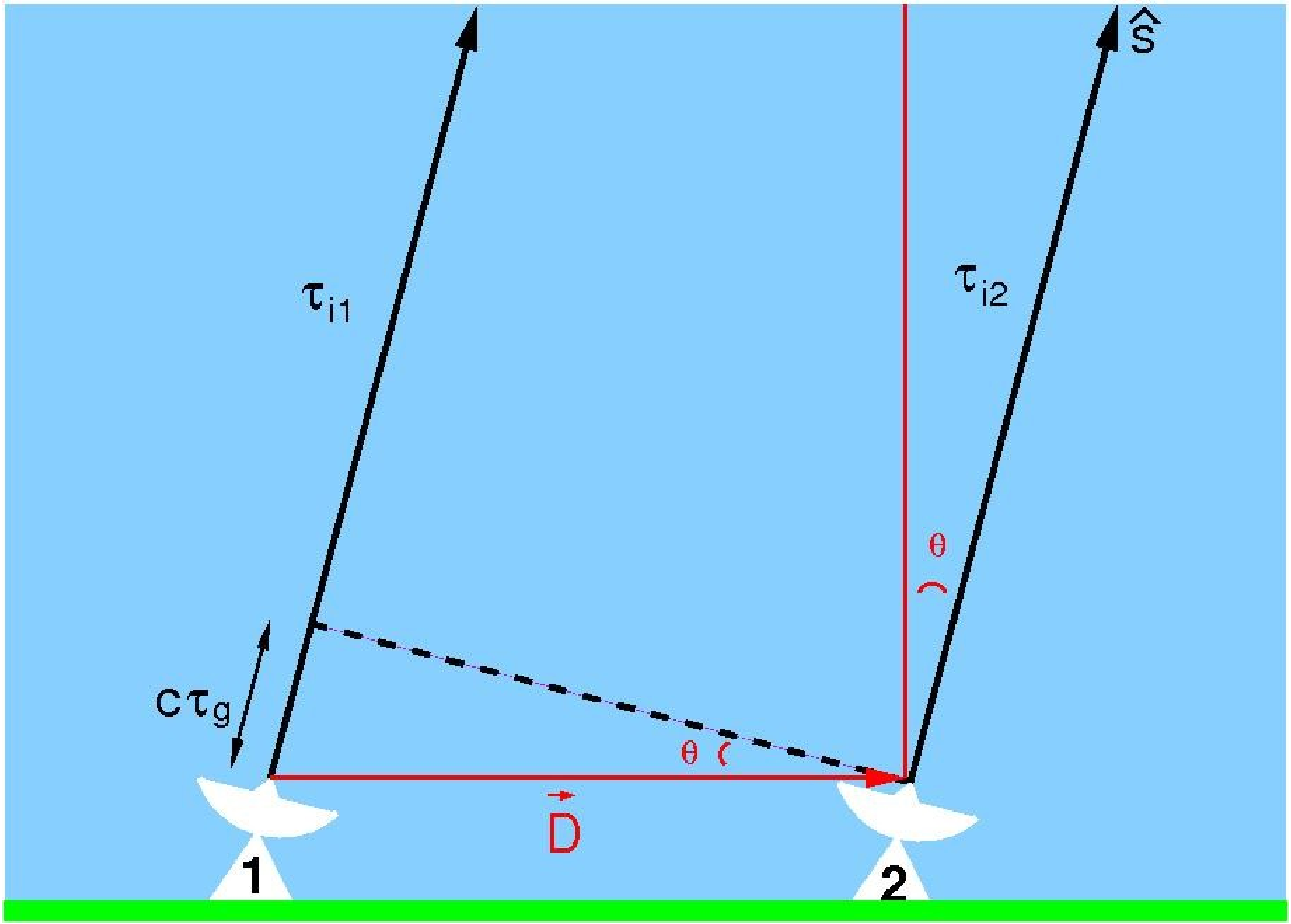


- Each slit represents an individual antenna element of an interferometer
- Distance d between antennas is the baseline length

$$E = E_0 \cos\left(\frac{\pi \nu d}{c} \sin\theta\right) \cos(2\pi\nu t)$$

$$I = I_0 \cos^2\left(\frac{\pi \nu d}{c} \sin\theta\right)$$

Delay Delay Delay



$$c\tau_{\mathrm{g}} = D \sin\theta = \boldsymbol{D} \cdot \hat{\boldsymbol{s}}$$

Simple Cosine Interferometer

Voltage from antenna 1 is: $V_1 \cos\left[2\pi\nu\left(t - \tau_g\right)\right]$

Voltage from antenna 2 is: $V_2 \cos\left[2\pi\nu\left(t\right)\right]$

Multiply the signals to get:

$$V_1 V_2 \cos\left[2\pi\nu\left(t\right)\right] \cos\left[2\pi\nu\left(t - \tau_g\right)\right]$$

This has many terms which vary rapidly with time. After averaging over many cycles of the signal, one is left with a term proportional to:

$$\cos\left[2\pi\nu\tau_{\mathrm{g}}\right] = \cos\left[2\pi\nu\frac{\boldsymbol{D}\cdot\hat{\boldsymbol{s}}}{c}\right]$$

Simple Sine and Complex Interferometer

Insert a delay equivalent to 90 degrees of phase before multiplying the signals from the two antennas. After averaging over time, the result is proportional to:

$$\cos\left[2\pi\nu\tau_{\mathrm{g}} + 90^\circ\right] = -\sin\left[2\pi\nu\tau_{\mathrm{g}}\right] = -\sin\left[2\pi\nu\frac{\boldsymbol{D}\cdot\hat{\boldsymbol{s}}}{c}\right]$$

The cosine and sine terms can be combined into a single complex term:

$$\cos\left[2\pi\nu\tau_{\mathrm{g}}\right] - i\sin\left[2\pi\nu\tau_{\mathrm{g}}\right] = e^{-i2\pi\nu\tau_{\mathrm{g}}}$$

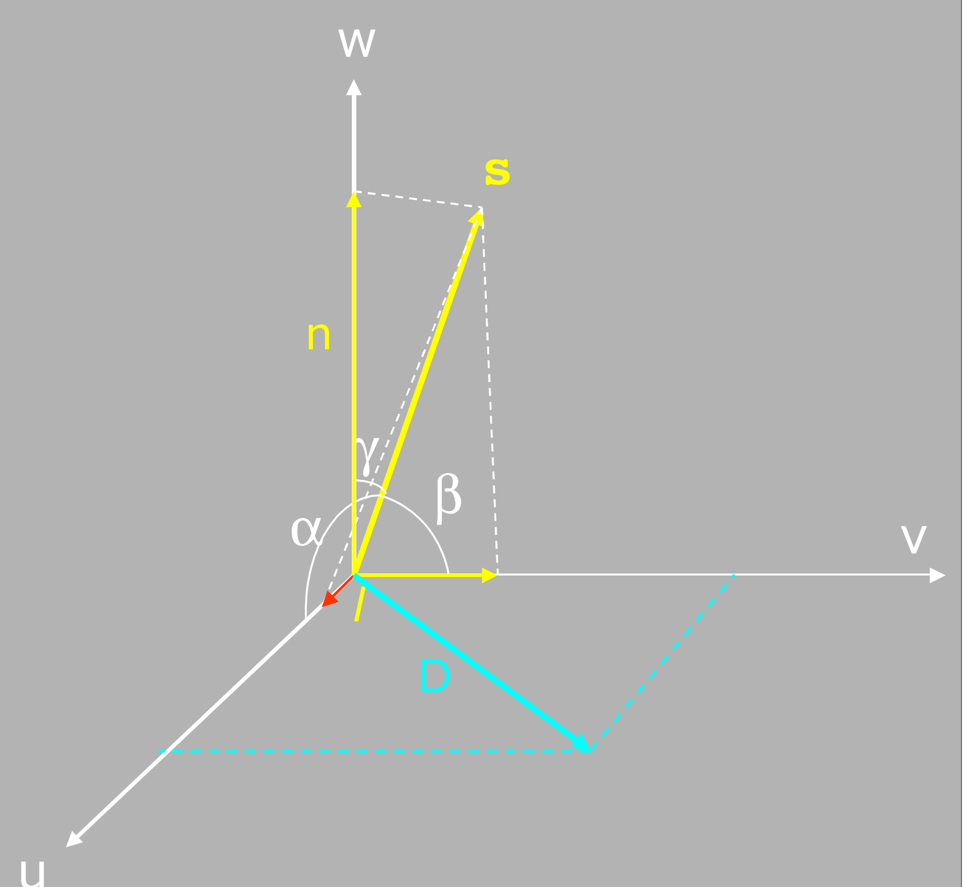
Coordinate Systems and Direction Cosines

The unit direction vector **s** is defined by its projections on the (u,v,w) axes. These components are called the **Direction Cosines**.

$$l = \cos(\alpha)$$

$$m = \cos(\beta)$$

$$n = \cos(\gamma) = \sqrt{1 - l^2 - m^2}$$



$$\boldsymbol{s} = (l, m, n) = \left(l, m, \sqrt{1 - l^2 - m^2}\right)$$

The baseline vector D is specified by its coordinates (u,v,w) (measured in wavelengths).

$$D = (\lambda u, \lambda v, \lambda w)$$

Perley 2006

(u,v,w) Coordinates

Define $\boldsymbol{D}_\lambda \equiv \boldsymbol{D}/\lambda$

So that $\nu\tau_{\mathrm{g}} = \boldsymbol{D}_\lambda \cdot \hat{\boldsymbol{s}} = \boldsymbol{D}_\lambda \cdot (\hat{\boldsymbol{s}}_0 + \boldsymbol{\sigma})$

where the direction $\mathbf{s}_0$ is some direction of interest and $\boldsymbol{\sigma}$ is an offset vector from that direction. The (u,v,w) coordinate system has the w axis pointing in the direction of $\mathbf{s}_0$ with the u axis toward East and the v axis North.

Then $\boldsymbol{D}_\lambda = (u, v, w)$

Integrating Over the Sky

The interferometer is sensitive to emission from all over the sky, attenuated by the antenna response. Let the sky intensity relative to the direction $\mathbf{s}_0$ be called $I(\boldsymbol{\sigma})$, and the normalized antenna response be called $A_N(\boldsymbol{\sigma})$. Then the interferometer response is

$$\int_{4\pi} A_N(\boldsymbol{\sigma}) I(\boldsymbol{\sigma}) e^{-i2\pi\nu\tau_\mathrm{g}}\, d\Omega$$

$$= e^{-i2\pi \boldsymbol{D}_\lambda \cdot \hat{\boldsymbol{s}}_0} \int_{4\pi} A_N(\boldsymbol{\sigma}) I(\boldsymbol{\sigma}) e^{-i2\pi \boldsymbol{D}_\lambda \cdot \boldsymbol{\sigma}}\, d\Omega$$

The Visibility

$$V(u,v,w) = |V|e^{i\phi} \equiv \int_{4\pi} A_N(\boldsymbol{\sigma}) I(\boldsymbol{\sigma}) e^{-i2\pi \boldsymbol{D}_\lambda \cdot \boldsymbol{\sigma}}\, d\Omega$$

$$= \int_{\infty}^{\infty} \int_{\infty}^{\infty} A_N(l,m) I(l,m)$$

$$\times \exp\left\{ -i2\pi \left[ul + vm + w\left(\sqrt{1-l^2-m^2} - 1 \right) \right] \right\} \frac{dl\, dm}{\sqrt{1-l^2-m^2}}$$

(the visibility integrand is defined to be zero for $l^2+m^2 \geq 1$)

2-D Visibilities

- For sufficiently small l and m offset directions of interest, $n \simeq 1$ and the w term can be ignored
- The visibility function then reduces to approximately a Fourier Transform of the sky brightness multiplied by the normalized antenna response.
- $V(u,v) \rightleftharpoons I(l,m)$ *(roughly speaking)*
- An interferometer measures an individual component of this Fourier Transform at one instant in time.

Comments on the Visibility (by R Perley)

- The Visibility is a function of the source structure and the interferometer baseline.
- The Visibility is NOT a function of the absolute position of the antennas (provided the emission is time-invariant, and is located in the far field).
- The Visibility is Hermitian: $V(u,v) = V^*(-u,-v)$. This is a consequence of the intensity being a real quantity.
- There is a unique relation between any source brightness function, and the visibility function.
- Each observation of the source with a given baseline length provides one measure of the visibility.
- Sufficient knowledge of the visibility function (as derived from an interferometer) will provide us a reasonable estimate of the source brightness.

Comments on Interferometers (by J M Anderson)

- Interferometers sample in Fourier space
  - Do not need to sample entire (u,v) plane to reconstruct adequate image of the sky
  - But large scale flux is "invisible" to an interferometer
- Interferometers are most sensitive to differences between telescopes
  - Delay, phase, and nearly all calibration parameters for an antenna are just relative to other antennas
- Interferometers are far less sensitive to RFI than direct measuring telescopes

Very Long Baseline Interferometry (VLBI)

- Introduction
- Science
- Comparison with standard interferometry
- Station operation
- Delay
- Correlators
- Calibration
- Smearing

Many slides and pictures taken from Craig Walker's (2004) and Ylva Pihlström and Craig Walker's (2006) Synthesis Imaging in Radio Astronomy presentations.

Very Long Baseline Interferometry

- Essentially, VLBI is just a technique to make the highest resolution observations
- As with all interferometers, the synthesized beam resolution goes as $\theta_s = \lambda / D$
  - For global arrays at GHz frequencies, resolution of order 1 milliarcsecond (mas)
  - For LOFAR with 1000 km baseline, resolution of 0.25" at 240 MHz, 1.2" at 50 MHz

The Quest for Resolution

Atmosphere gives 1" limit without corrections which are easiest in radio

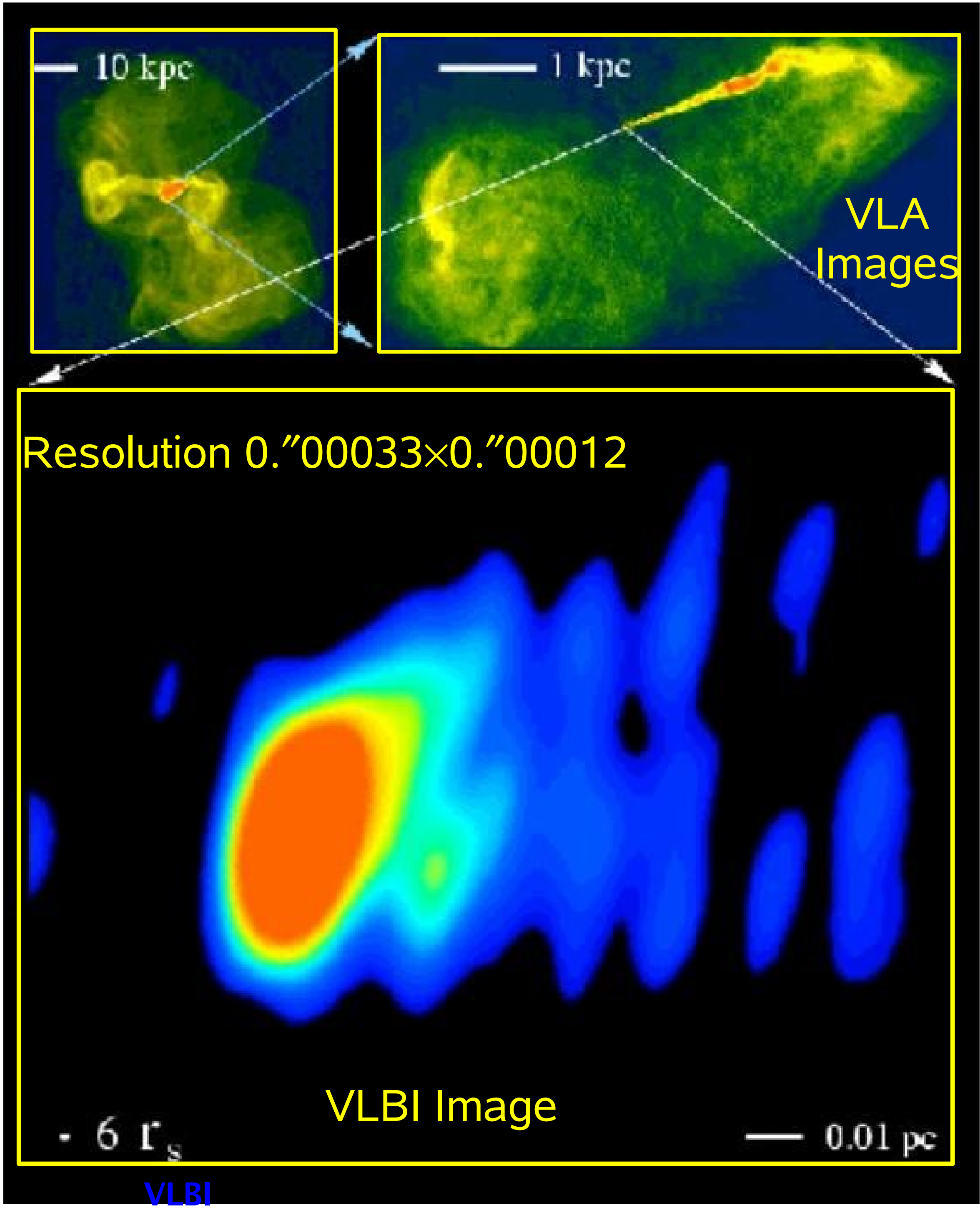
Jupiter and Io as seen from Earth

1 arcmin | 1 arcsec | 0.05 arcsec | 0.001 arcsec

Simulated with Galileo photo

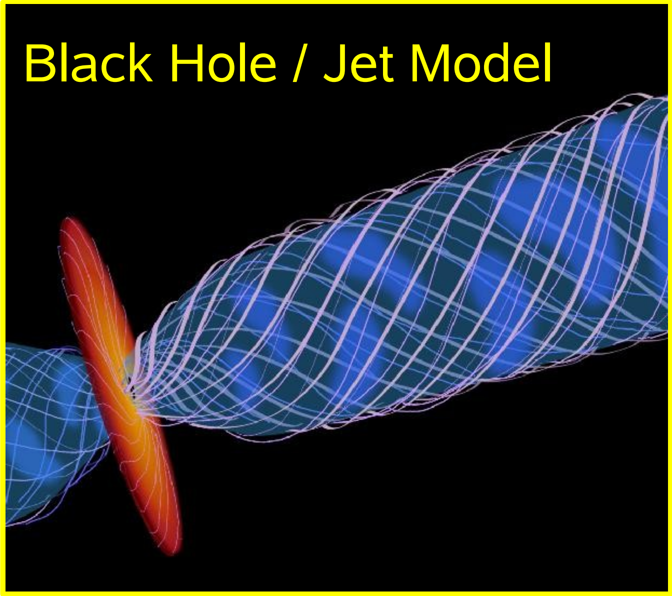
Walker 2004

Example 1:Jet Formation: Base Of M87 Jet



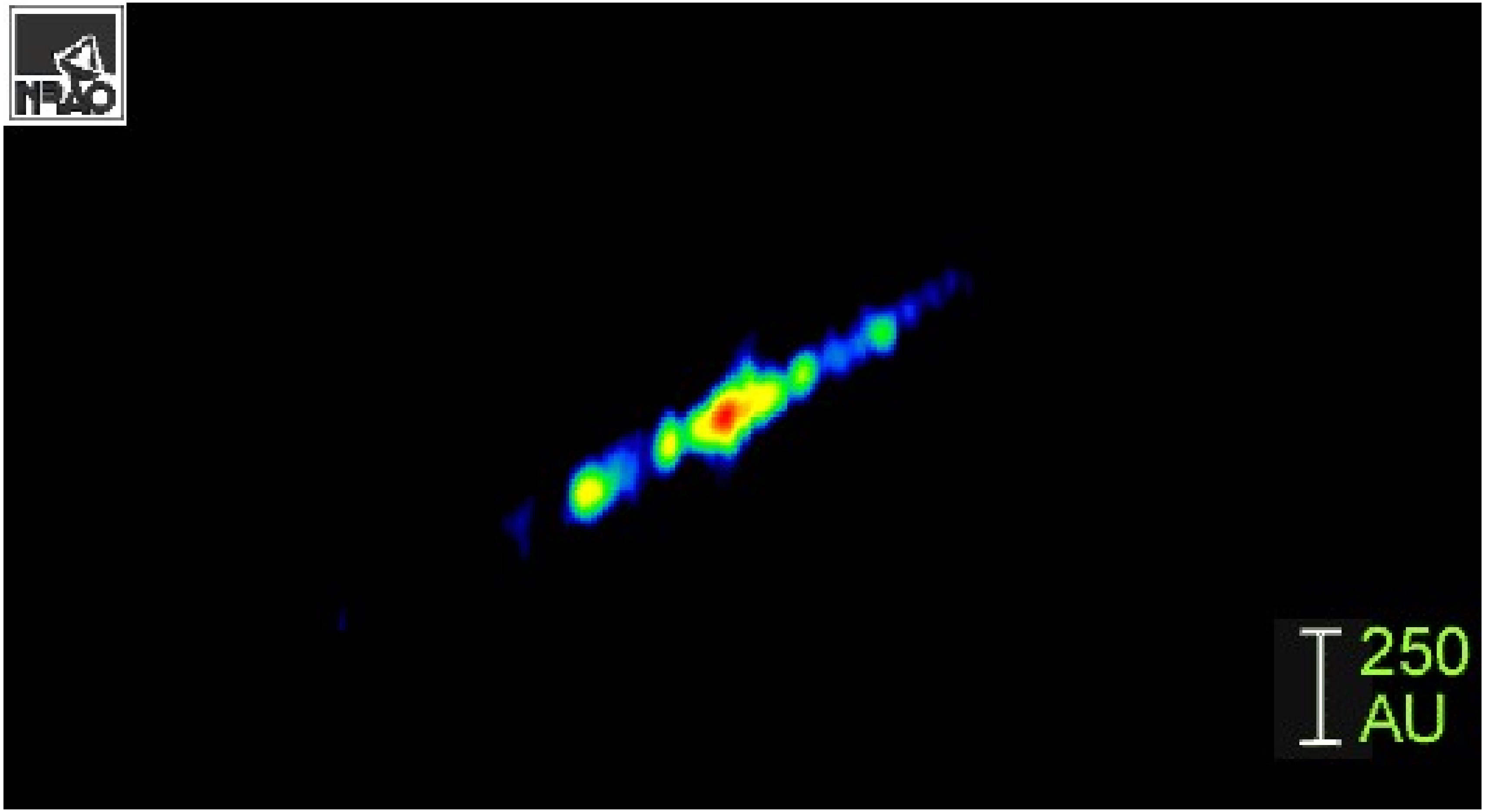
43 GHz Global VLBI
Junor, Biretta, & Livio
Nature, 401, 891

Shows hints of jet collimation region



Walker 2004

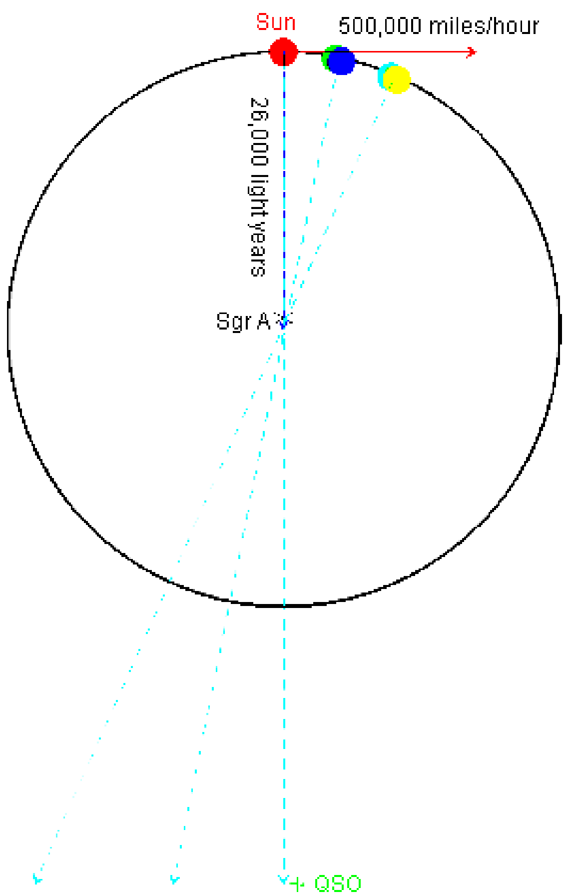
Example 2: Jet Dynamics: The SS433 Movie

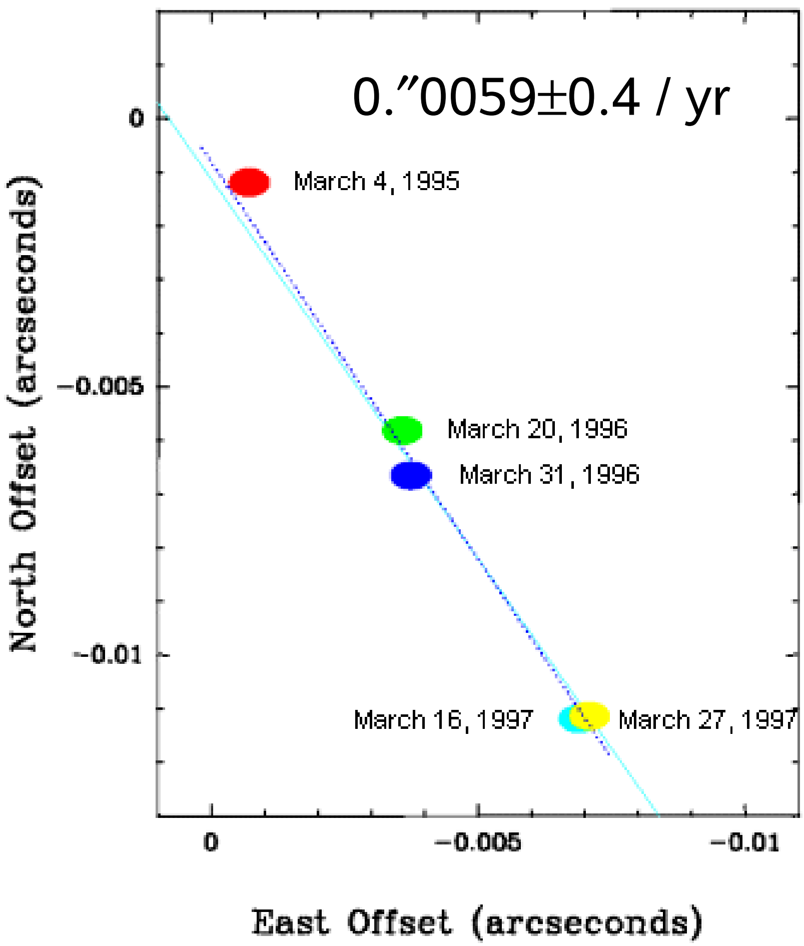


- Two hour snapshot almost every day for 40 days on VLBA at 1.7 GHz
  - Mioduszewski, Rupen, Taylor, and Walker

Walker 2004

Example 3: Motions Of Sgr A*

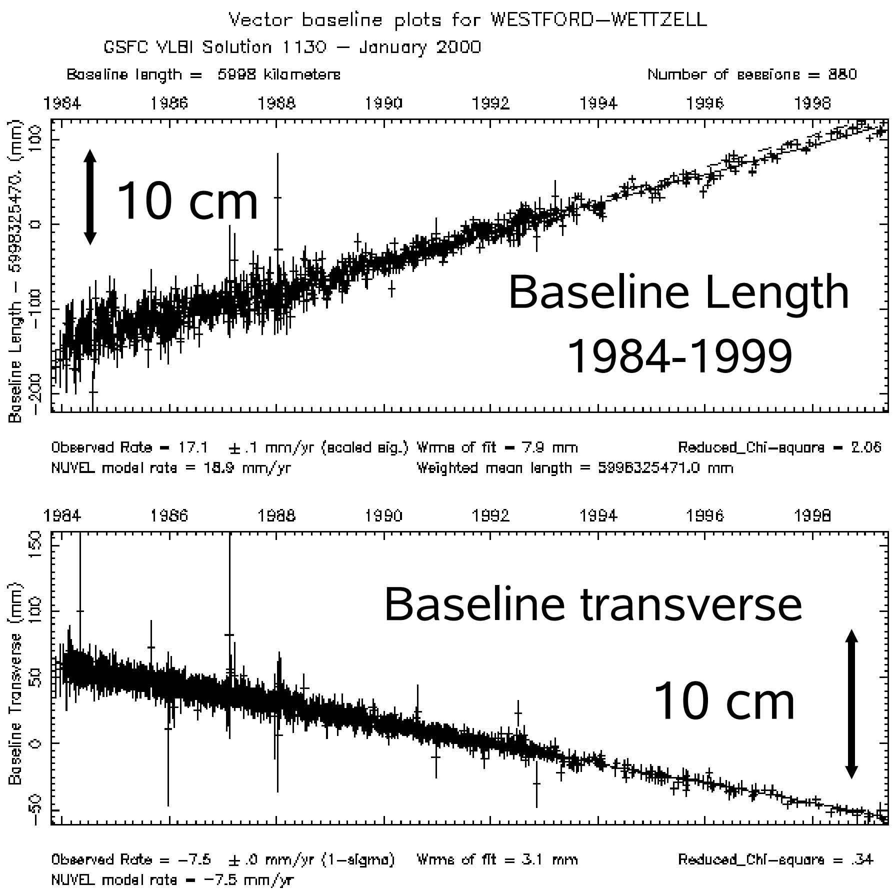




Walker 2004 Reid et al. 1999, Ap. J. 524, 816

Example 4: Geodesy and Astrometry

- Fundamental reference frames
  - International Celestial Reference Frame (ICRF)
  - International Terrestrial Reference Frame (ITRF)
  - Earth rotation and orientation relative to inertial reference frame of distant quasars
- Tectonic plate motions measured directly
- Earth orientation data used in studies of Earth's core and Earth/atmosphere interaction
- General relativity tests
  - Solar bending significant over whole sky



Walker 2004

Example 5: Other Science

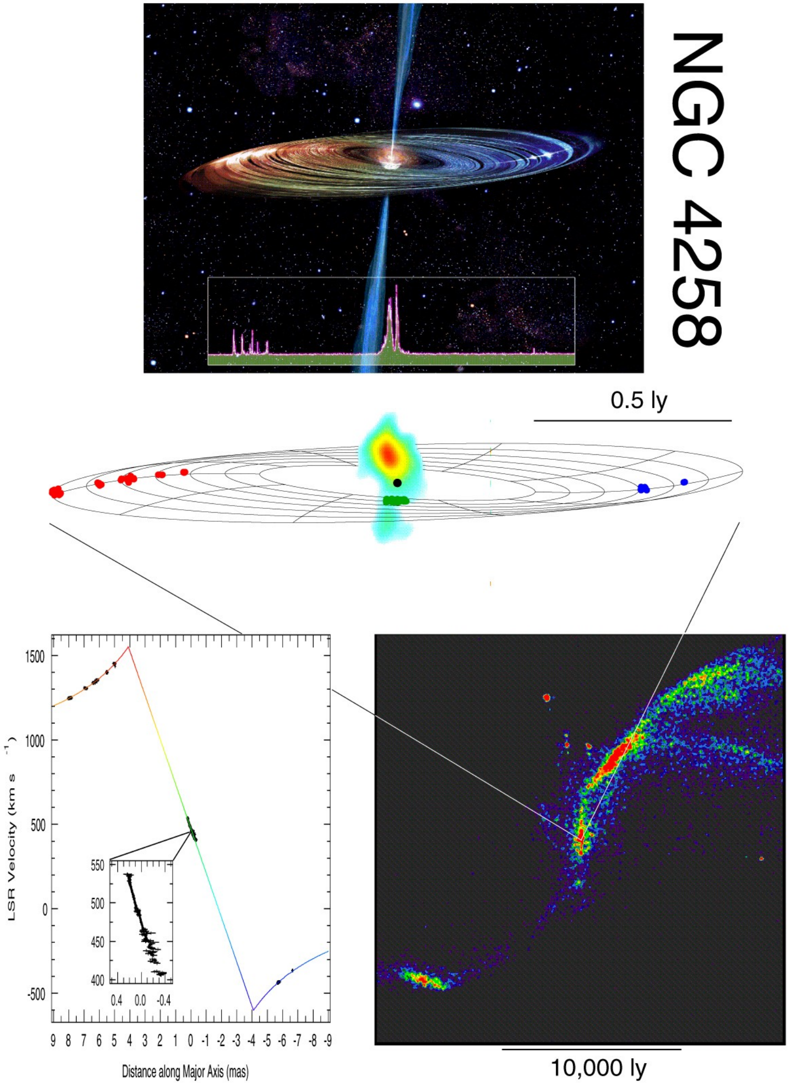


Image courtesy: L. Greenhill

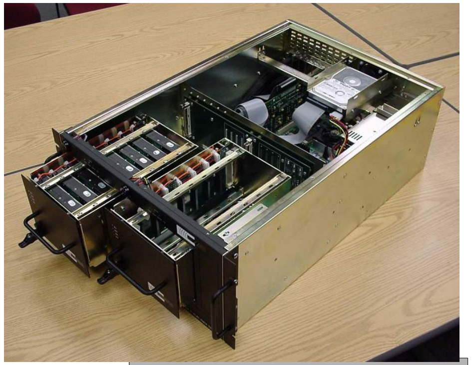
- Spacecraft tracking
  - Huygens descent onto Titan
  - Mars missions
- Jupiter/Io torus
  - LOFAR ITS–Nançay
- HI absorption
- Maser emission

Drawbacks to VLBI

- Need bright sources
- Time average smearing and bandwidth smearing (discussed in a few slides) greatly reduce field of view
- VLBI produces huge datasets
  - The JIVE VLBI correlator is producing datasets for individual observations approaching 1 TB
    - My recent 4 hour 320 MHz experiment with only 8 telescopes is ~ 300 GB
  - LOFAR and the SKA will produce far more data
- Ionosphere, troposphere, clocks, and so on are all different, so calibration "difficult"

What Is VLBI?

- Not fundamentally different from linked interferometry
- Radio interferometry with unlimited baselines
  - High resolution – milliarcsecond (mas) or better
  - Baselines up to an Earth diameter for ground based VLBI
  - Can extend to space (HALCA)
  - ~~Sources must have high brightness temperature~~
- Traditionally uses no IF or LO link between antennas
  - Data recorded on tape or disk then shipped to correlator
  - Atomic clocks for time and frequency– usually hydrogen masers
  - Correlation occurs days to years after observing
  - Real time over fiber is an area of active development
- Can use antennas built for other reasons



Mark5 recorder

Maser

Walker 2004

VLBI and Connected Interferometry Differences

VLBI is not fundamentally different from connected interferometry

- Differences are a matter of degree.
  - Separate clocks – Cause phase variations
  - Independent atmospheres (ionosphere and troposphere)
  - Phase fluctuations not much worse than VLA A array
  - Gradients are worse – affected by total, not differential atmosphere
  - Ionospheric calibration useful – dual band data or GPS global models
- Calibrators poor
  - Compact sources are variable – Calibrate using Tsys and gains
  - All bright sources are at least somewhat resolved – need to image
  - There are no simple polarization position angle calibrators
- Geometric model errors cause phase gradients
  - Source positions, station locations, and the Earth orientation are difficult to determine to a small fraction of a wavelength

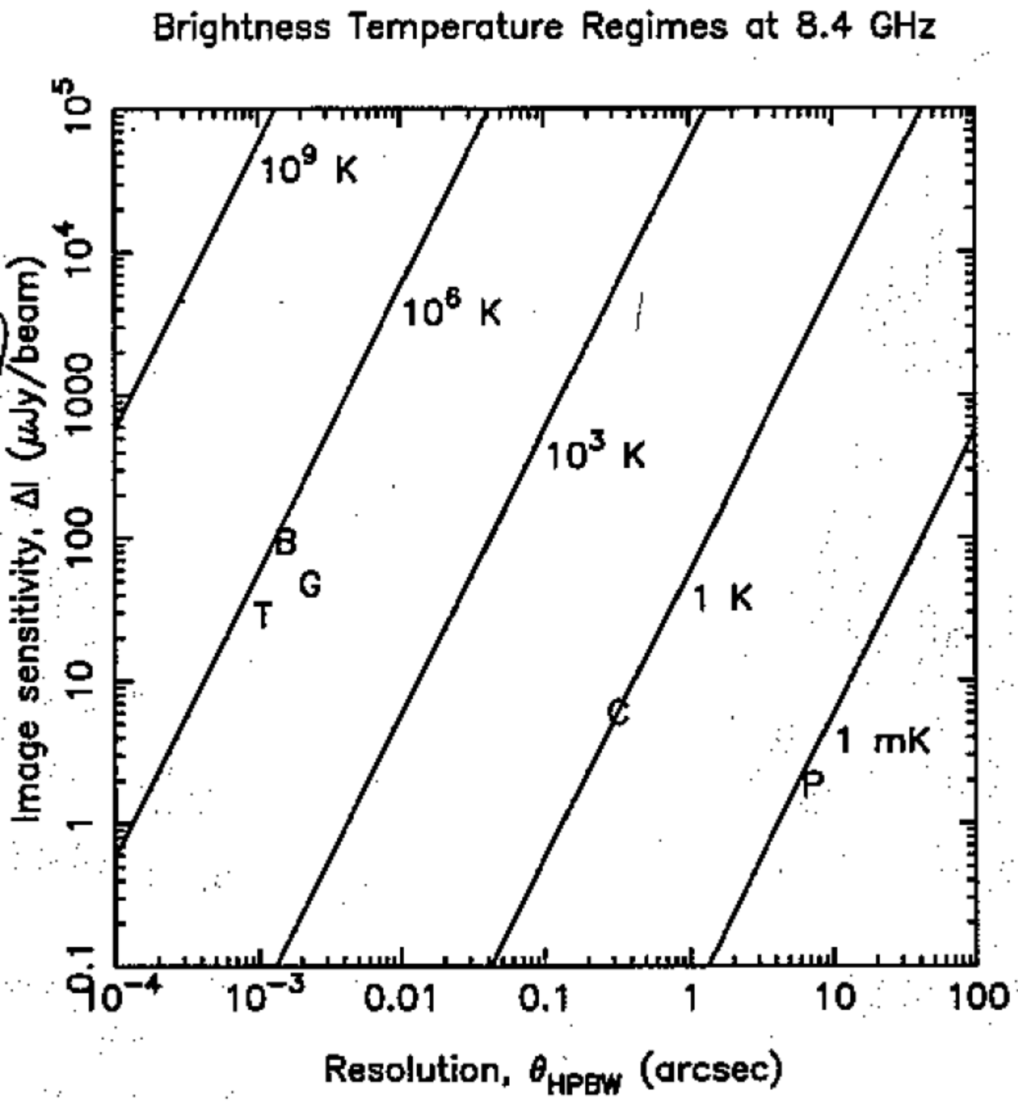
Walker 2004

VLBI and Connected Interferometry Differences II

- Phase gradients in time and frequency need calibration – fringe fit
- ~~VLBI is not sensitive to thermal sources~~
  - ~~10^6 K brightness temperature limit~~
  - ~~This limits the variety of science that can be done~~
- Hard to match resolution with other bands like optical
  - ~~An HST pixel is a typical VLBI field of view~~
- Even extragalactic sources change structure on finite time scales
  - VLBI is a movie camera
- Networks have inhomogeneous antennas – hard to calibrate
- Much lower sensitivity to RFI
- Primary beam is not usually an issue for VLBI

Walker 2004

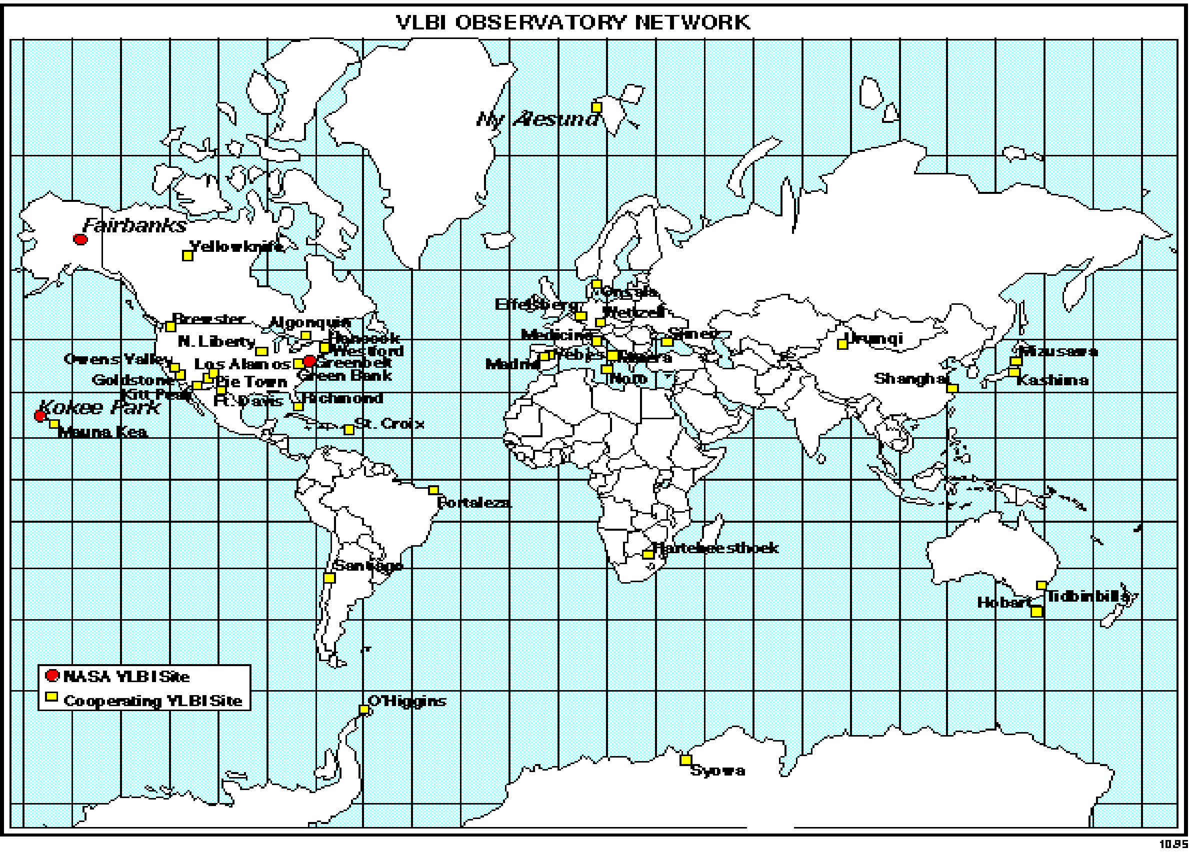
Brightness Temperature Limit



- VLBI has traditionally been limited to high brightness temperatures ($T_B > 10^6$ K)
- But current VLBI systems with wide bandwidths and phase referencing are sensitive to $T_B < 10^4$ K
- EVN, LOFAR, HSA, VLBA

$$T_{\rm B,min} = \frac{c^2 I_{\nu,\rm min}}{2k\nu^2} = \frac{c^2 S_{\nu,\rm min}}{2k\nu^2\theta_{\rm s}^2} \sim 0.4 S_{\nu,\rm min}(\rm mJy)\,[D(\rm km)]^2$$

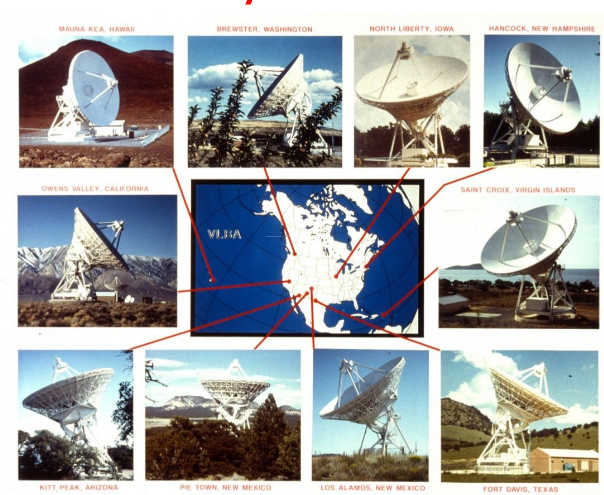
VLBI Stations

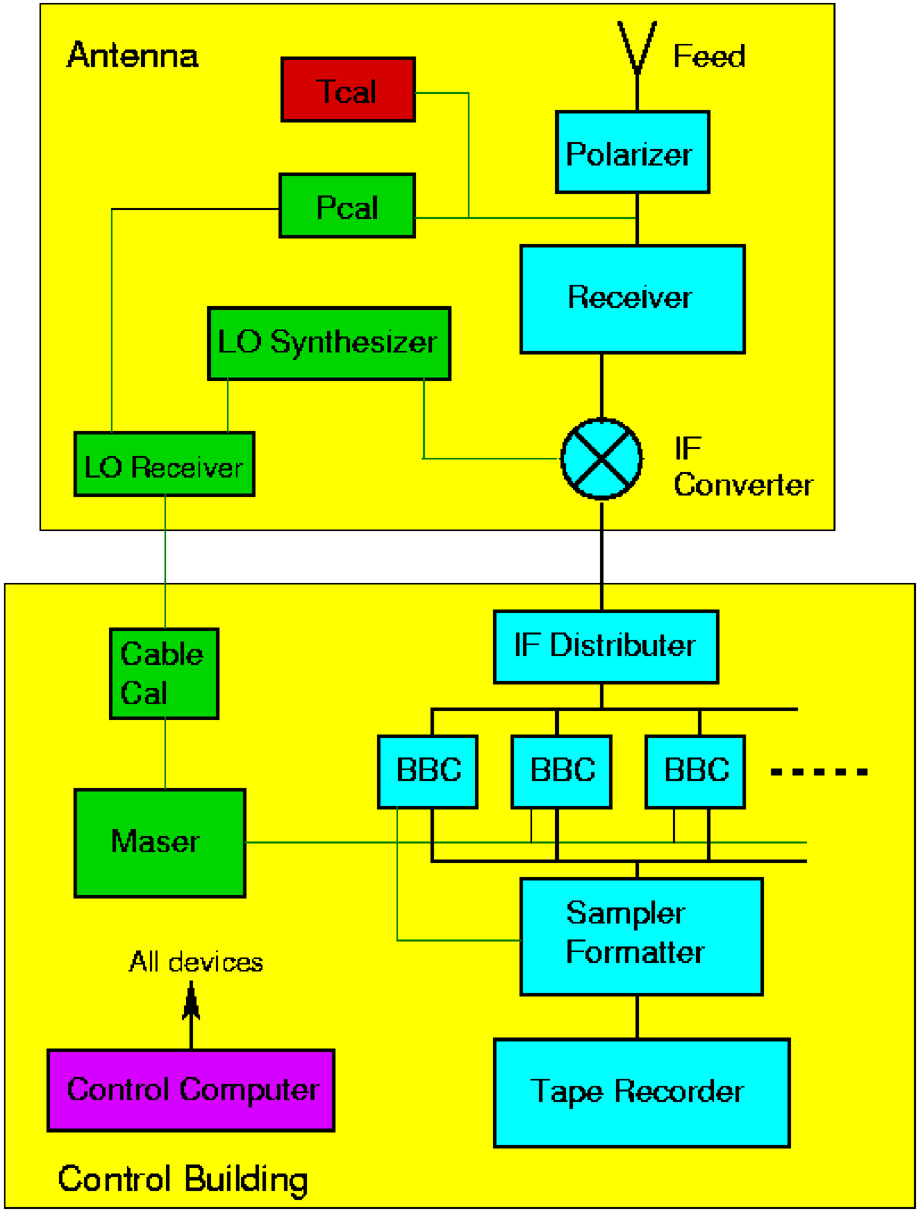


- Map from GSFC
- Many stations missing from
  - Europe
  - China
  - Japan
  - Korea
  - Australia
  - Puerto Rico
-

VLBA Station Electronics

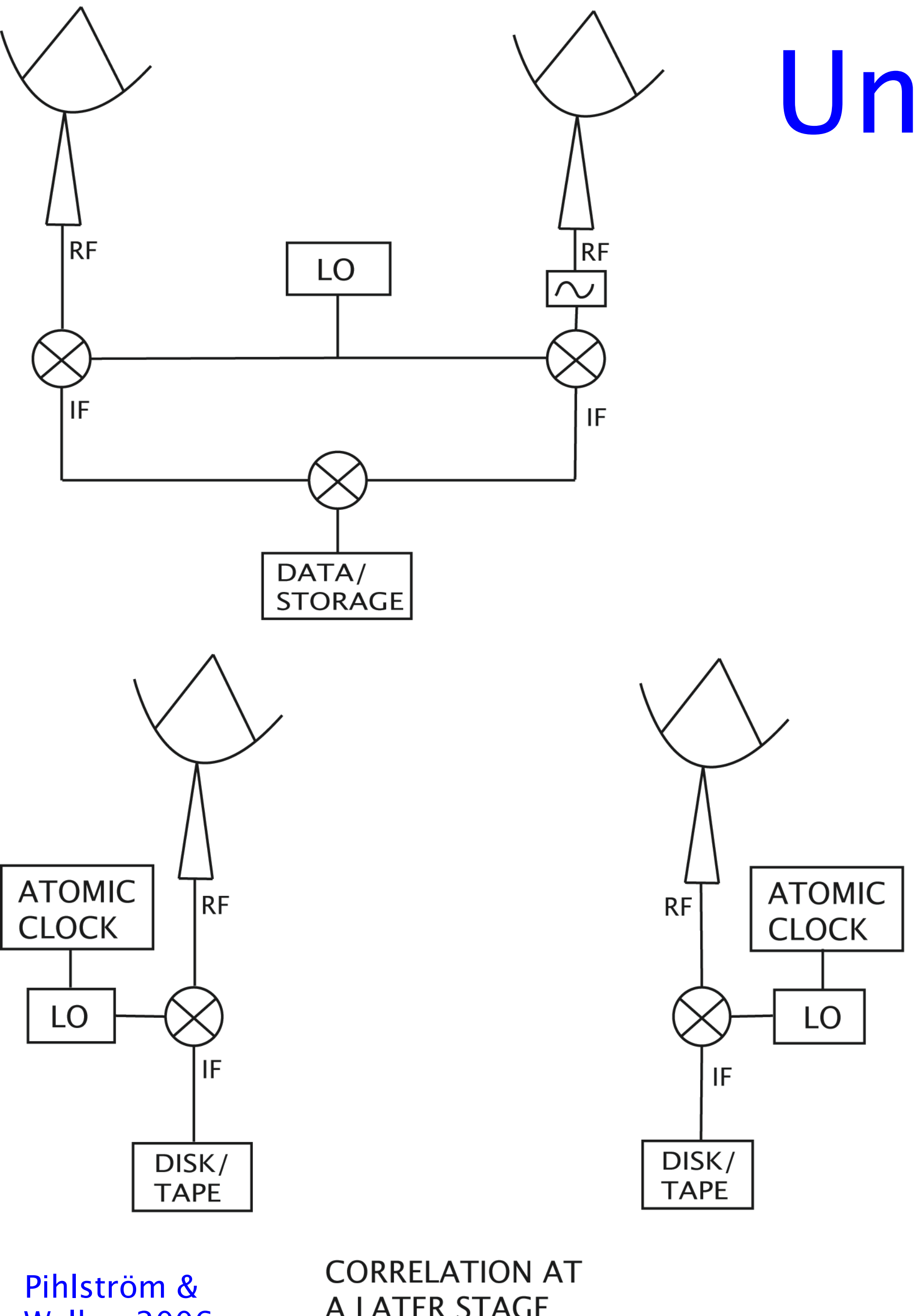
- At antenna:
  - Select RCP and LCP
  - Add calibration signals
  - Amplify
  - Mix to IF (500-1000 MHz)
- In building:
  - Distribute to baseband converters (8)
  - Mix to baseband
  - Filter (0.062 – 16 MHz)
  - Sample (1 or 2 bit)
  - Format for tape (32 track)
  - Record
  - Also keep time and stable frequency
- Other systems conceptually similar
  - LOFAR eliminates many of these elements





Walker 2004

VLBI Telescopes Are Unconnected



Pihlström & Walker 2006

- "Standard" radio interferometers have local oscillators controlled by a single clock
- VLBI stations have independent clocks
  - Stations too far apart
  - Or signal path does not allow coherent propagation
  - Timekeeping critical

VLBI Is Essentially About Time

An interferometer baseline gives information about amplitude and phase. The simple equation for phase was

$$\phi = 2\pi\nu\tau_{\rm g}$$

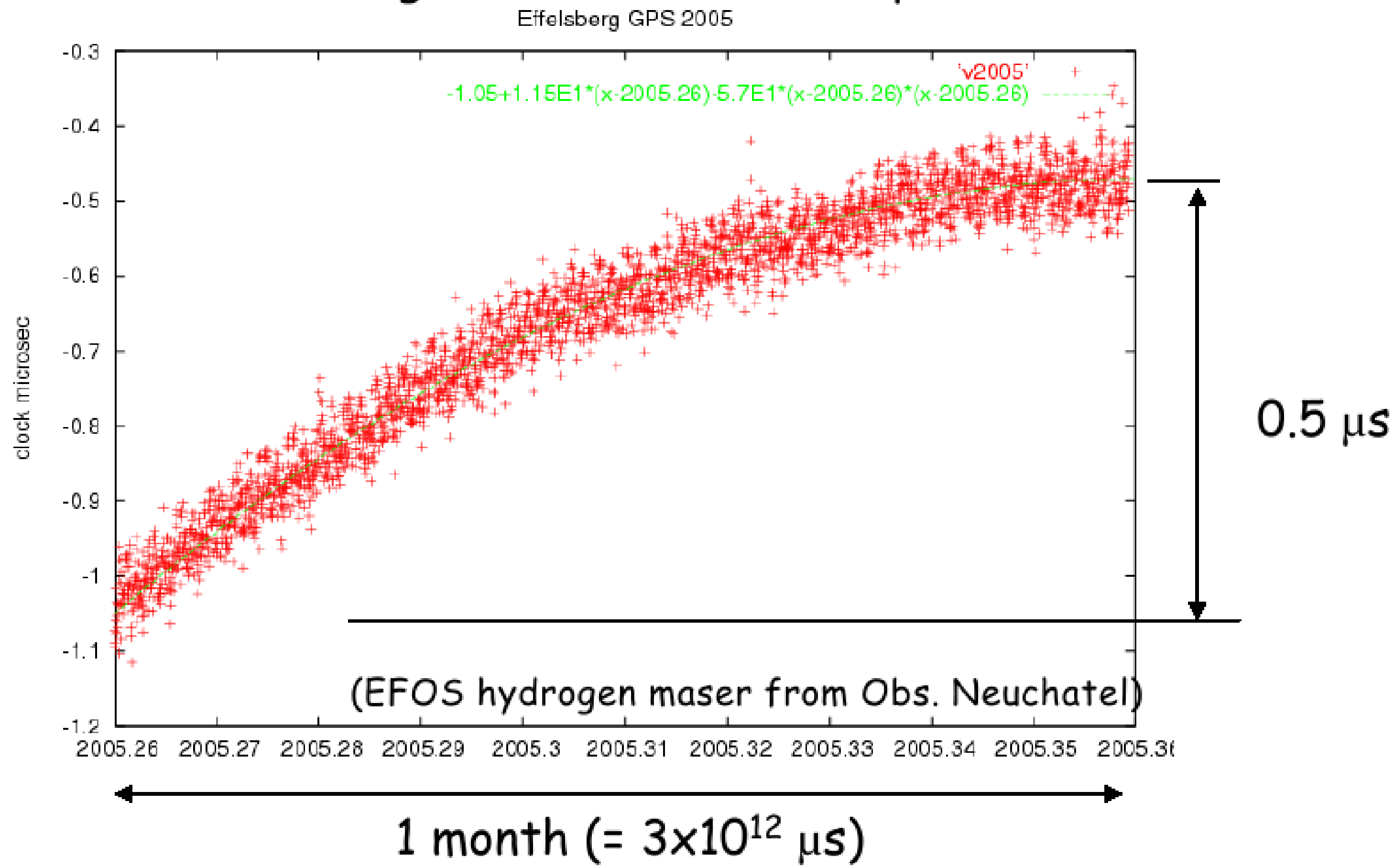
where τ_g was the difference in geometrical delay. For small interferometers, other terms are small enough to be ignored. But for VLBI, this must be expanded to

$$\phi_{1,2}(\nu,t) = 2\pi\nu\left[(\tau_{1,\rm g} + \tau_{1,\rm c} + \tau_{1,\rm tele} + \tau_{1,\rm tropo,dry} + \tau_{1,\rm tropo,wet} + \tau_{1,\rm iono}) - (\tau_{2,\rm g} + \tau_{2,\rm c} + \tau_{2,\rm tele} + \tau_{2,\rm tropo,dry} + \tau_{2,\rm tropo,wet} + \tau_{2,\rm iono})\right]$$

where τ_g is the geometric delay relative to some reference (typically the center of the Earth), τ_c is the clock offset, τ_{tele} is an additional instrumental delay (cable and electronic delays, antenna flexure, and so on), τ_{tropo} is atmospheric delay, and τ_{iono} is ionospheric delay. All τ's depend on ν and t.

Clock Accuracy and Stability

Effelsberg maser - GPS time, April 2005



Rate = 0.5 μs / 3×10^{12} μs = 1.7×10^{-13} s/s

Compare to correlator delay window: ~ 1 μs

Drift due to cavity frequency change (due temperature, ...)

Roy 2005

The Delay Model (CALC)

For 8000 km baseline
1 mas = 3.9 cm
= 130 ps

Adapted from Sovers, Fanselow, and Jacobs Reviews of Modern Physics, Oct 1998

Item	Approx Max.	Time scale
Zero order geometry.	6000 km	1 day
Nutation	$\sim$ 20”	< 18.6 yr
Precession	$\sim$ 0.5 arcmin/yr	years
Annual aberration.	20"	1 year
Retarded baseline.	20 m	1 day
Gravitational delay.	4 mas @ 90° from sun	1 year
Tectonic motion.	10 cm/yr	years
Solid Earth Tide	50 cm	12 hr
Pole Tide	2 cm	$\sim$1 yr
Ocean Loading	2 cm	12 hr
Atmospheric Loading	2 cm	weeks
Post-glacial Rebound	several mm/yr	years
Polar motion	0.5 arcsec	$\sim$ 1.2 years
UT1 (Earth rotation)	Several mas	Various
Ionosphere	$\sim$ 2 m at 2 GHz	All
Dry Troposphere	2.3 m at zenith	hours to days
Wet Troposphere	0 – 30 cm at zenith	All
Antenna structure	<10 m. 1cm thermal	—
Parallactic angle	0.5 turn	hours
Station clocks	few microsec	hours
Source structure	5 cm	years

Walker 2004

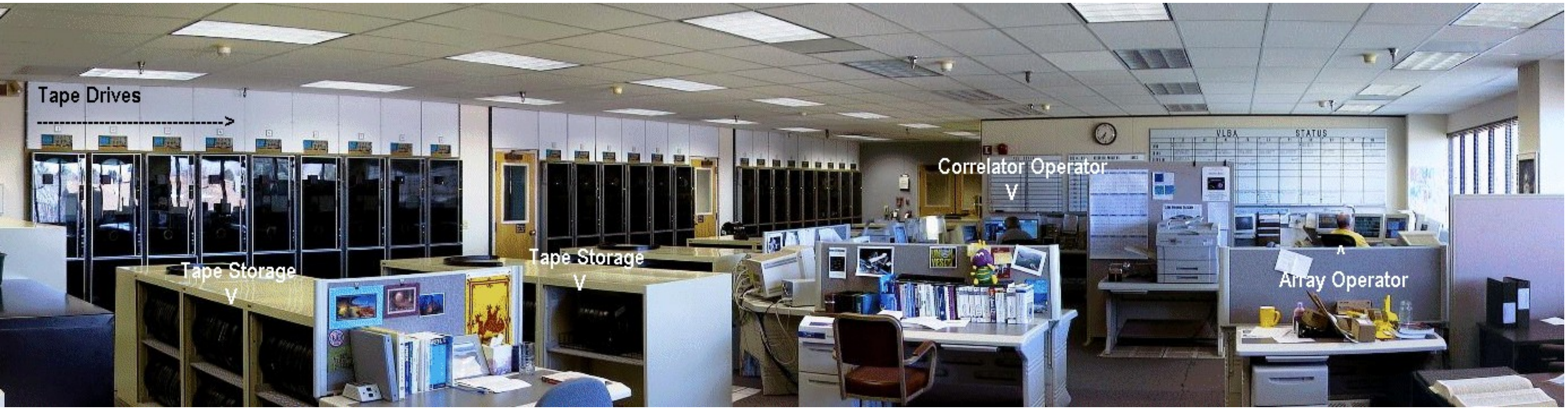
VLBI Correlator

- Read tapes or disks or get signals in real-time over network
- Synchronize data
  - Apply delay model
  - Correct for known Doppler shifts
    - Mainly from Earth rotation
    - This is the total fringe rate and is related to the rate of change of delay
- Generate cross and auto correlation power spectra
  - FX: FFT or filter, then cross multiply (VLBA, Nobeyama, ATA, GMRT, LOFAR)
  - XF: Cross multiply lags. FFT later (JIVE, Haystack, VLA, EVLA, ALMA …)
- Accumulate and write data to archive
- Some corrections may be required in postprocessing
  - Data normalization and scaling (Varies by correlator)

Walker 2004

VLBI Correlators

JIVE (Dwingeloo, current)



VLBA (Socorro)

VLBI Amplitude Calibration

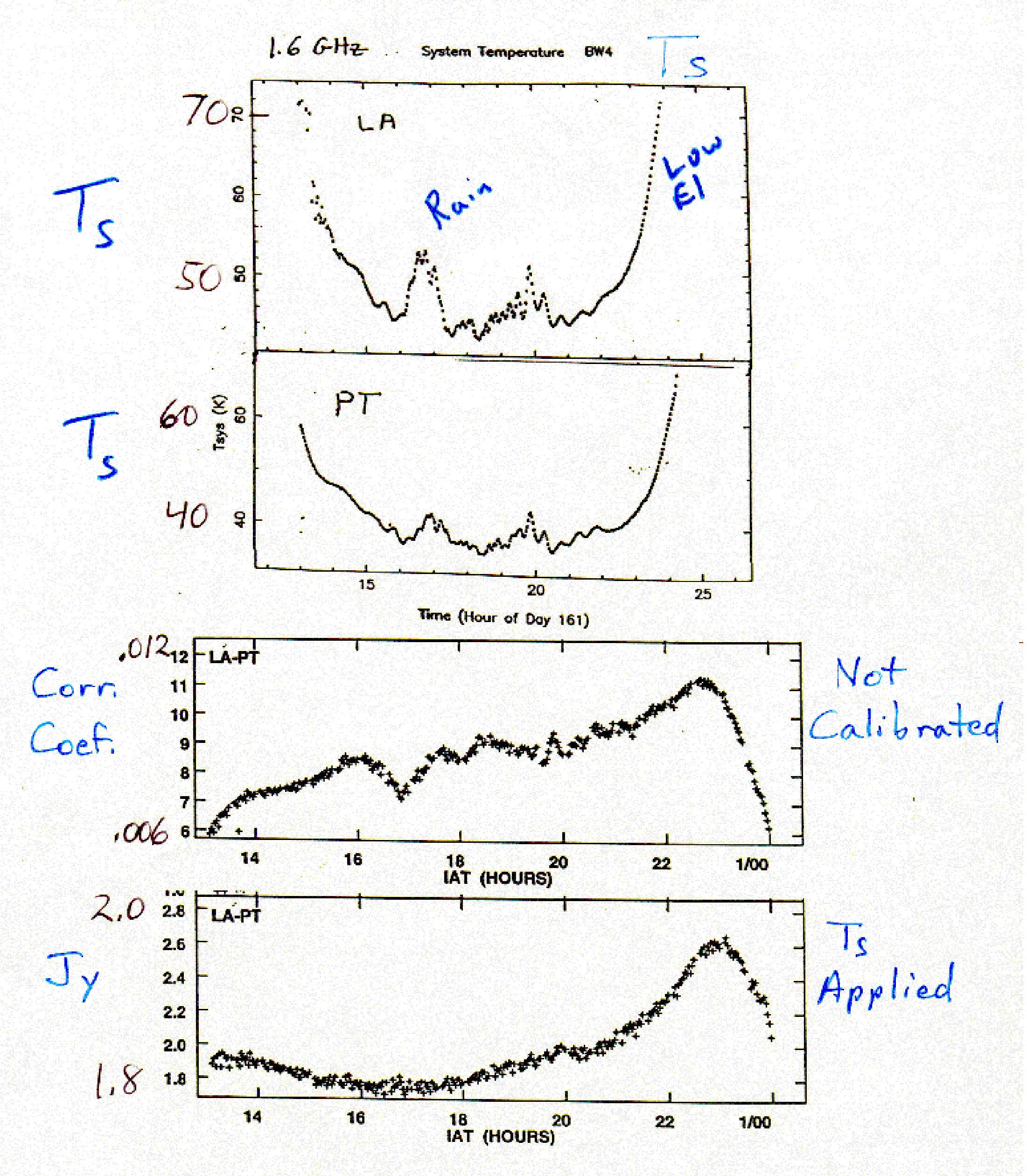
$$S_{cij} = \rho \frac{A}{\eta_s} \sqrt{\frac{T_{si} T_{sj}}{K_i K_j e^{-\tau_i} e^{-\tau_j}}}$$

- S_{cij} = Correlated flux density on baseline i – j
  ρ = Measured correlation coefficient
- A = Correlator specific scaling factor
  η_s = System efficiency including digitization losses
- T_s = System temperature
  - Includes receiver, spillover, atmosphere, blockage
- K = Gain in degrees K per Jansky
  - Includes gain curve
- $e^{-\tau}$ = Absorption in atmosphere plus blockage
- Note $T_s/K = SEFD$ (System Equivalent Flux Density)

LOFAR may require different calibration method as T_{sys} is not really measured

Walker 2004

Calibration With Tsys



Example shows removal of effect of increased Ts due to rain and low elevation

Walker 2004

Gain Curves

VLBA:

Caused by gravitationally induced distortions of antenna

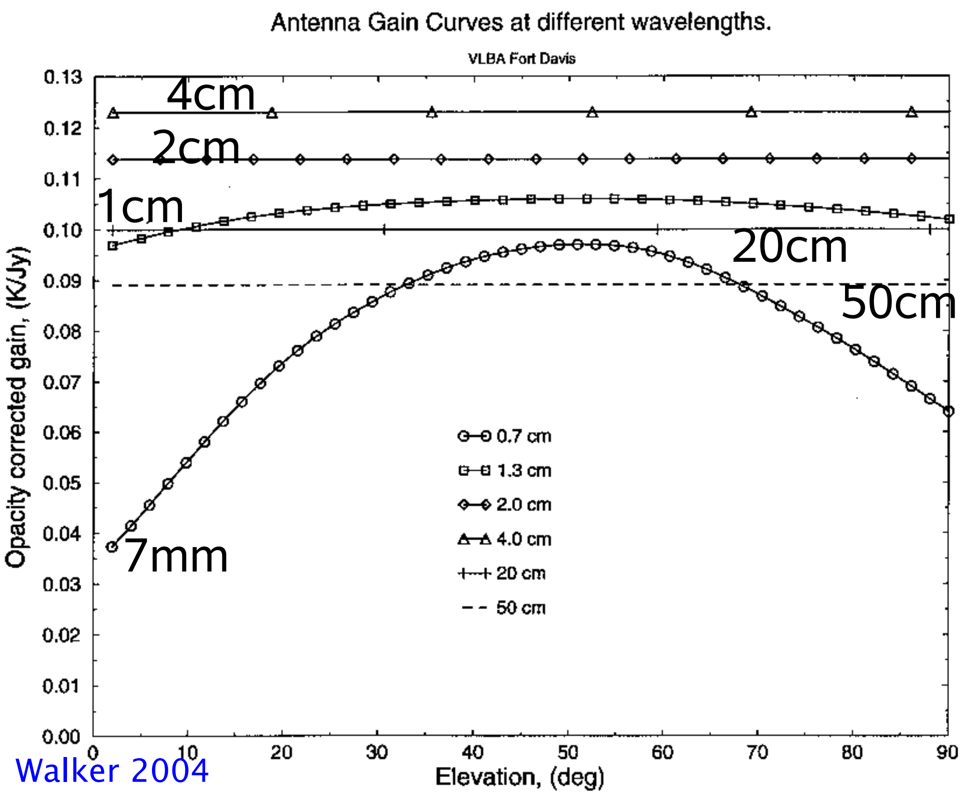
Function of elevation, depends on frequency

LOFAR:

From S. Yatawatta

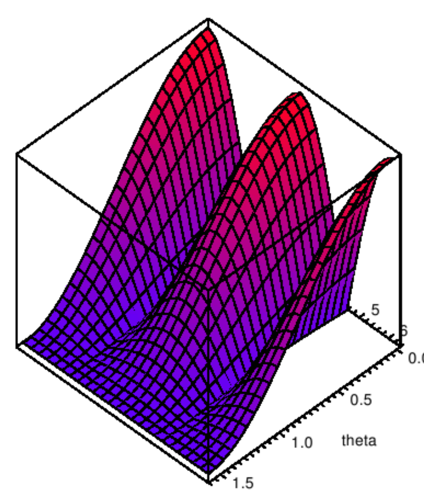
Bent dipole response

Function of azimuth, elevation, depends on frequency

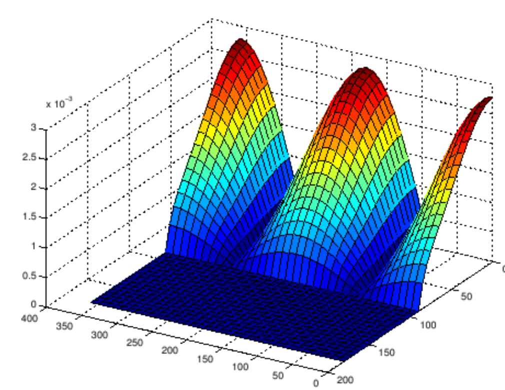


Walker 2004

Droopy Dipole Beam



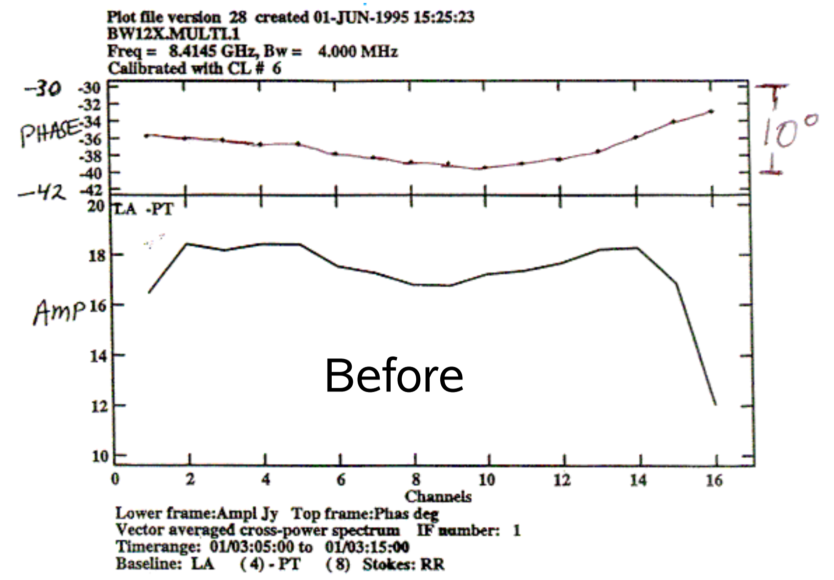
$|E_\theta|$

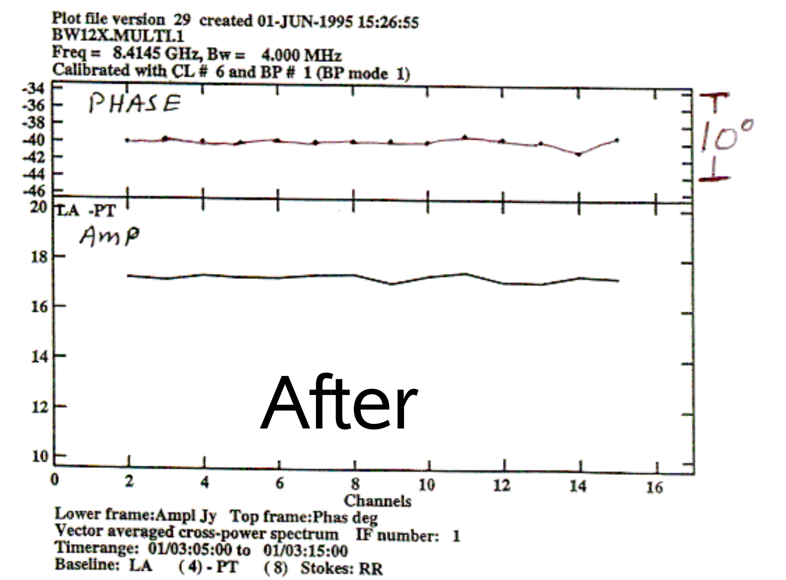


$|E_\theta|$ (M. Arts)

Bandpass Calibration

- Based on bandpass calibration source
  - Effectively a self-cal on a per-channel basis
  - Needed for spectral line calibration
  - May help continuum calibration by reducing closure errors
  - Affected by high total fringe rates
    - Fringe rate shifts spectrum relative to filters
    - Bandpass spectra must be shifted to align filters when applied
    - Will lose edge channels in process of correcting for this.





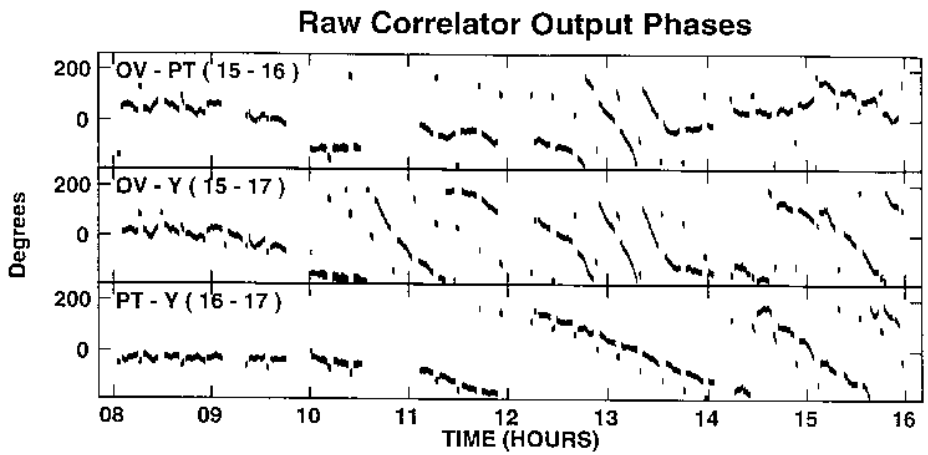
Walker 2004

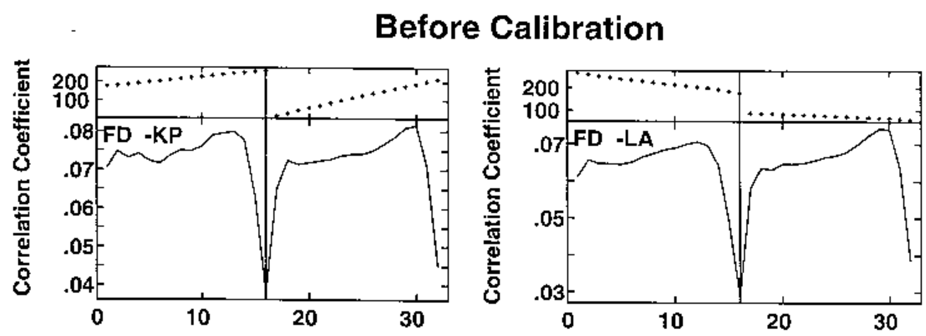
Fringe Fitting

Raw correlator output has phase slopes in time and frequency

- Slope in time is “fringe rate”
  - Usually from imperfect troposphere or ionosphere model
- Slope in frequency is “delay”
  - A phase slope because $\phi = 2\pi\upsilon\tau$
  - Fluctuations worse at low frequency because of ionosphere
  - Troposphere affects all frequencies equally ("nondispersive")

Fringe fit is self calibration with first derivatives in time and frequency





Walker 2004

Fringe Fitting: Why

- For Astronomy:
  - Remove clock offsets and align baseband channels
    - Done with 1 or a few scans on a strong source
    - Could use bandpass calibration if smearing corrections were available
  - Fit calibrator to track most variations (optional)
  - Fit target source if strong (optional)
  - Used to allow averaging in frequency and time
    - Allows higher SNR self calibration (longer solution, more bandwidth)
  - Allows corrections for smearing from previous averaging
  - Fringe fitting weak sources rarely needed any more
- For geodesy:
  - Fitted delays are the primary "observable"
  - Slopes are fitted over wide spanned frequency range
    - "Bandwidth Synthesis"
  - Correlator model is added to get "total delay", independent of models

Walker 2004

Fringe Fitting: Theory

Interferometer phase	$\phi_{t,\nu} = 2\pi\nu\tau_t$
Phase error	$d\phi_{t,\nu} = 2\pi\nu d\tau_t$
Linear phase model	$\Delta\phi_{t,\nu} = \phi_0 + (\delta\phi/\delta\nu)\Delta\nu + (\delta\phi/\delta t)\Delta t$

- Determining the delay and rate errors is called "fringe fitting"
- Fringe fit is self calibration with first derivatives in time and frequency

Pihlström & Walker 2006

Fringe Fitting: How

- Two step process (usually)
  - 2D FFT to get estimated rates and delays to reference antenna
    - Required for start model for least squares
    - Can restrict window to avoid high sigma noise points
    - Can use just baselines to reference antenna or can stack 2 and even 3 baseline combinations
  - Least squares fit to phases starting at FFT estimate
- Baseline fringe fit
  - Not affected by poor source model
  - Used for geodesy. Noise more accountable.
- Global fringe fit
  - One phase, rate, and delay per antenna
  - Best SNR because all data used
  - Improved by good source model
  - Best for imaging and phase referencing
- Standard fringe fitting algorithms are extremely computationally expensive
  - LOFAR and SKA must find improved algorithms to cope with data rate

Walker 2004

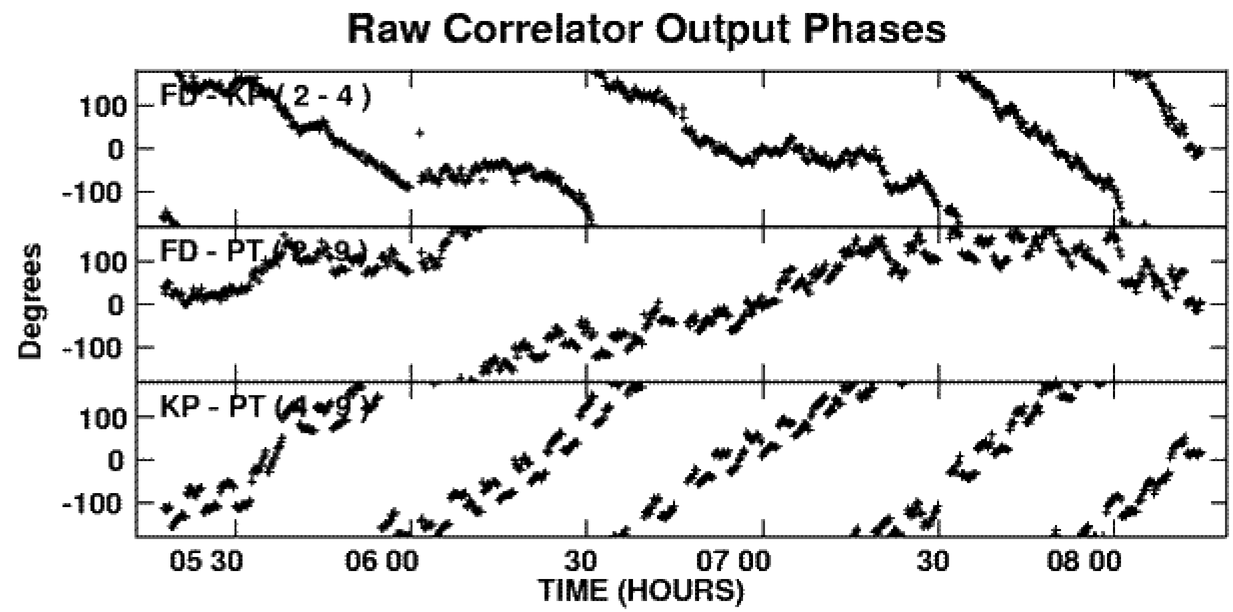
Phase Referencing

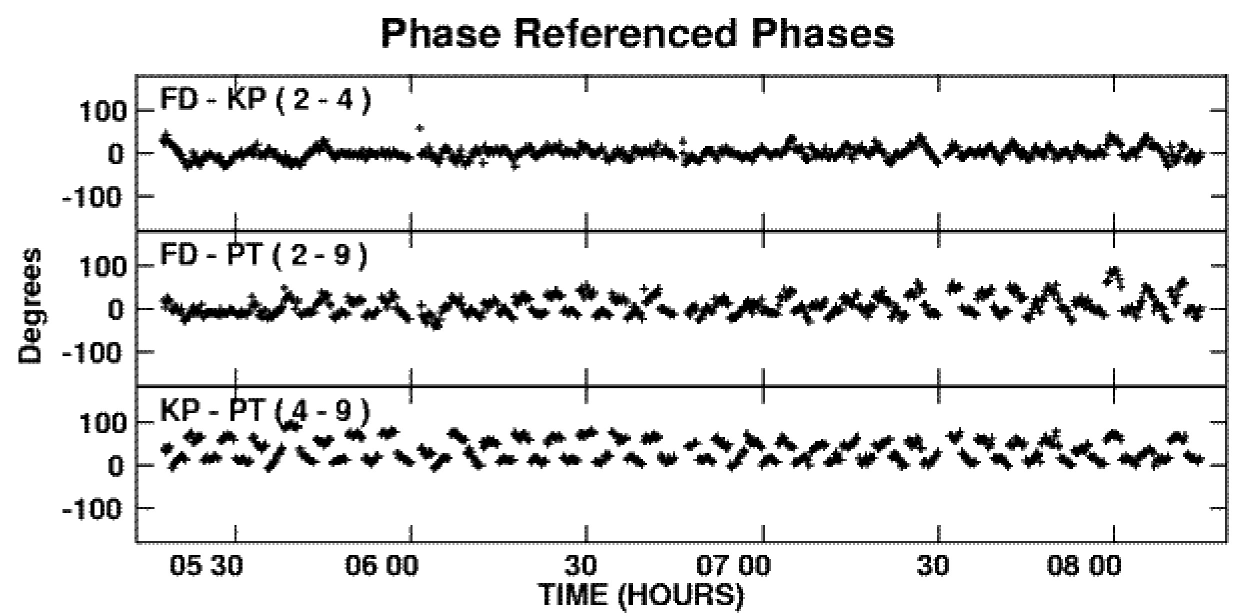
- Calibration using phase calibrator outside target source field
  - Nodding calibrator (move antennas)
  - In-beam calibrator ~~(separate correlation pass)~~
  - Multiple calibrators for most accurate results – get gradients
- Similar to VLA calibration except:
  - Geometric and atmospheric models worse
    - Affected by totals between antennas, not just differentials
    - Model errors usually dominate over fluctuations
    - Errors scale with total error times source-target separation in radians
  - Need to calibrate often (5 minute or faster cycle)
  - Need calibrator close to target (< 5 deg)
- Biggest problems:
  - Wet troposphere at high frequency
  - Ionosphere at low frequencies (20 cm is as bad as 1cm)
- Used for weak sources and for position measurements
  - Increases sensitivity by 1 to 2 orders of magnitude
  - Used by about 30-50% of VLBA observations

Walker 2004

Phase Referencing Example

- 6 min cycle – 3 on each source
- Phases of one source self-calibrated (near zero)
  - Fourier transform of point source at center has zero phase
- Other source shifted by same amount





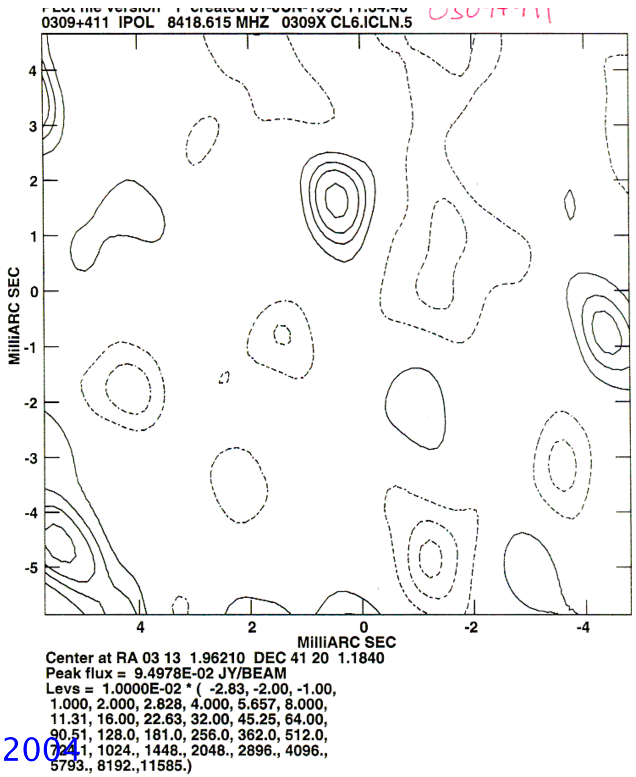
Walker 2004

Phase Referencing Example II

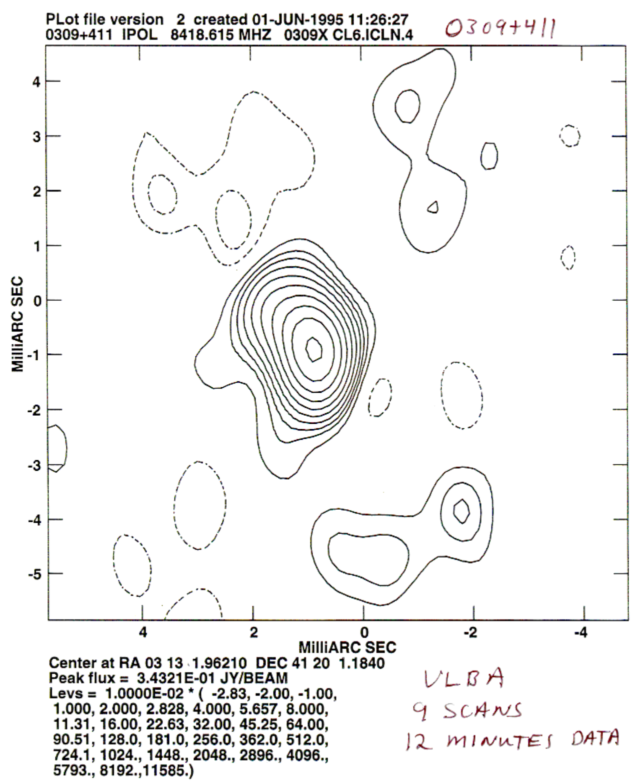
With no phase calibration, source is not detected (no surprise)

With reference calibration, source is detected, but structure is distorted (target-calibrator separation is probably not small)

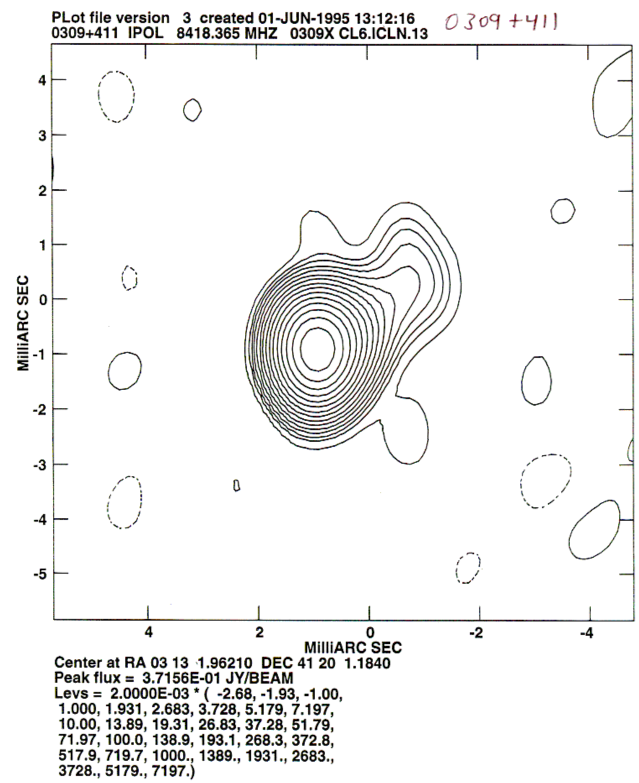
No Phase Calibration



Reference Calibration



Self-calibration



Walker 2004

Self Calibration for Imaging

- Iterative procedure to solve for both image and gains:
  - Use best available image to solve for gains (can start with point)
  - Use gains to derive improved image
  - Should converge quickly for simple sources
    - Many iterations (~50–100) may be needed for complex sources
    - May need to vary some imaging parameters between iterations
    - Should reach near thermal noise in most cases
  - Can image even if calibration is poor or nonexistent
- Possible because there are N antenna gains and $N(N-1)/2$ baselines
  - Need at least 3 antennas for phase gains, 4 for amplitude gains
  - Works better with many antennas
- Does not preserve absolute position or flux density scale
  - Gain normalization usually makes this problem minor
- Is required for highest dynamic ranges on all interferometers

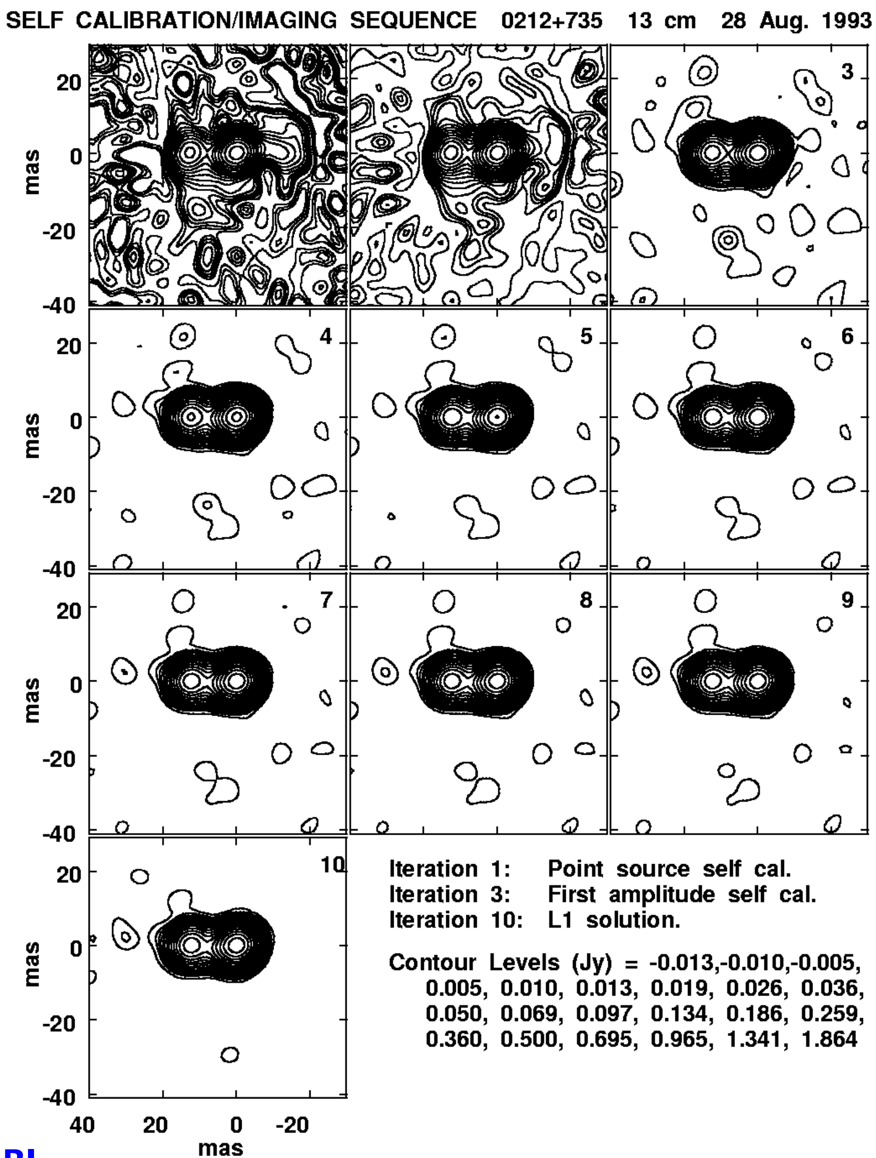
Walker 2004

Self Calibration Imaging Sequence

Iterative procedure to solve for both image and gains:

- Use best available image to solve for gains
  - One often starts with a point source model
- Use gains to derive improved image
- Should converge quickly for simple sources

Does not preserve absolute position or flux density scale



Walker 2004

Questions Before Lunch?

LUNCH

Outline For After Lunch
(for those whose stay awake)

- Interferometry fundamentals
- VLBI
- lunch
- More VLBI
- Ionospheric calibration
- Wide fields of view

The Effect of Bandwidth

(from R Perley)

Real interferometers must accept a range of frequencies (amongst other things, there is no power in an infinitesimal bandwidth)! So we now consider the response of our interferometer over frequency.

To do this, we first define the frequency response functions, G(ν), as the amplitude and phase variation of the signals paths over frequency.

Then integrate:

$$V = \frac{1}{\Delta\nu} \int_{\nu-\Delta\nu/2}^{\nu+\Delta\nu/2} I_\nu(s) G_1(\nu) G_2^*(\nu) e^{2\pi i \nu \tau_g} d\nu$$

The Effect of Bandwidth II

If the source intensity does not vary over frequency width, we get

$$V = \iint I_\nu(\boldsymbol{s}) \sin c(\tau_g \Delta\nu) e^{-2i\pi\nu_0 \tau_g} d\Omega$$

where it is assumed the G(ν) are square, real, and of width Δν. The frequency ν_0 is the mean frequency within the bandwidth.

Perley 2006

The Bandwidth/FOV limit

This shows that the source emission is attenuated by the function sinc(x), known as the 'fringe-washing' function. Noting that $\tau_g \sim (D/c)\sin(\theta) \sim D\theta/\lambda\nu \sim (\theta/\theta_s)/\nu$, we see that the attenuation is small when

$$\frac{\Delta\nu}{\nu}\frac{\theta}{\theta_s} << 1$$

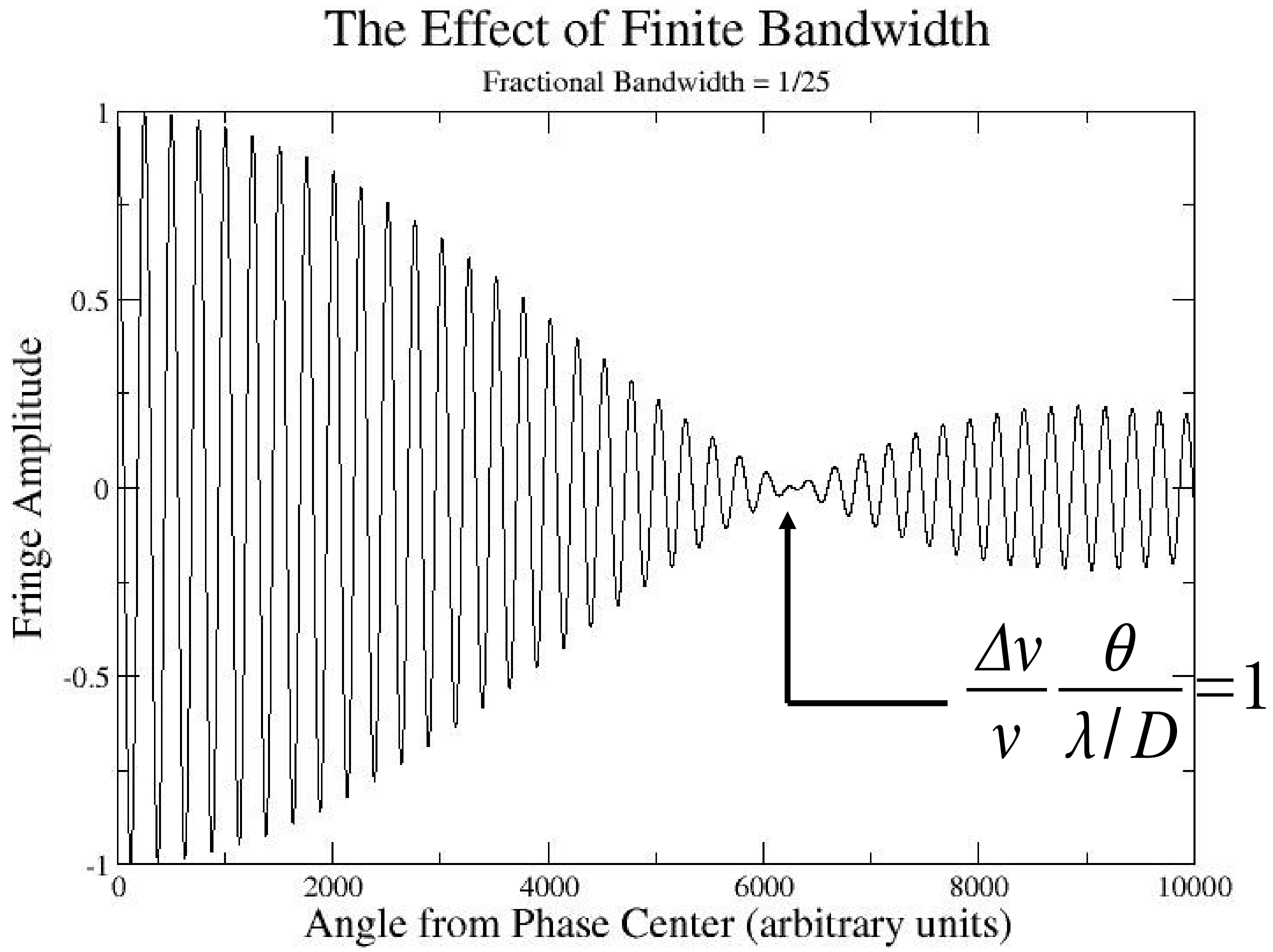
The ratio $\Delta\nu/\nu$ is the fractional bandwidth. The ratio θ/θ_s is the source offset in units of the fringe separation, λ/D.

In words, this says that the attenuation is small if the fractional bandwidth times the angular offset in resolution units is less than unity. Significant attenuation of the measured visibility is to be expected if the source offset is comparable to the interferometer resolution divided by the fractional bandwidth.

Perley 2006

Bandwidth Effect Example

Finite Bandwidth causes loss of coherence at large angles, because the amplitude of the interferometer fringes are reduced with increasing angle from the delay center.



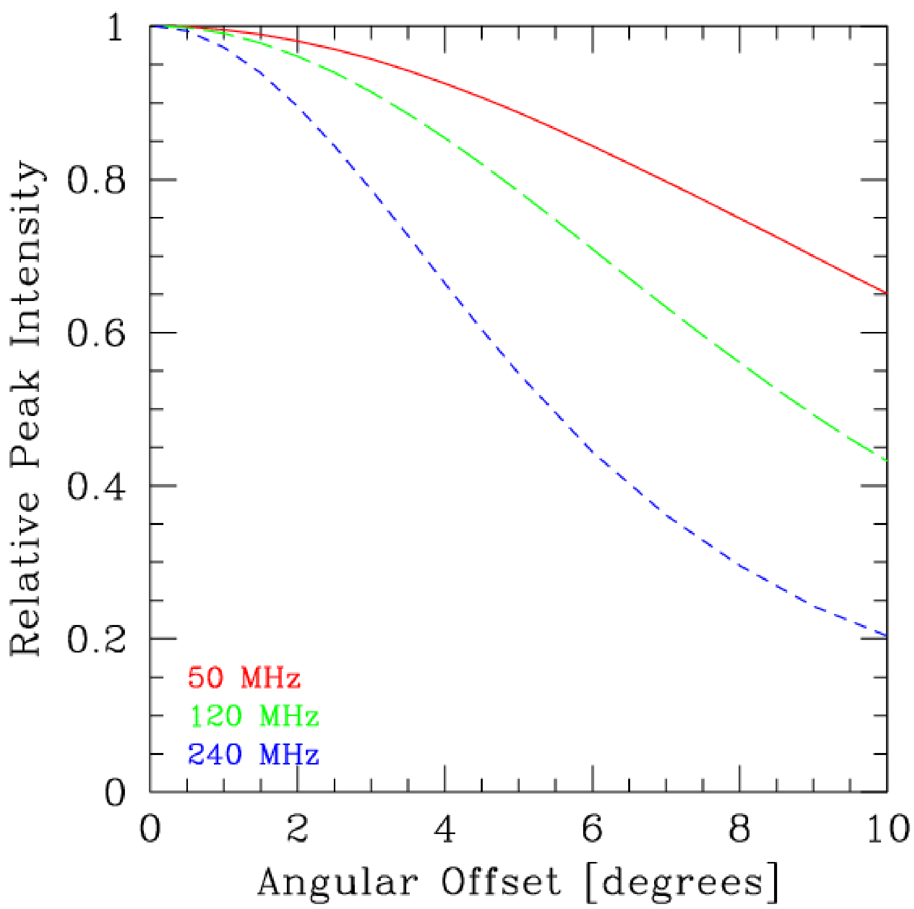
Perley 2006

Avoiding Bandwidth Losses

- Although there are computational methods which allow recovery of the lost amplitude, the loss in SNR is unavoidable.
- The simple solution is to observe with a small bandwidth. But this causes loss of sensitivity.
- So, the best (but not cheapest!) solution is to observe with LOTS of narrow channels.
- Modern correlators will provide tens to hundreds of thousands of channels of appropriate width. (Huge datasets!)
  - Long baseline LOFAR will probably need channel widths of 1 kHz to image full station beams

Perley 2006

Long Baseline LOFAR Bandwidth Smearing



- Simulations for proposed LOFAR long baseline configuration
- 1 kHz channels
- From Vogt & Anderson draft

Time Average Losses

For sources away from the phase tracking center, the visibility phase rotates with time. Because real interferometers must integrate over finite time intervals (τ_a) in order to measure signals, the visibility amplitudes will be reduced during the averaging.

$$V = \frac{V_0}{\delta t} \int_{-\delta t/2}^{\delta t/2} e^{i 2 \pi \upsilon_f t} dt = \operatorname{sinc}(\upsilon_f \delta t)$$

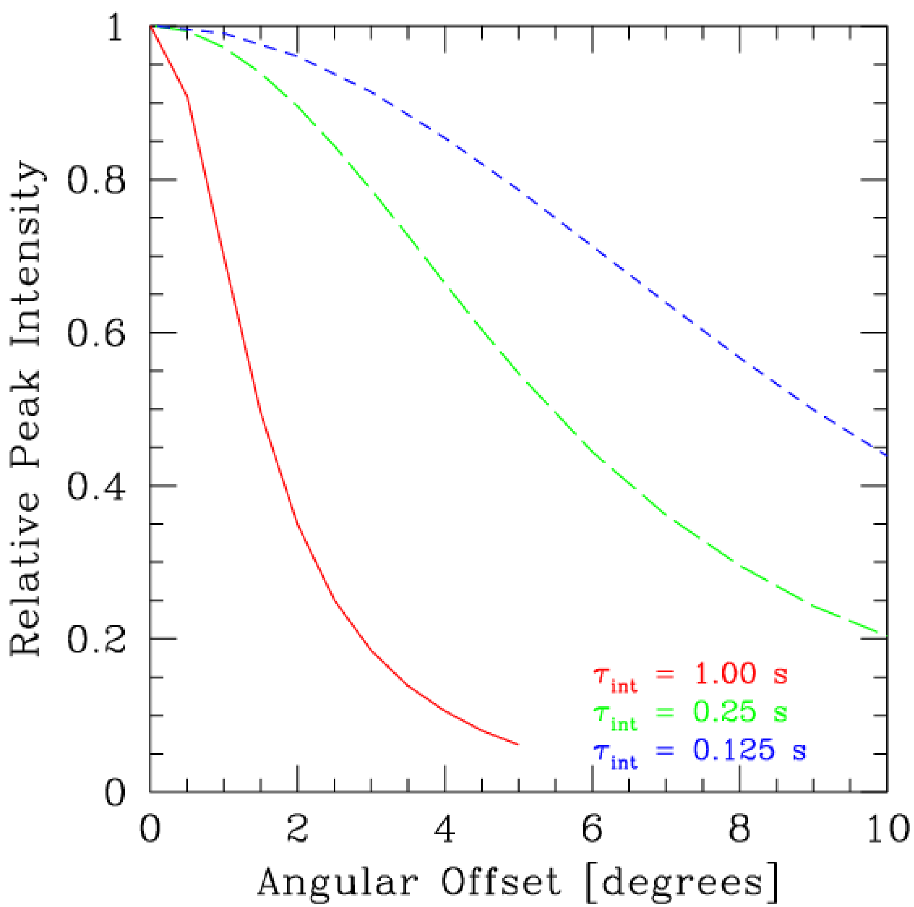
The fractional coherence loss for a source near the pole is:

$$\mathrm{CL} = \frac{\alpha \pi^2}{12} \omega_{\mathrm{E}}^2 \tau_{\mathrm{a}}^2 \left(\frac{\theta}{\theta_{\mathrm{s}}} \right)^2 \sim 10^{-9} \left(\frac{\theta \tau_{\mathrm{a}}}{\theta_{\mathrm{s}}} \right)^2$$

where $\alpha \sim 1$, ω_E is the rotational velocity of the Earth, and θ is the angular distance away from the tracking center. To have coherence losses below 10%, one can image out to $\theta = 10^4/\tau_a$ synthesized beamwidths from the tracking center.

Perley 2006

Long Baseline LOFAR Time-Average Smearing



- Simulations for proposed LOFAR long baseline configuration
- For 240 MHz observations
- From Vogt & Anderson draft

Why Observe to Edge of Primary (Station) Beam?

- Low frequency sky is filled with sources
  - Need to subtract nearby (and far!) bright sources to minimize synthesized sidelobe confusion
  - Lots of sources/flux available to improve calibration
    - More flux means higher S/N
    - Map out spatial gradients of ionosphere, etc.
- Someone else may later use observations to image other sources in beam

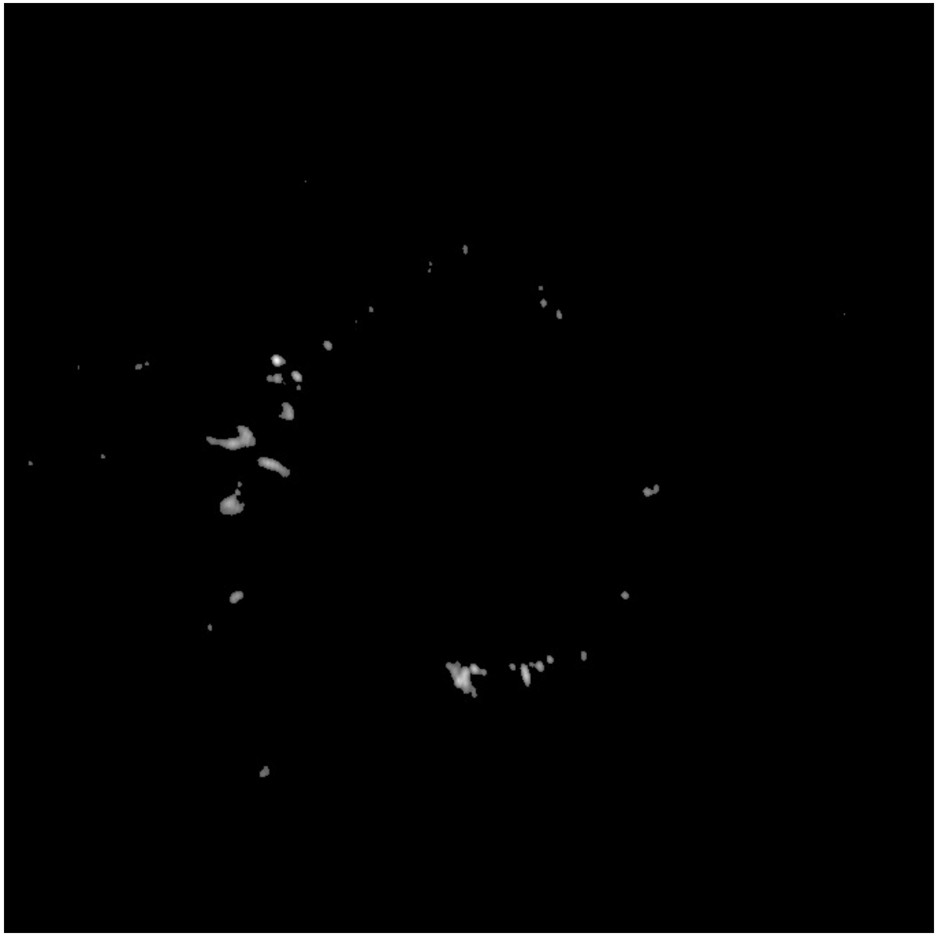
Upgrading the JIVE Correlator

- Efforts are underway to improve the capabilities of the EVN and global VLBI experiments at JIVE
- A new data storage system (PCInt) has been added to handle increased data output rates, allowing for shorter integration times (1/16 s tested already) and higher frequency resolution
- Want to correlate and store data for full primary beam for all experiments
  - Can average data down in time and frequency if user requests
- EXPReS is opening up real-time e-VLBI
  - 512 Mb/s data rates achieved for some stations
  - Network connections to more stations (including Effelsberg) underway
- Planning to build new correlator to handle higher data rates and more telescopes in future

Long Baseline LOFAR

- Baselines of up to ~1000 km
- May need frequency resolution of 1 kHz and time resolution of 0.25 s to image far enough into station beam pattern
- Naively, this results in ~4 GB/s coming out of the correlator for full LOFAR array
- Challenge to handle/process full data rate
- Required data rate orders of magnitude lower for smaller arrays or if channel widths and averaging times relaxed

VLBI Treat: TX Cam Movie



- 73 epoch movie by Gonidakis and Diamond
- 43 GHz SiO maser emission surrounding a Mira variable

The Ionosphere

- Introduction
- Delay
- Calibrating with existing models
- MIM

Based partially on work done for the ALBUS project
In collaboration with R.M. Campbell, M. Mevius, J. Noordam, and H.J. van Langevelde

Comments

- The following discussion is from the point of view of an astronomer who is trying to remove the effects of the ionosphere
- I am not an atmospheric scientist
- I focus mainly on interferometry, which is affected somewhat differently than direct detection ("single-dish") observations.

Why the Ionosphere Matters: Some Math

Following R.M. Campbell's notes:

$$\begin{aligned} n^2 &= (\mu - i\chi)^2 \\ &= 1 - \frac{X}{1 - iZ - \frac{Y_T^2}{2(1-X-iZ)} \pm \left[\frac{Y_T^4}{4(1-X-iZ)^2} + Y_L^2\right]^{1/2}}, \end{aligned}$$

where $X \equiv \nu_{\rm pl}^2/\nu^2$, $Y_T \equiv (\nu_B/\nu)\sin\theta$, $Y_L \equiv (\nu_B/\nu)\cos\theta$, and $Z \equiv \nu_{\rm c}/\nu$. θ is the angle between $\mathbf{B}_\oplus$ and the direction of propagation, $\nu_{\rm pl}$ is the plasma frequency, ν_B is the cyclotron frequency, and $\nu_{\rm c}$ is the collision frequency.

Ignoring collisions, and defining $Y \equiv \nu_B/\nu$,

$$n \simeq \mu \simeq 1 - \frac{X}{2} - \frac{X^2}{8} \pm \frac{XY\cos\theta}{2} - \frac{XY^2}{2}\left(1 - \frac{\sin^2\theta}{2}\right) \pm \frac{X^2 Y\cos\theta}{4} + \cdots$$

Phase Delay Math

Let $K_{\text{pl}}^2 \equiv \frac{e^2}{4\pi^2 \epsilon_0 m_e}$, so that $\nu_{\text{pl}}^2 = K_{\text{pl}}^2 n_e$. Then the phase difference between a signal in vacuum and one traveling through the ionosphere is

$$\Delta\phi \simeq \frac{2\pi\nu}{c} \int_{h=\infty}^{h_0} (1-n)\, d\ell \approx \frac{\pi K_{\text{pl}}^2}{2c\nu} \int_{h=\infty}^{h_0} n_e\, d\ell$$

For 2D ionosphere, make the following assumptions: for a vertical line of sight,

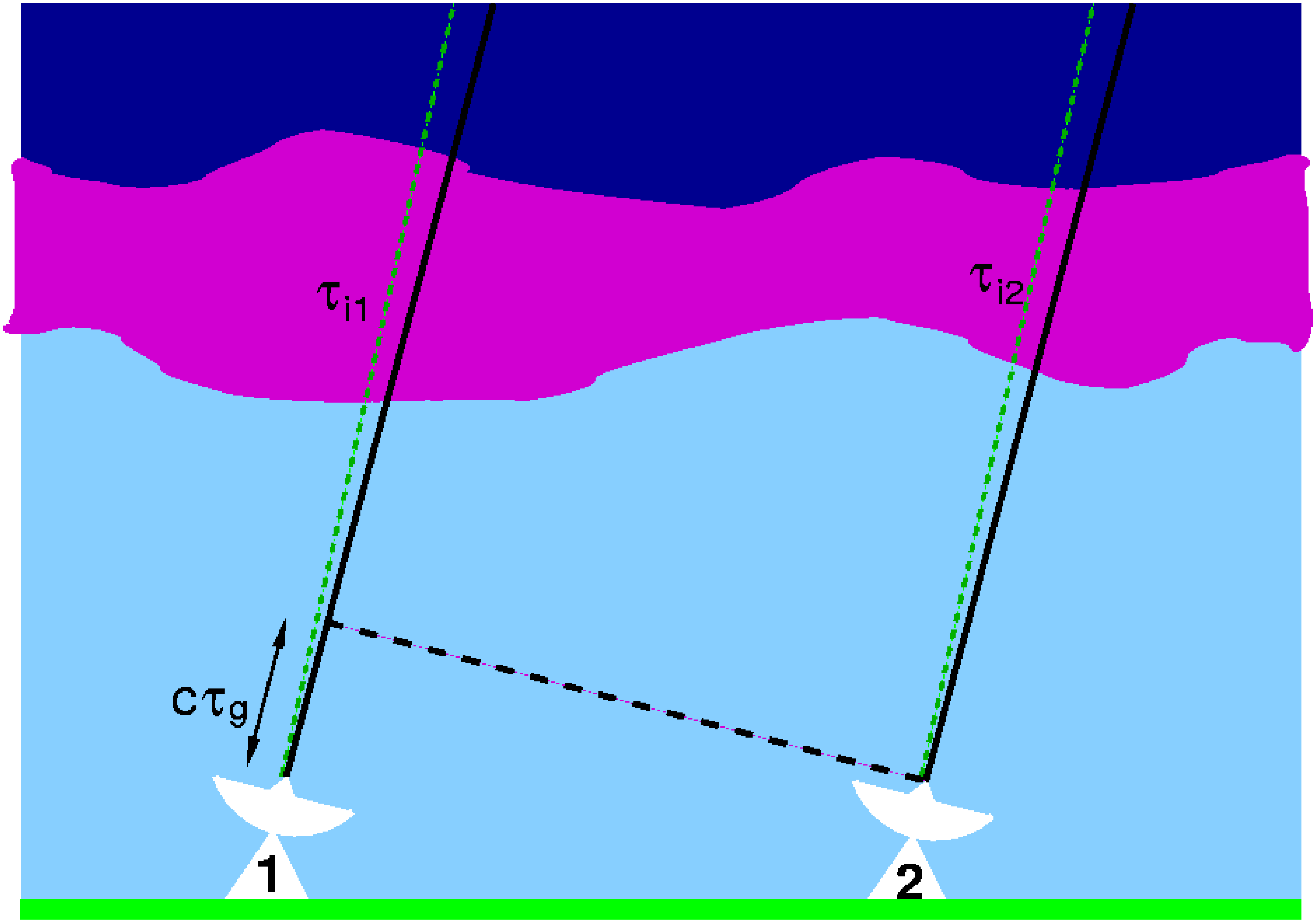
$$\Delta\phi_{\text{vertical}} \approx \frac{\pi K_{\text{pl}}^2}{2c\nu} \int_{h=h_0}^{\infty} n_e\, dh = \frac{C_0}{\nu} VTEC(lat, lon).$$

for a slant line of sight

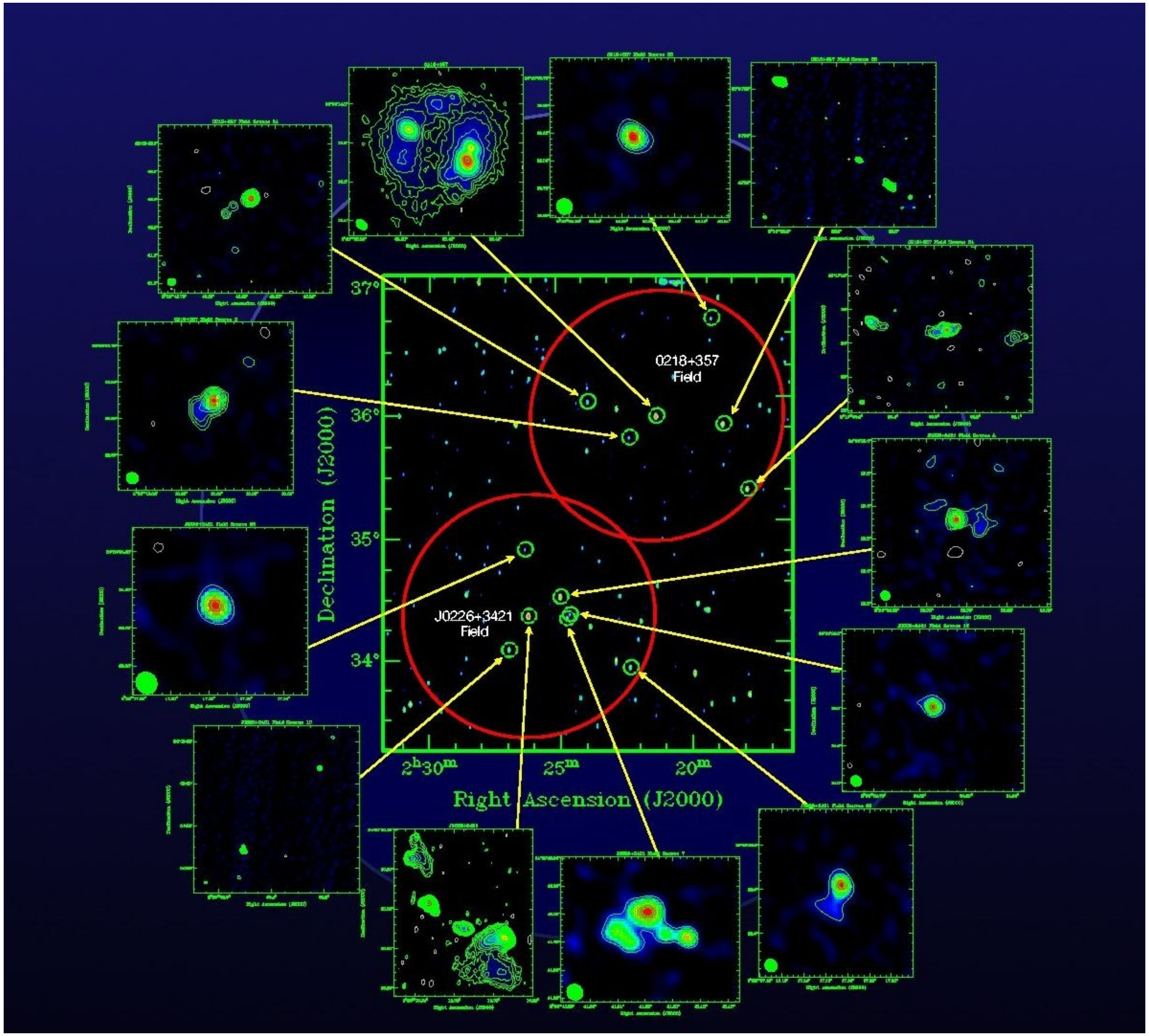
$$\Delta\phi_{\text{slant}} \sim \frac{C_0}{\nu} \quad \frac{VTEC(lat, lon)}{\left\{1 - \left[\frac{(r_\oplus + h_0)\cos(El)}{r_\oplus + h_{\text{iono}}}\right]^2\right\}^{1/2}}$$

Free electrons alter the propagation speed of radio waves, and for interferometry, delay determines apparent direction. So ionospheric delay affects measurements.

Interferometry Basics Plus Ionosphere

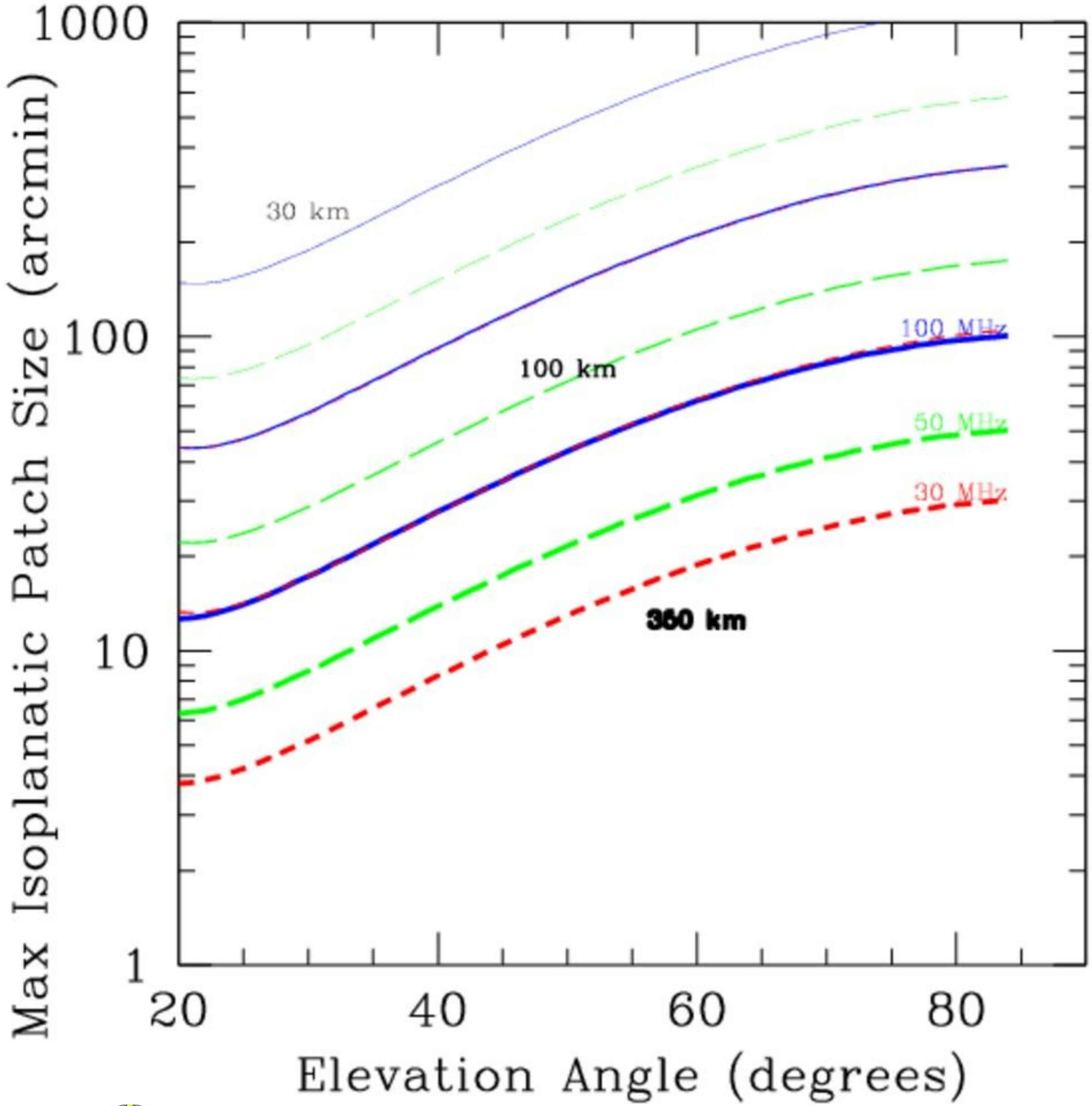


Ionospheric Distortions in Wide-Field Imaging



- 320 MHz observations using the VLBA
- Fringe fitting removed ionospheric delay at field center
- Coherence for other detected sources minimal --- self-calibration needed
- Wide-field/ ionospheric calibration with E. Lenc

LOFAR Isoplanatic Patch Size Model



- Patch size for modest daytime observations
- Overpredicts VLA 74 MHz isoplanatic patch size by factor ~2
- Small scale structure will reduce this size

Ionospheric Effects on Observations

- Radio sources selected from a deep VLA 74 MHz image.
- The individual 30-second maps were compiled as animations of the nine hour measurement, running from nighttime through two hours past sunrise.
- The variations in position, peak intensity, and sidelobe structure show the effects of differential ionospheric effects across the field.

Lazio 2005

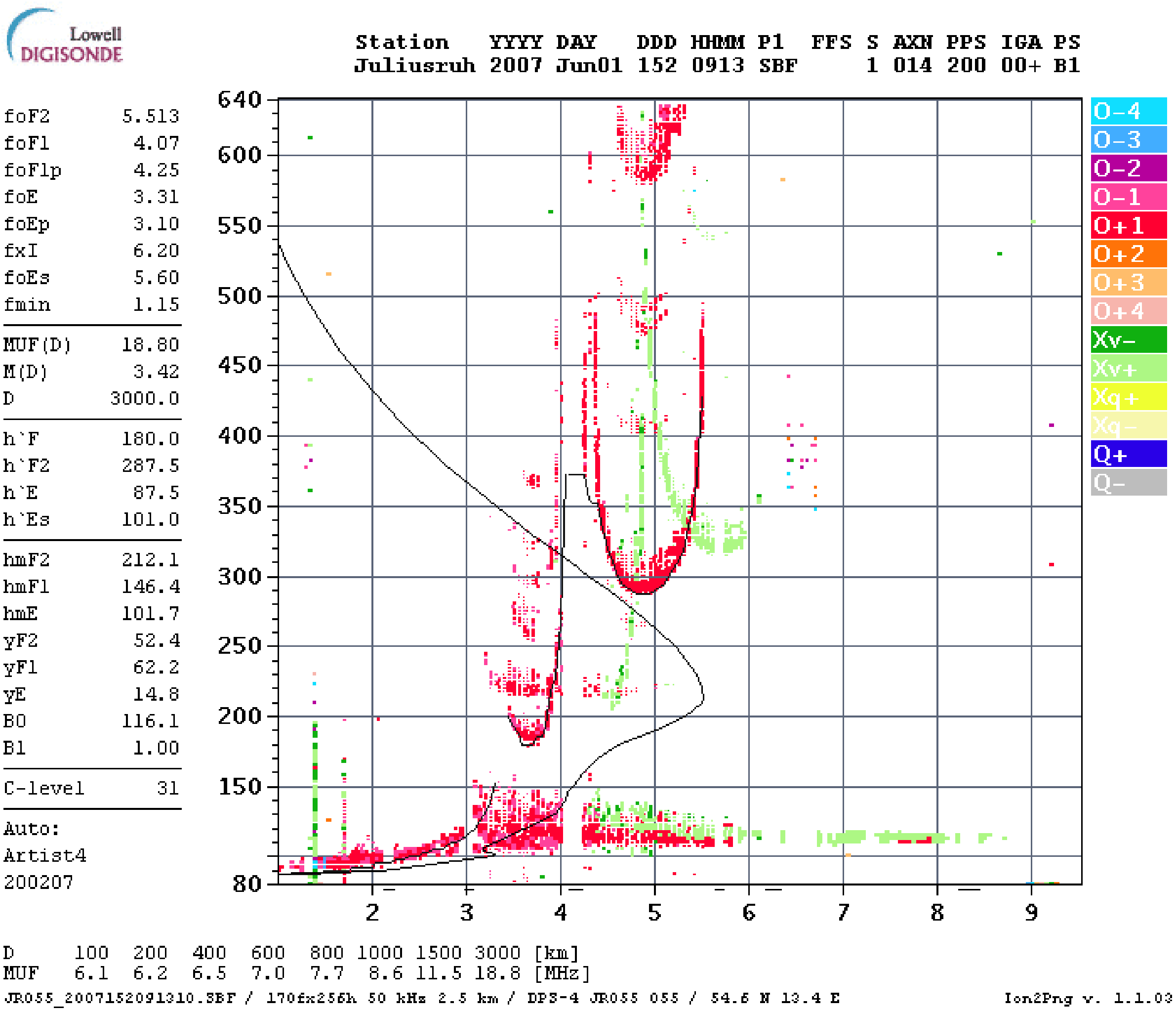
Movie and text from J. Lazio

Calibration Strategy

- Strong Sources
  - Self-calibration
  - Multi-frequency analysis
- Weak Sources
  - Phase Referencing
  - Modeling

At low frequencies modeling the ionosphere is essential to enable imaging of weak objects

Sources of Ionospheric Data: Ionosondes



http://www.iap-kborn.de/radar/Radars/Ionosonde/iono_plots.php

- Low frequency radio waves transmitted into the ionosphere
- Waves reflected when group velocity becomes zero
- Measure density as a function of height

Sources of Ionospheric Data: GPS

http://www.gps.gov

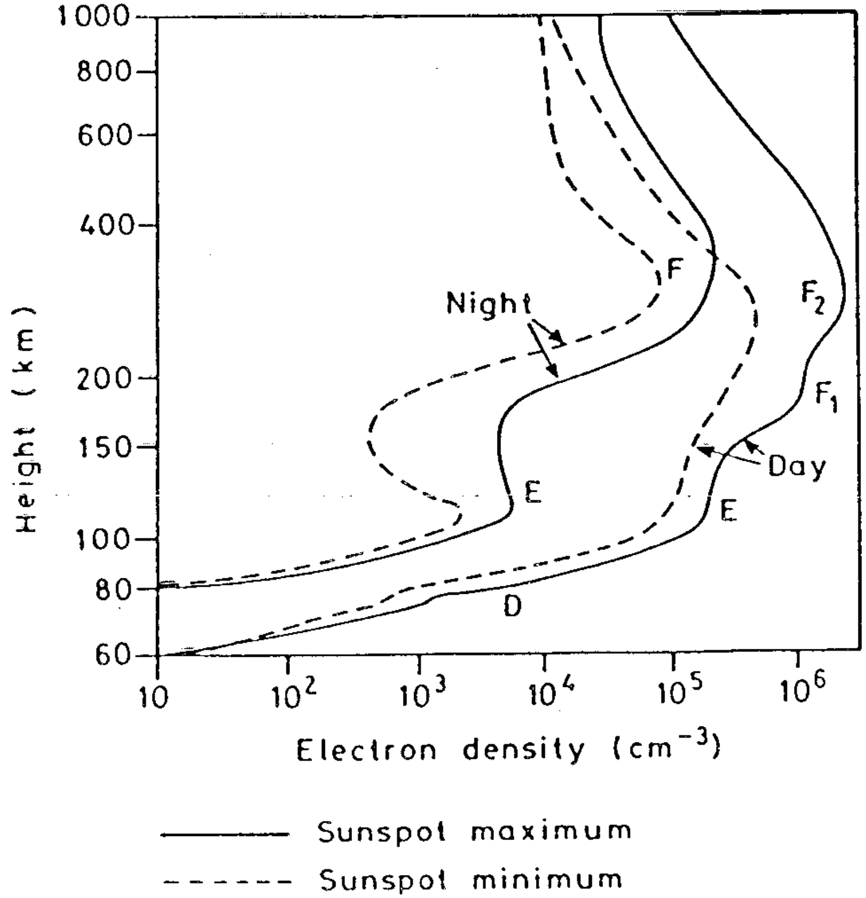
- Global Positioning System satellites broadcast signals at several frequencies in the GHz region
- Special dual frequency receivers can measure differential delay caused by the ionosphere

•Networks of hundreds of such receivers used to measure ionosphere

Sources of Ionospheric Data: Other

- Radar observations
  - Passive radar using existing radio sources (FM radio/television stations)
  - Directed radar and heating experiments
- Optical line imaging
- Lightning research
- Atmospheric chemistry

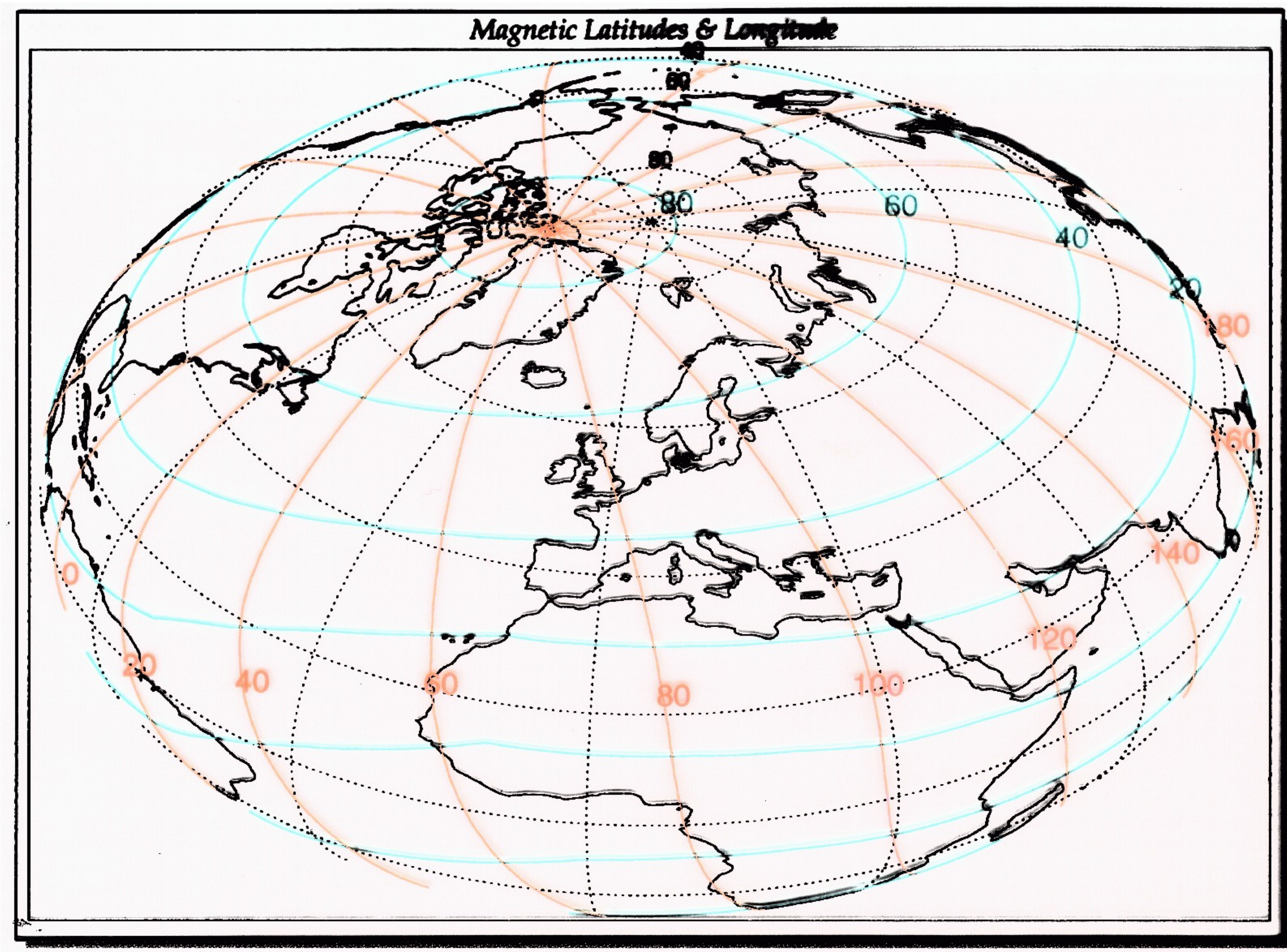
Ionosphere Profile



From R.M. Campbell presentation: authorship unknown

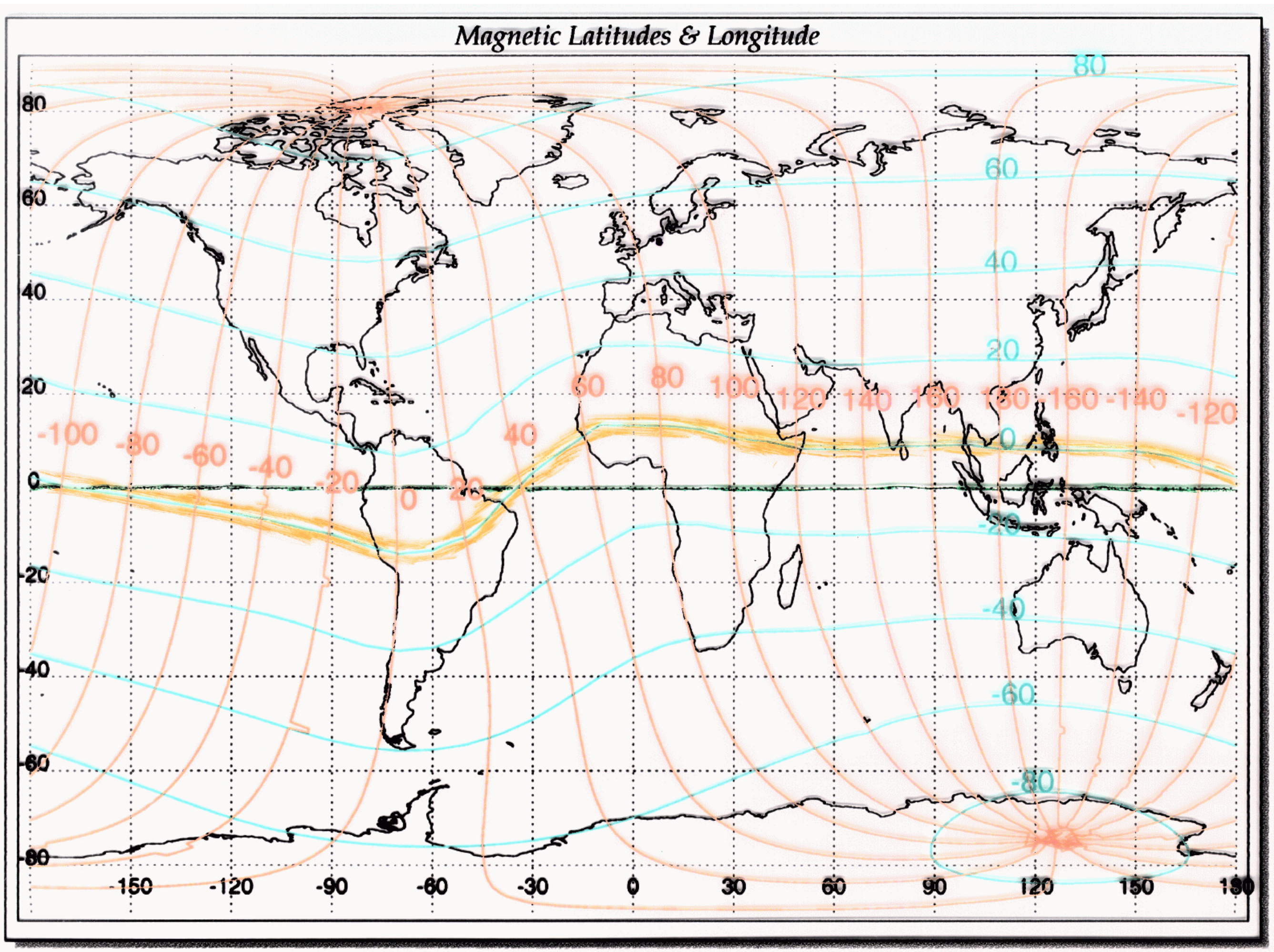
- Solar radiation ionizes atmospheric particles during daytime
- Recombination reduces the electron density during the nighttime
- Number density of neutral particles many orders of magnitude higher
- Peak density around 300 km, but extends well above and below this height

Geographic and Magnetic Coordinates I



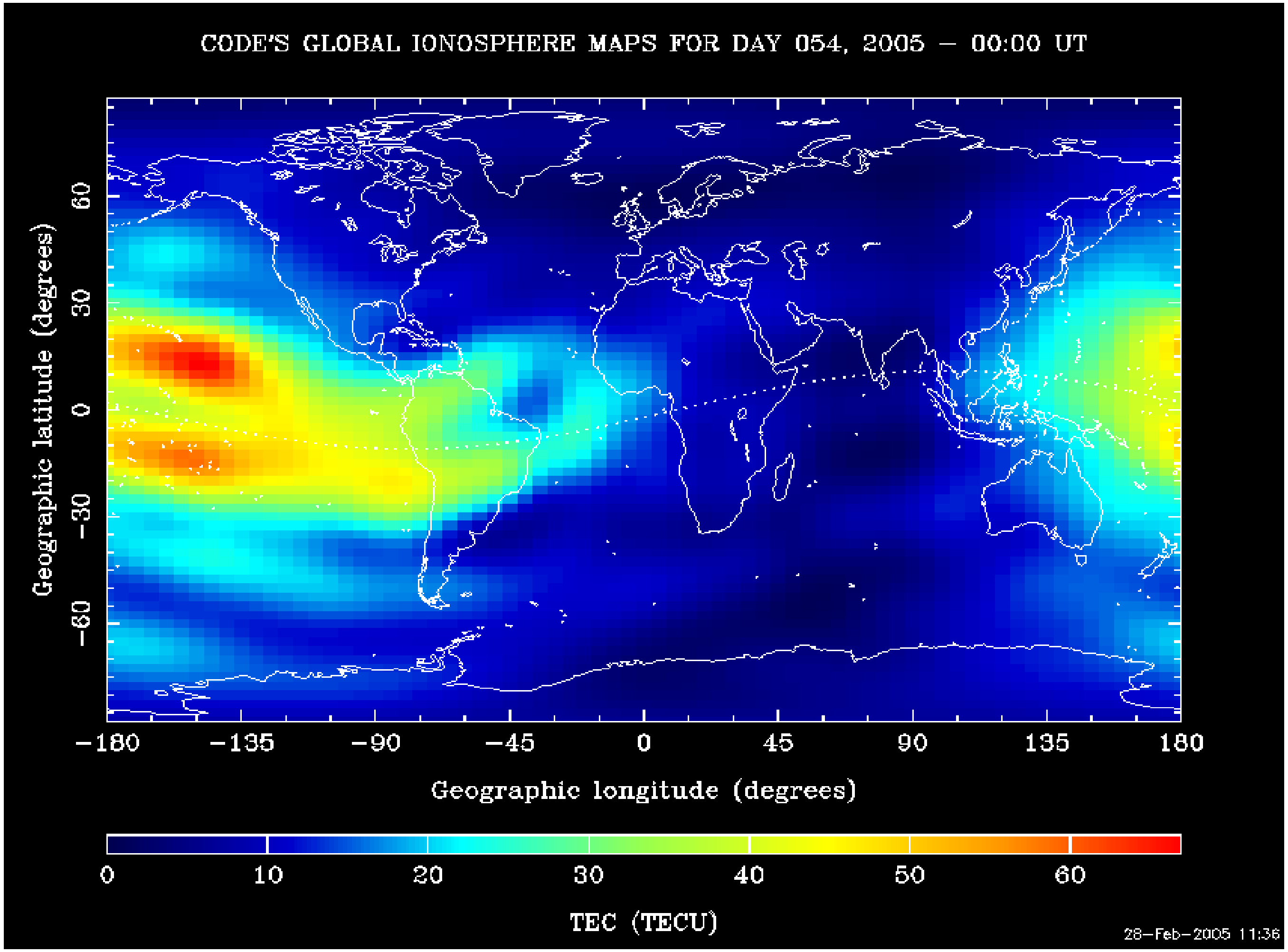
- Electrons constrained by magnetic field
- Ionizing radiation follows geographic coordinates and season
- Charged particles then constrained to follow magnetic flux lines

Geographic and Magnetic Coordinates II



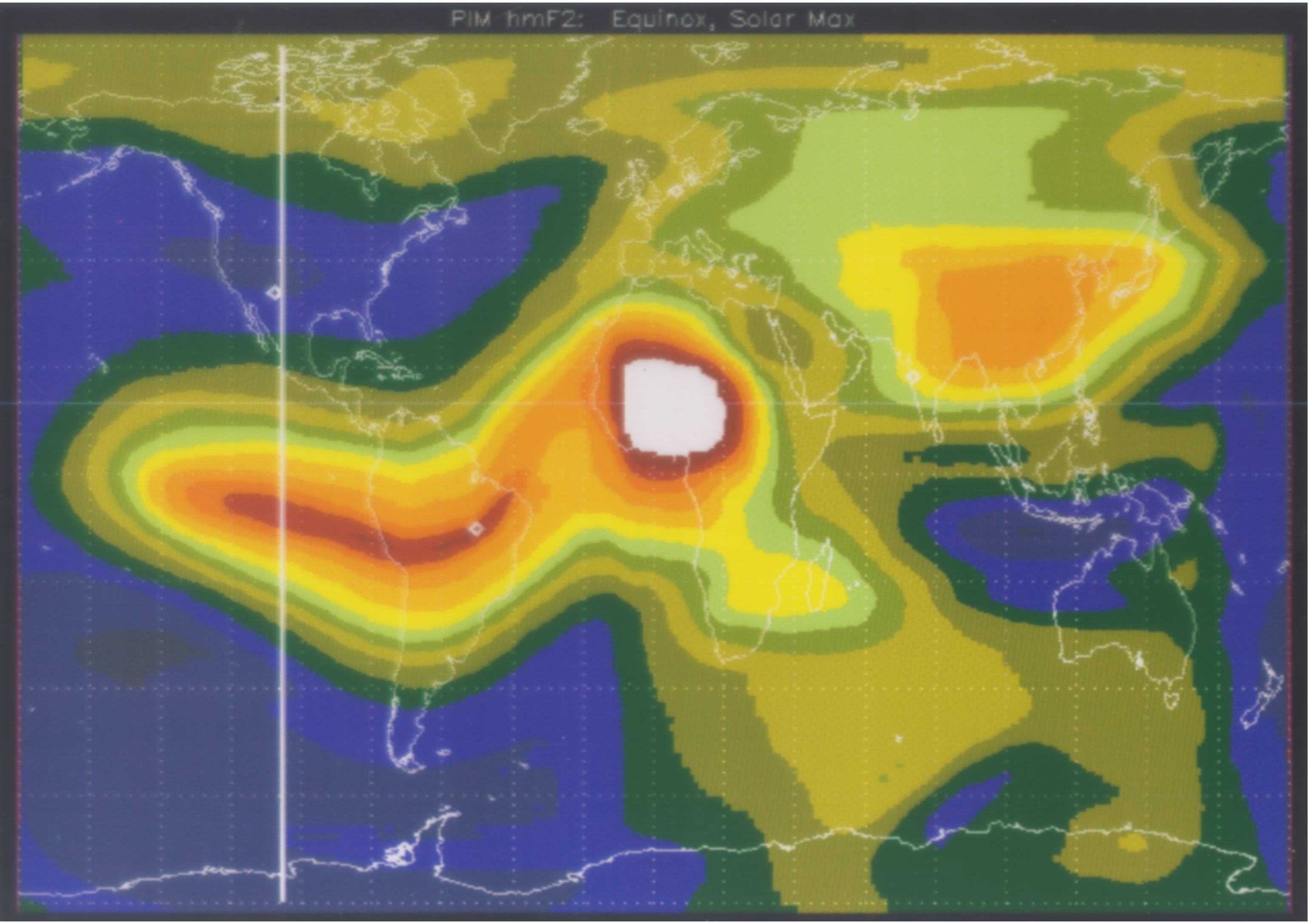
- Magnetic equator shifted toward Europe
- European antennas located at magnetic mid–latitudes
- Same for US, Australia, South Africa

Vertical Total Electron Content Behavior



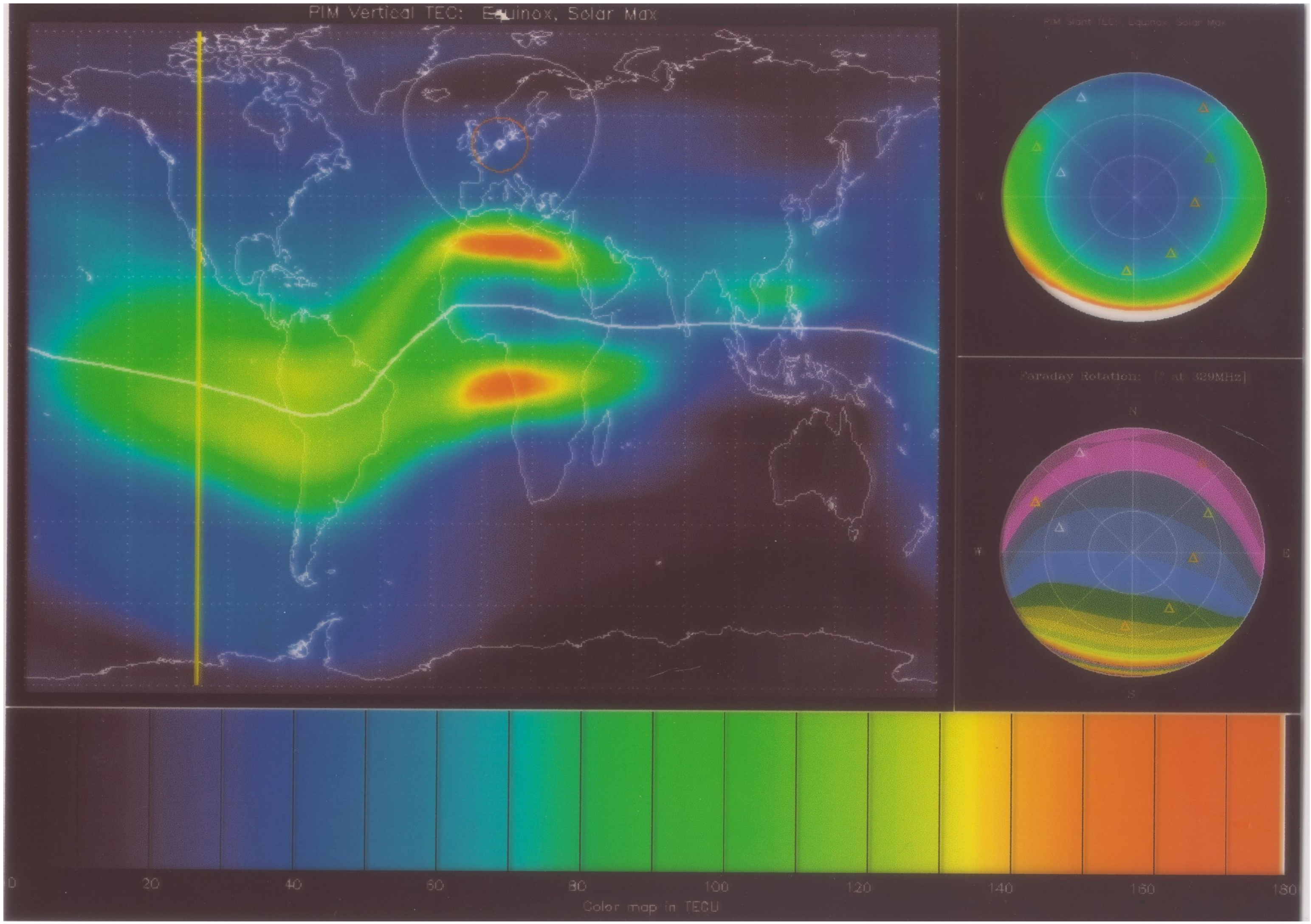
- 1 TECU = 10^{16} m^{-2}
- 1 TECU ≈ 4/3 turn of phase at 1 GHz, or 40/3 turns at 100 MHz
- Ionization fraction lags Solar noon
- Electrons raised in equatorial fountain fall along flux lines to either side of equator

Electron F2 Peak Height



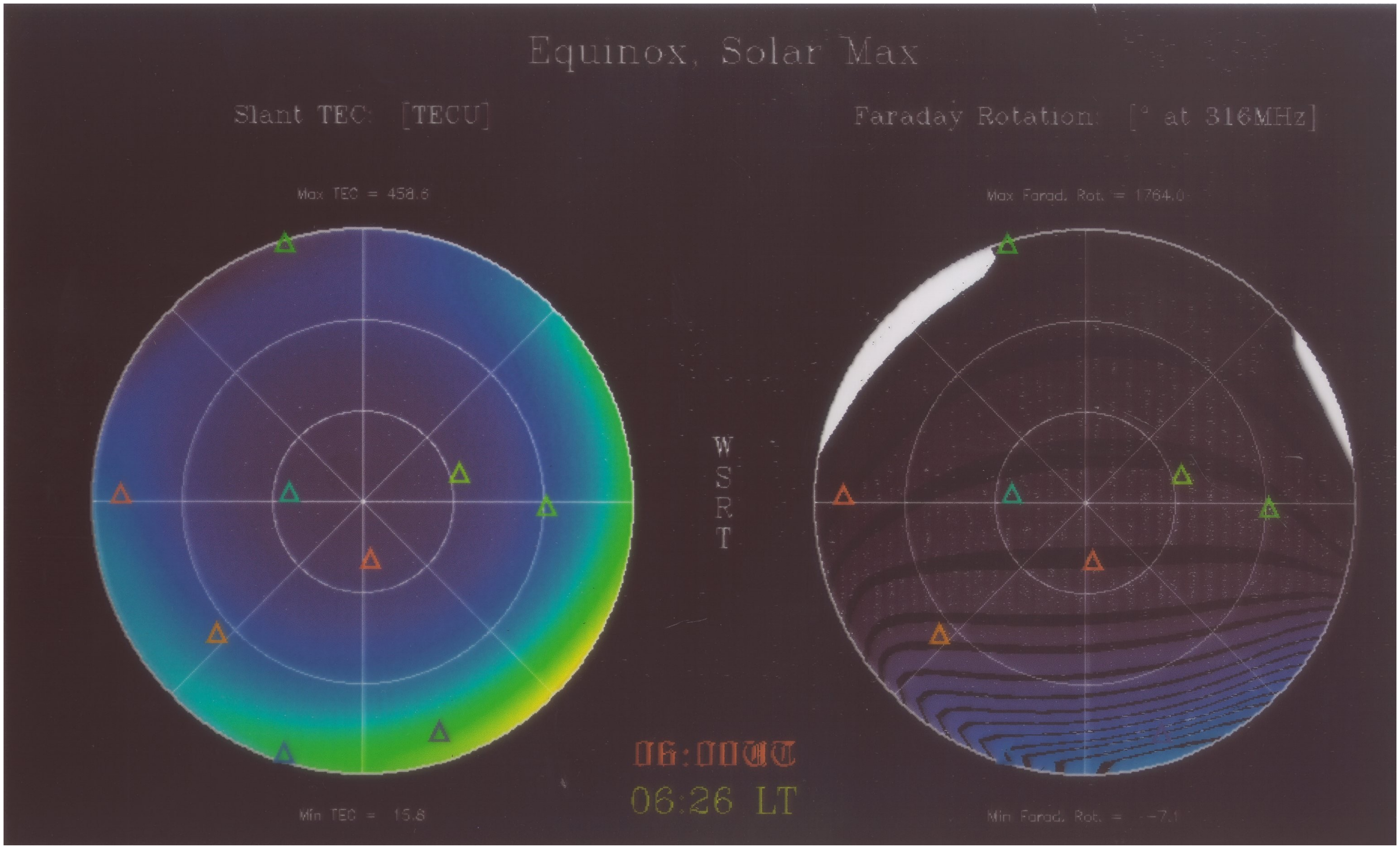
- Solar noon given by vertical white line
- F2 peak height increases by 25 km per color step

Slant Total Electron Content for Westerbork



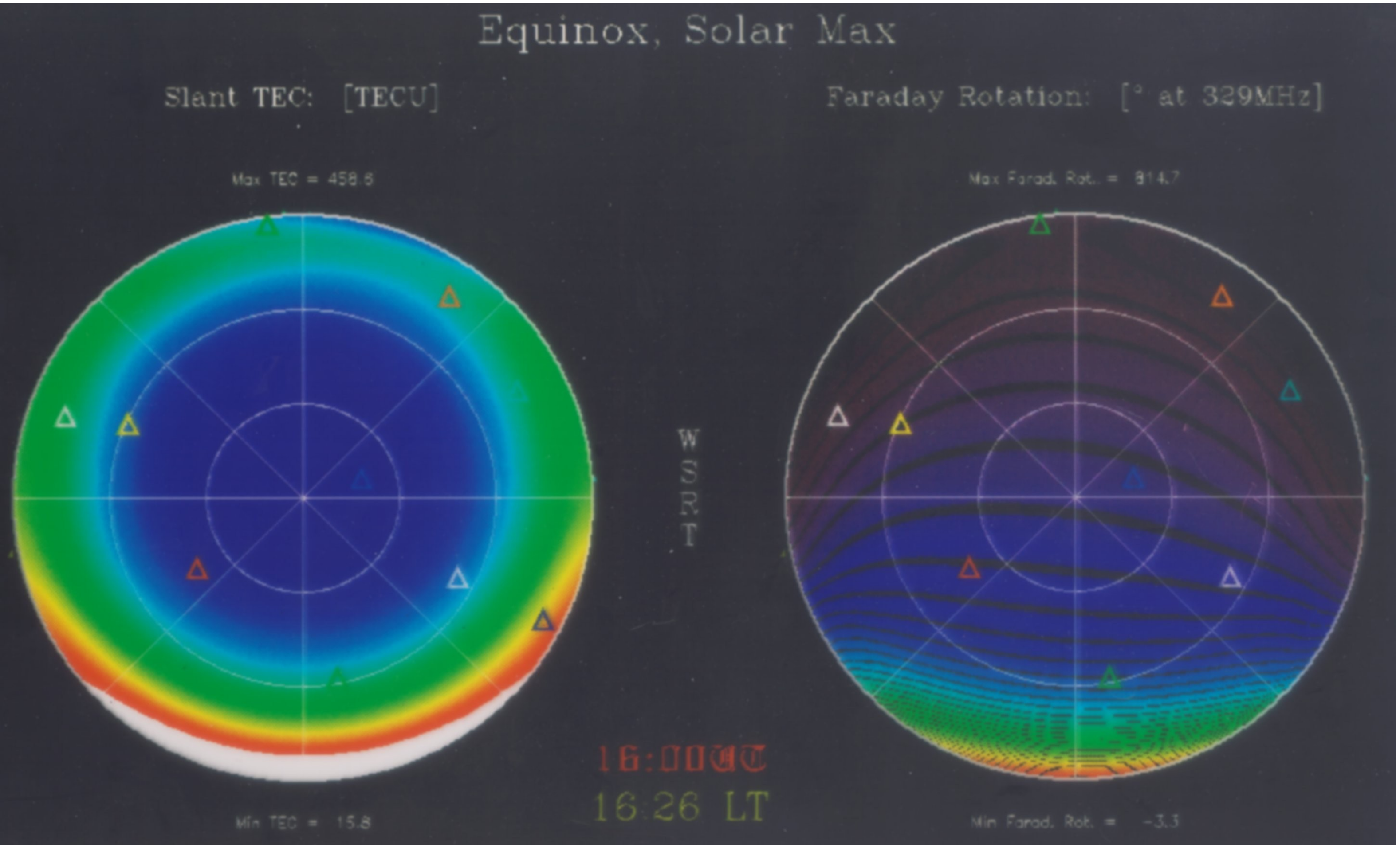
- Same electron model as last slide
- Vertical TEC at left
- Slant TEC upper right
- TEC values very large near horizon

Slant Total Electron Content for Westerbork: Morning



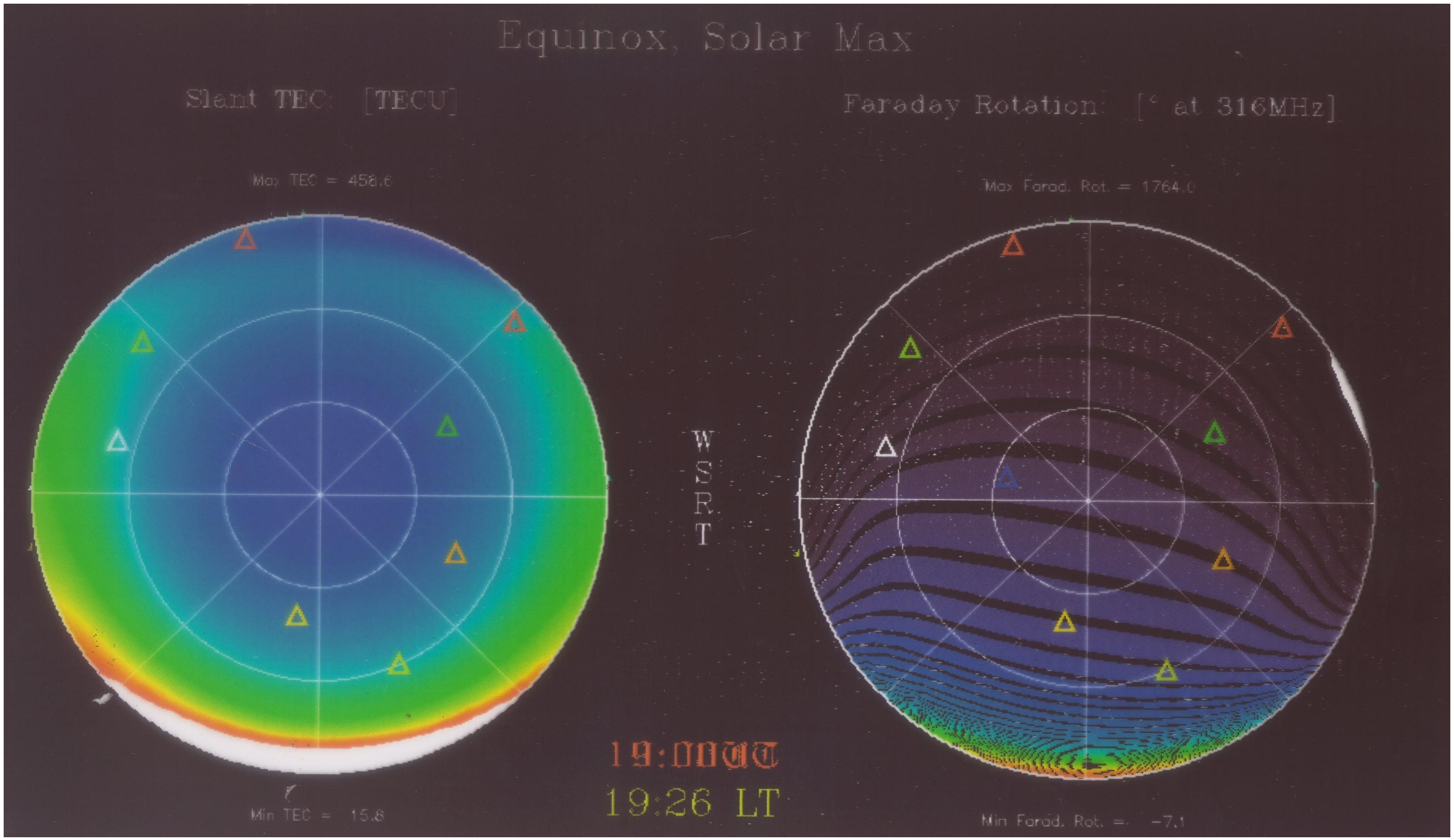
- Slant TEC at left
- Triangles show locations of GPS satellites

Slant Total Electron Content for Westerbork: Afternoon



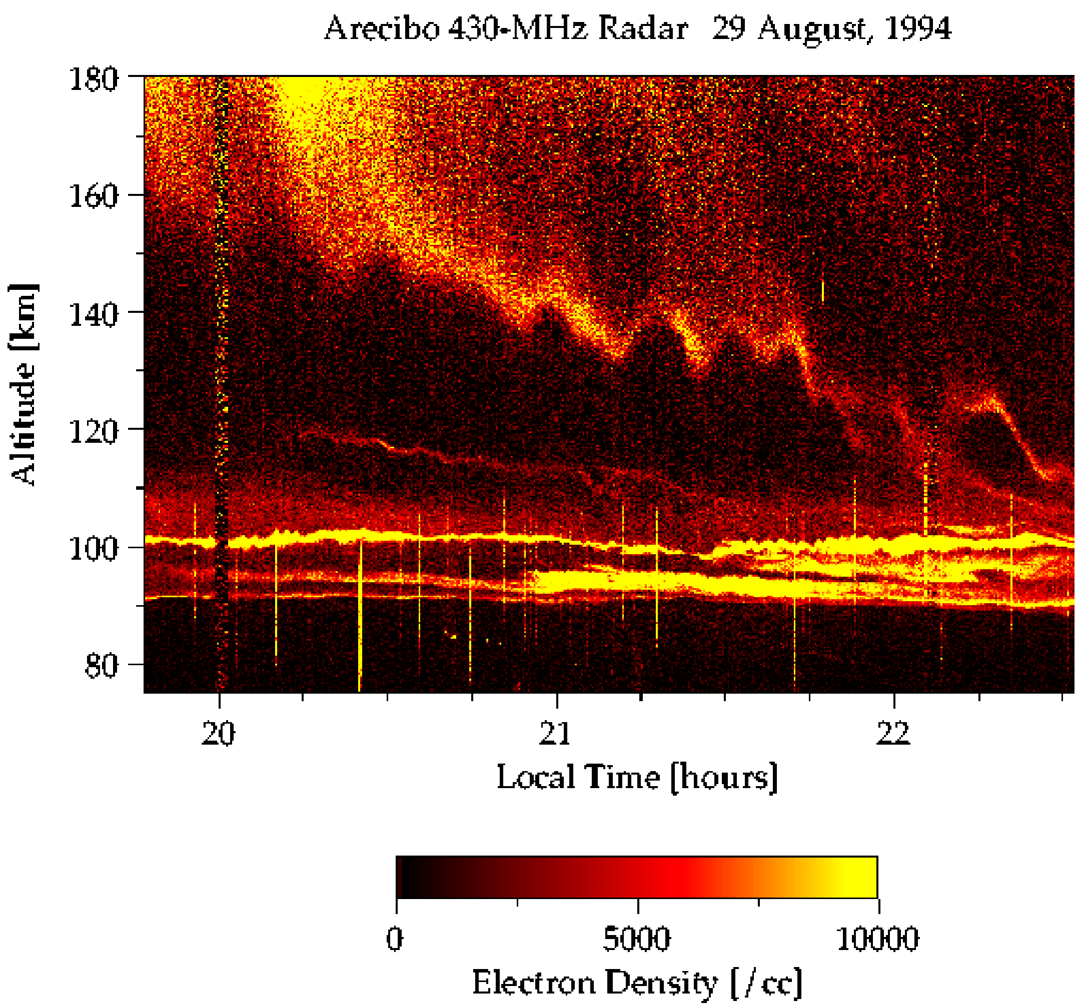
- Slant TEC at left
- Triangles show locations of GPS satellites

Slant Total Electron Content for Westerbork: Night



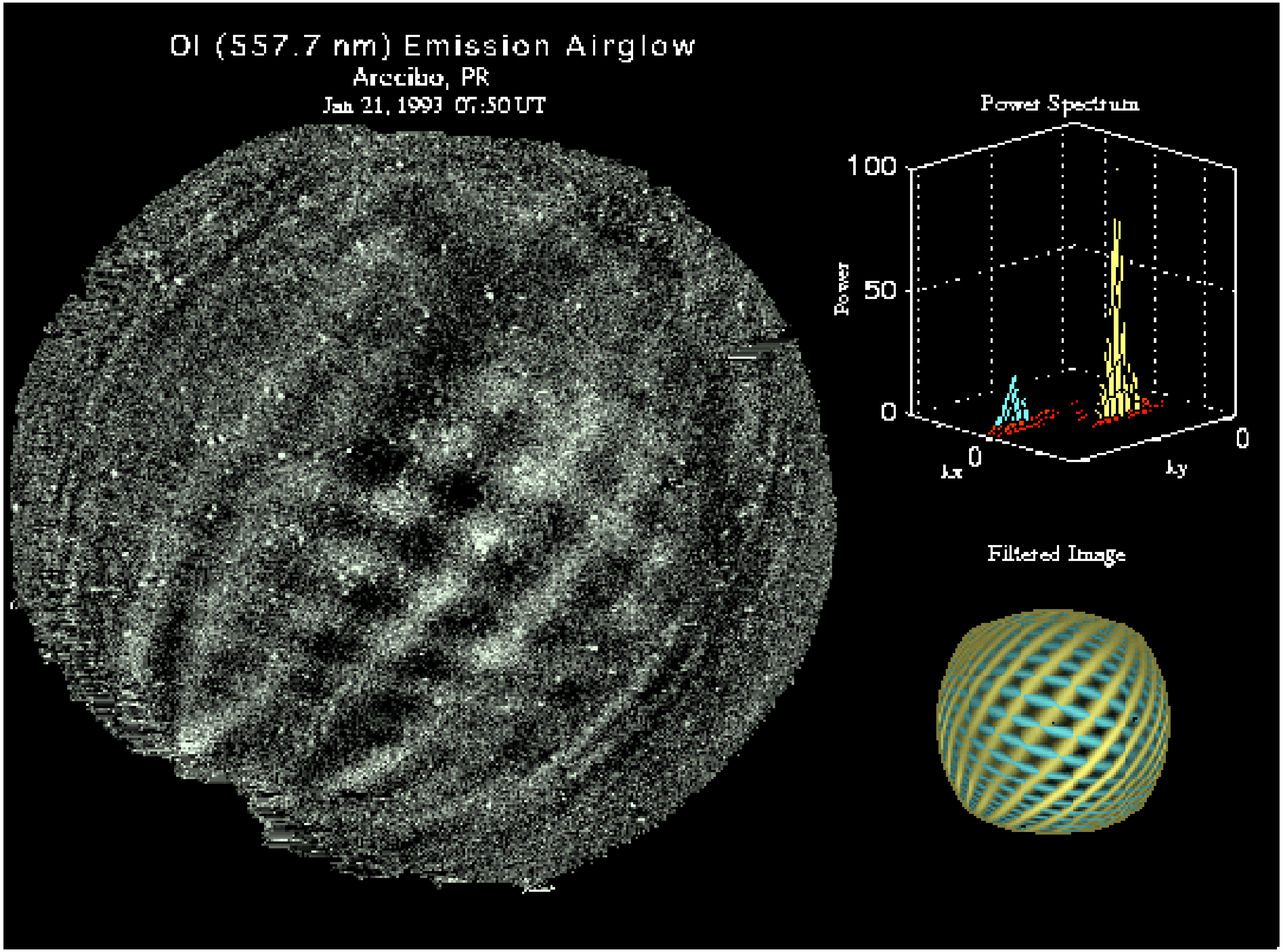
- Slant TEC at left
- Triangles show locations of GPS satellites

Buoyancy Waves



- Vertical structure important
- Waves occur throughout atmosphere, but often seen in ionosphere around 100 km
- Vertical streaks from meteors

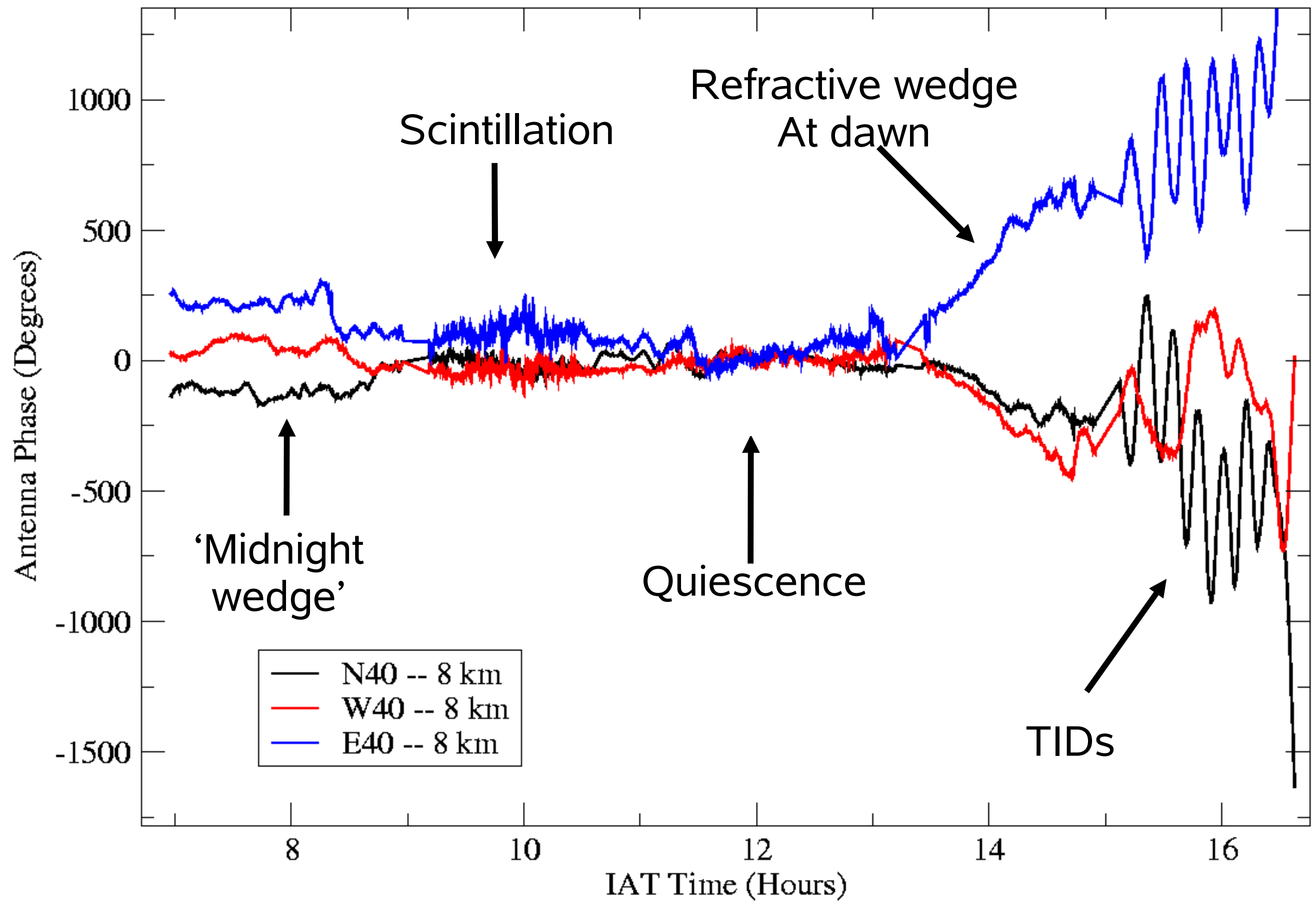
Airglow Above Arecibo



- Left: optical emission showing buoyancy waves
- Right: simple model of two interfering waves
- Typical wavelength: 30 km (10—100 km)

Ionospheric Delay Over the VLA

- Phase variation on three 8-km VLA spacings at 3 different azimuths
- Wide range of ionospheric phenomena seen
- Some of the ionospheric phase fluctuations arise from the sporadic E-layer of the ionosphere?



data from Perley

Lazio 2005

Ionospheric Calibration Requirements

- Many current European interferometers sensitive at GHz frequencies (Westerbork, MERLIN, EVN)
  - 1.4 GHz observations need ~0.03 TECU calibration
- LOFAR will operate from 30 to 240 MHz, the SKA will operate at low frequencies, GMRT operates down to 150 MHz, the VLA down to 74 MHz, Westerbork LFFE
- Ionosphere normally the dominant source of phase errors at low frequencies
- LOFAR requires ionospheric calibration to level of 10^{-3} TECU (one part in $10^{4.5}$ during daytime!)
- Must be able to calibrate large areas of the sky, many degrees across, over several thousand kilometers on Earth

Quote From an Ionospheric Scientist

"Hi James,

Global IONEX data with a considerably higher resolution in time and position doesn't make sense ... . So, I start from the assumption that you will not find the desired (global) data."

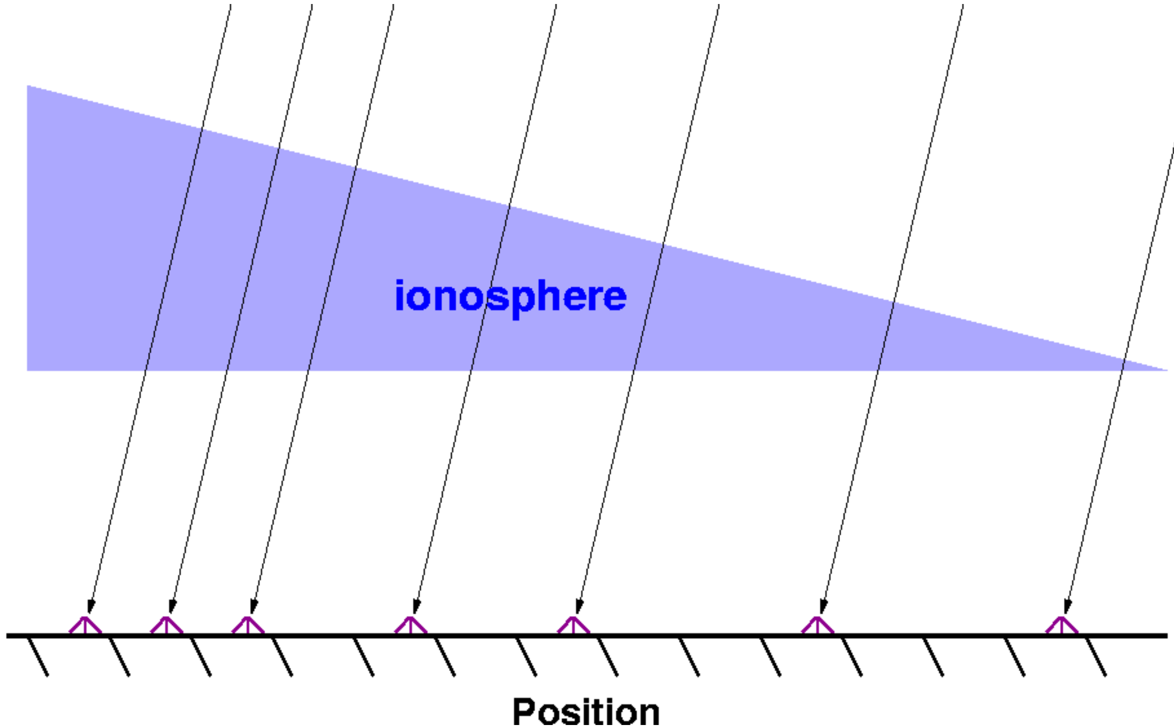
Dr. Stefan Schaer, CODE Analysis Center, 2005 May 19

Ionospheric calibration at the precision needed by LOFAR for long baselines is hard!

Calibration for Short Baselines Has Already Been Achieved

- Works for VLA, Westerbork
- Traditionally a wedge model for the ionosphere used
- Field-based calibration scheme (Bill Cotton) successfully used at the VLA out to B array (~10 km baselines)
- Current methods fail for longer baselines

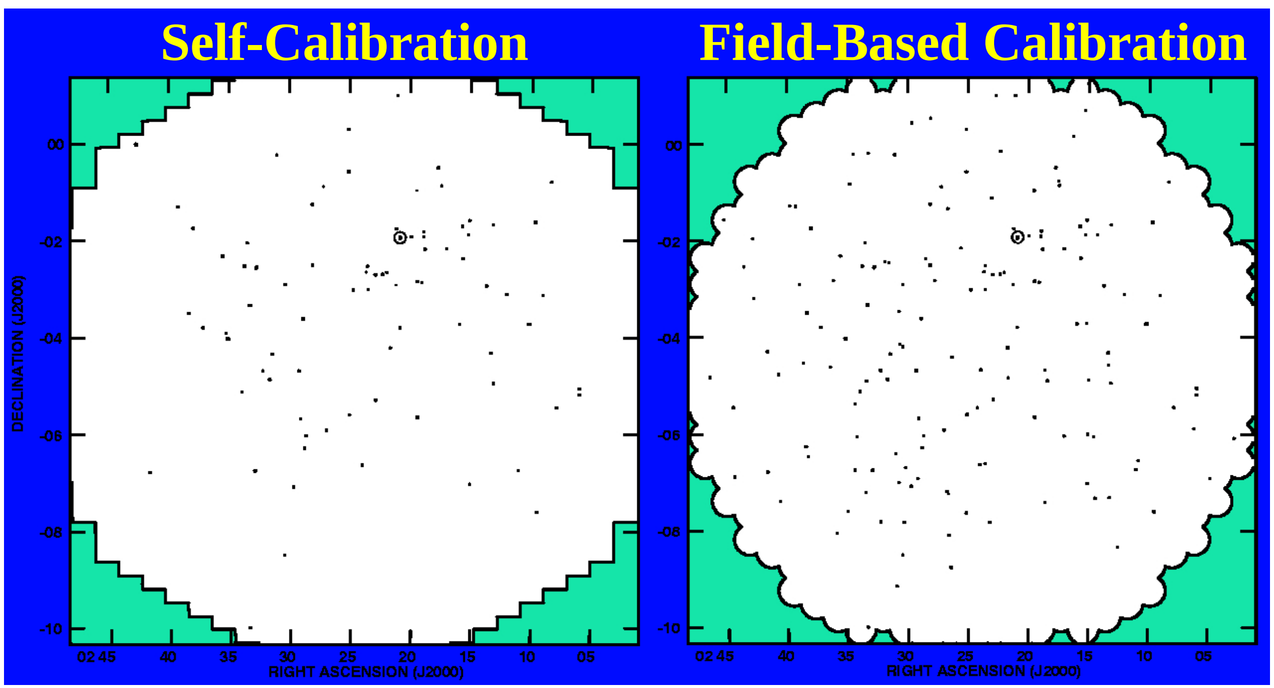
Ionospheric Wedge Model

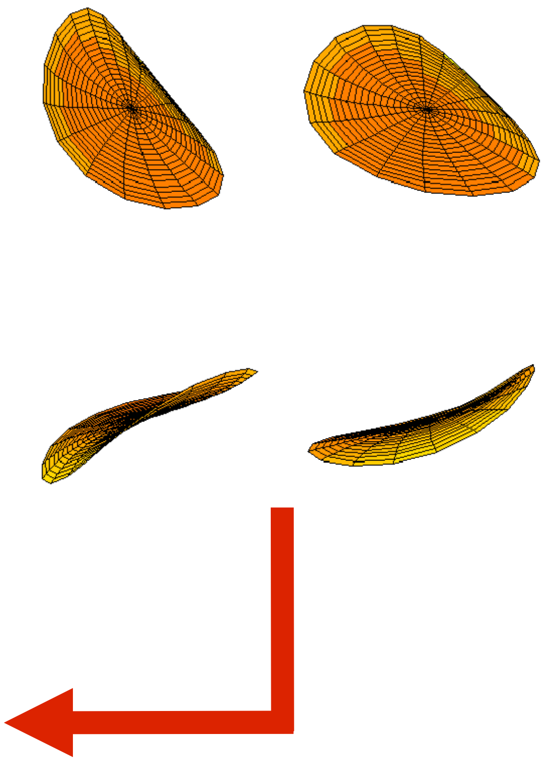


- Assume differential delay related to ionospheric density GRADIENT, so
- $\varphi = (x_1 - x_2) * K$
- Depends on BASELINE length, not station or ionosphere POSITION

Field-based Calibration

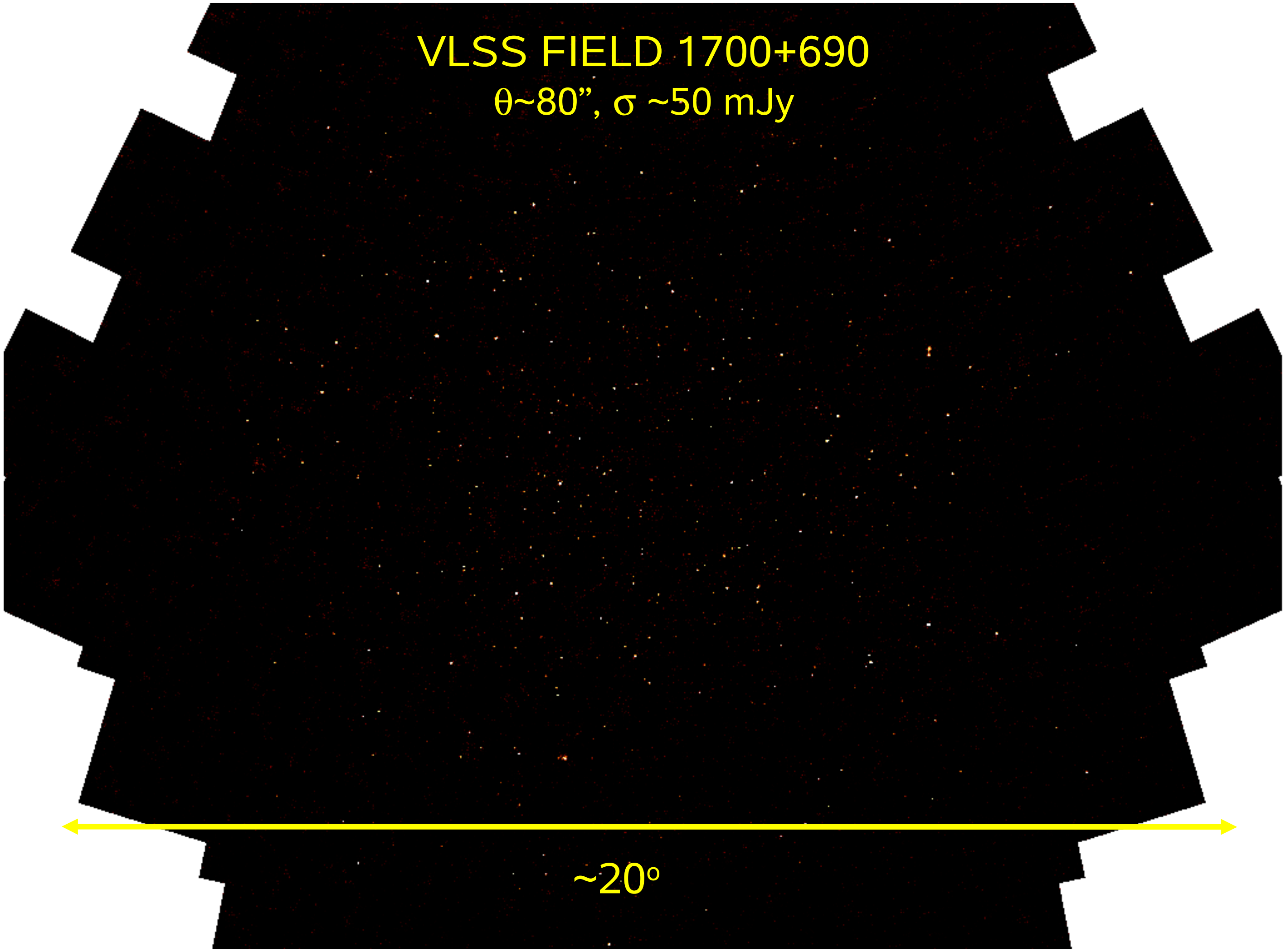
- Take snapshot images of bright sources in the field and compare to known positions.
- Fit to a 2nd order Zernike polynomial phase delay screen for each time interval.
- Apply time variable phase delay screen to produce corrected image.





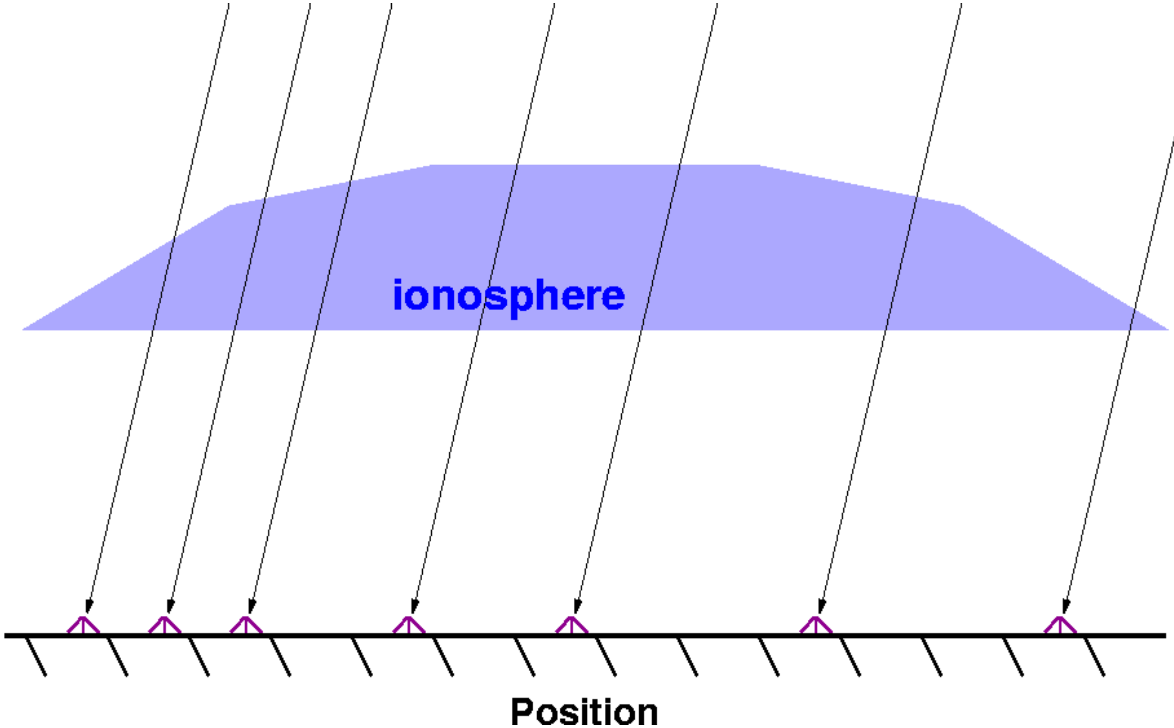
Lazio 2005

Cosmic Evolution
The First Black Holes



Lazio 2005

Gradient Model Breaks Down for Long Baselines

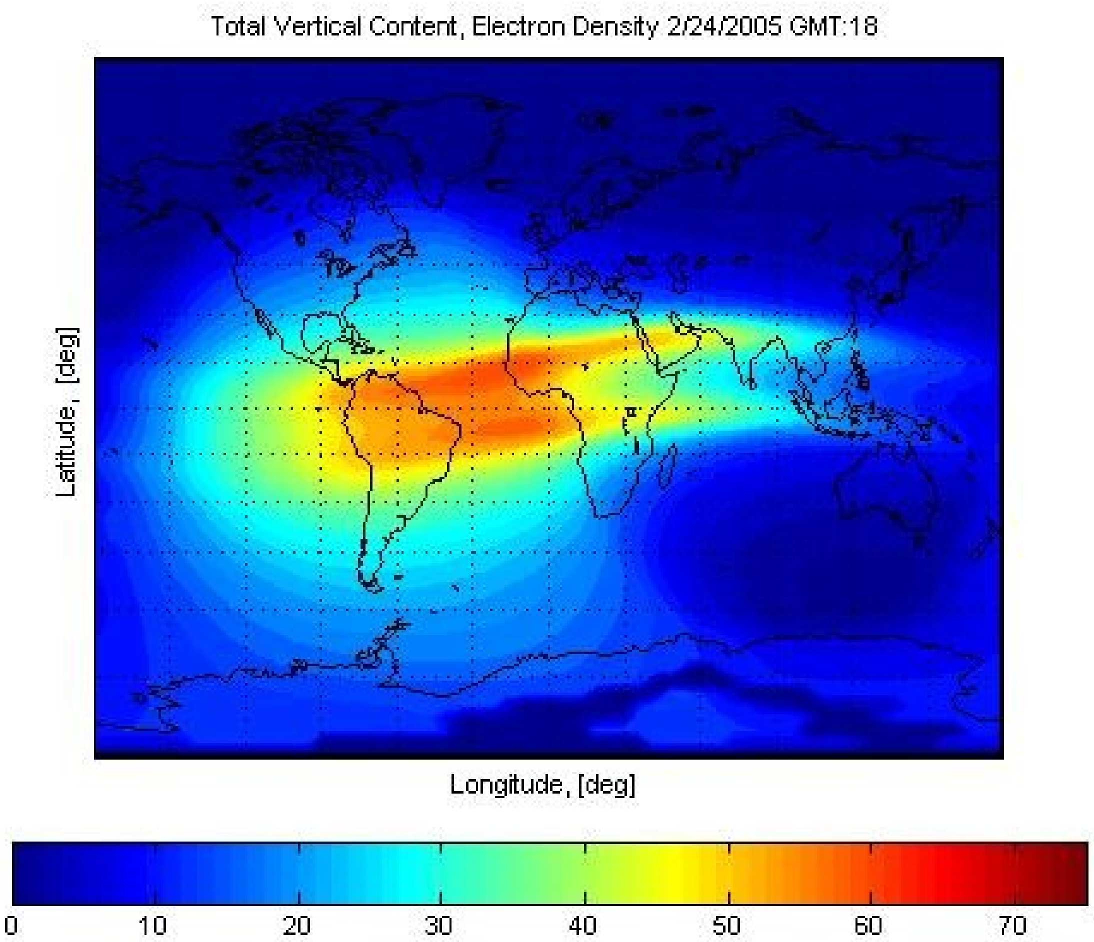


- For stations at great distances, large-scale ionospheric structure and ionospheric waves cause gradient approach to fail
- Gradient approach also fails for large angular separations on sky

Large-Scale Ionosphere Models: IONEX

- AIPS TECOR task uses IONEX format files
- standard IONEX files sampled at 2 hour intervals
- grid spacing 5° by 2.5° (lon x lat)
- effectively 2-D model ignoring height information
- Probably better than nothing, but still insufficient for VLBI calibration

Realtime MHD Modeling

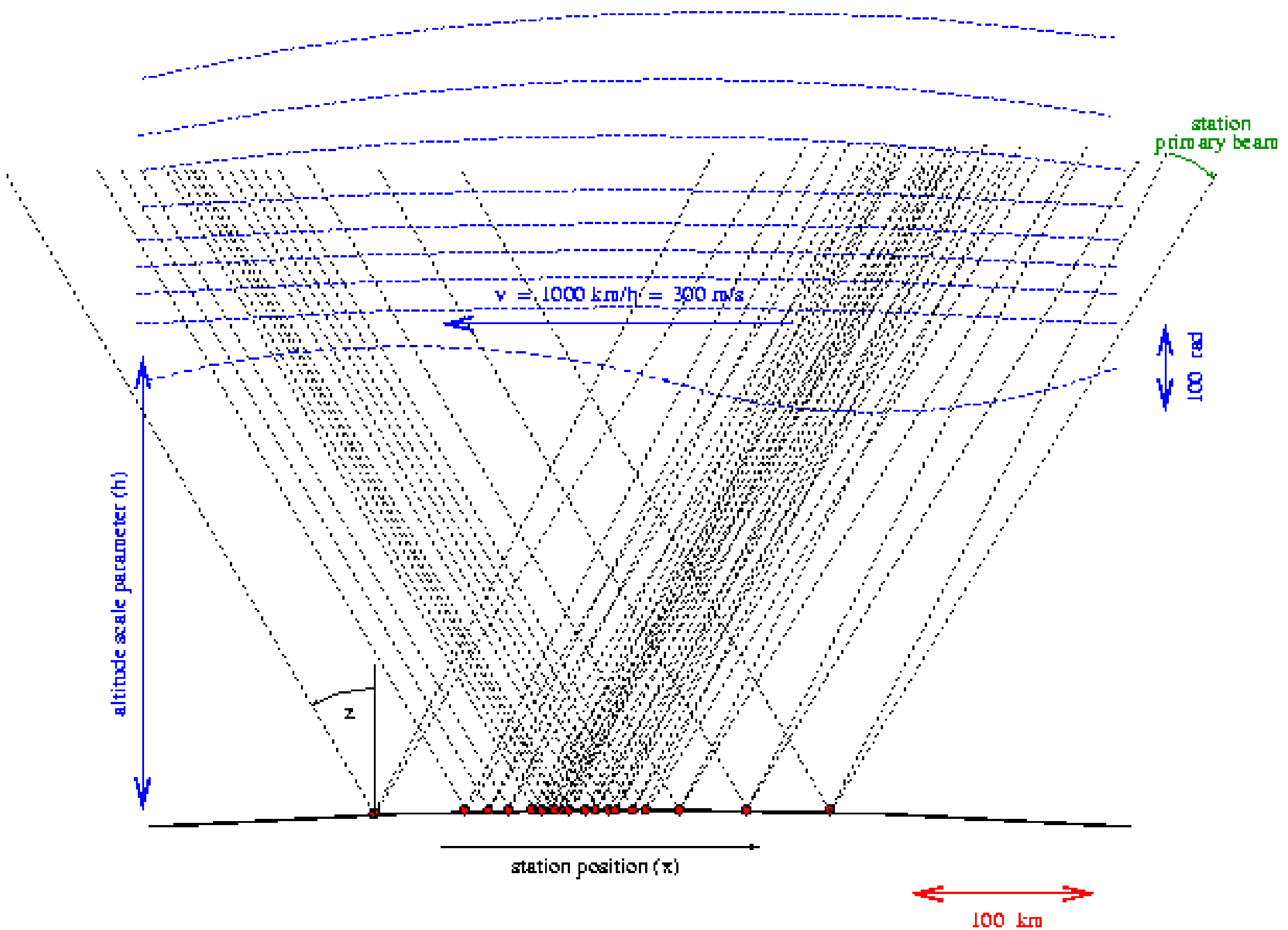


- Model made by FusinNumerics
- Example of current computational modeling incorporating GPS data
- 2° by 2° grid
- 3-D model includes height effects for slant paths
- Improvement over TECOR, but still not high enough resolution

Minimum Ionospheric Model (MIM) as Expressed by Noordam

- Only try to fit over telescope array, not over entire globe
- Use minimum number of parameters (few bright sources available, sometimes none)
- Only deal with observables (astronomers not interested in internal structure of ionosphere in general)
- Assume large-scale (>100 km) structure and go progressively smaller, until ….

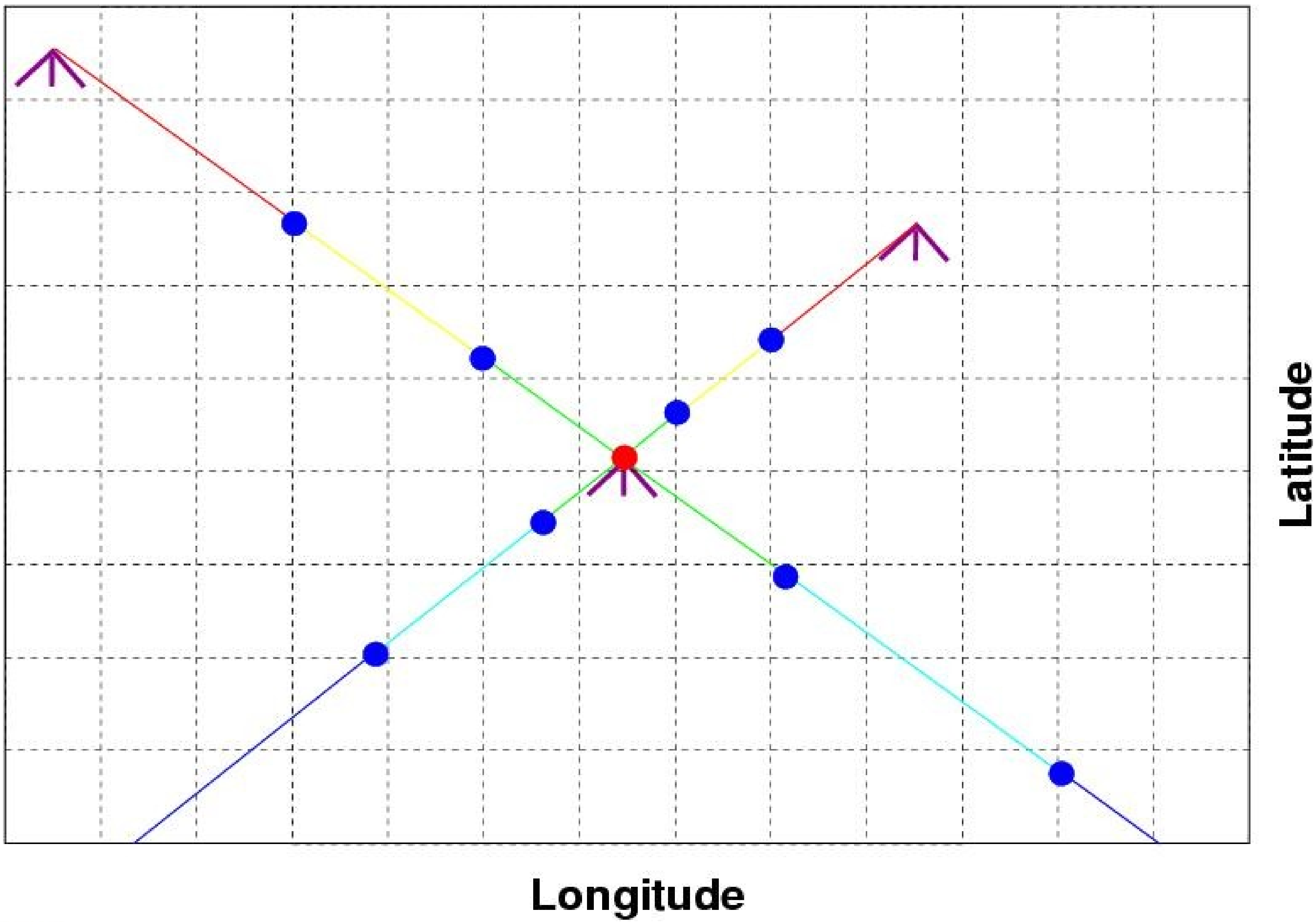
Ionospheric Blanket
With Many Piercing Points



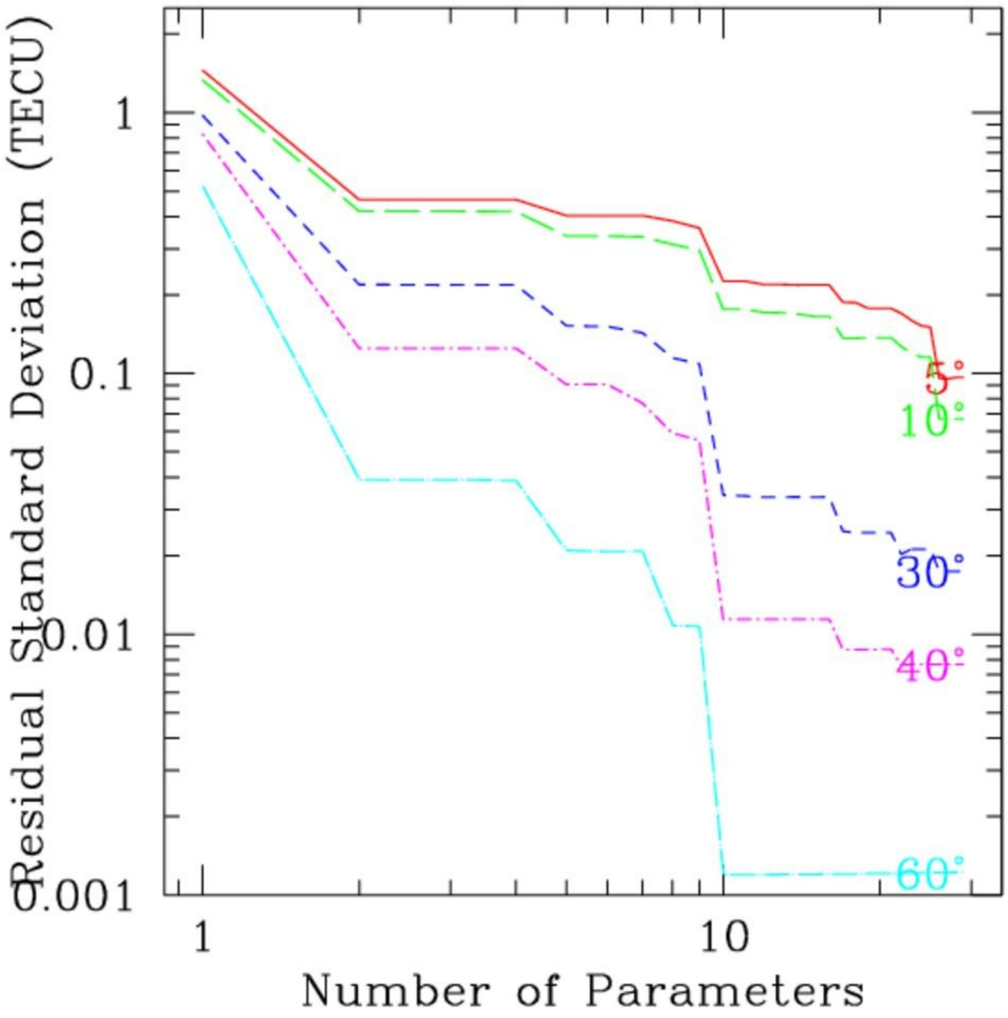
Example 2D MIM Form: Polynomials

- $\mathrm{VTEC}(x,y,t) = \Sigma_{i=0 \text{ to } m}\Sigma_{j=0 \text{ to } n}\ c_{i,j}(t)\ x^i\ y^j$
- $c_{i,j}(t) = \Sigma_{k=0 \text{ to } p}\ a_k\ t^k$
- Scale vertical electron content VTEC by elevation angle term to get slant TEC
- x,y could be Latitude,Longitude or RA,Dec and so on

2D MIM --- Lat,Lon



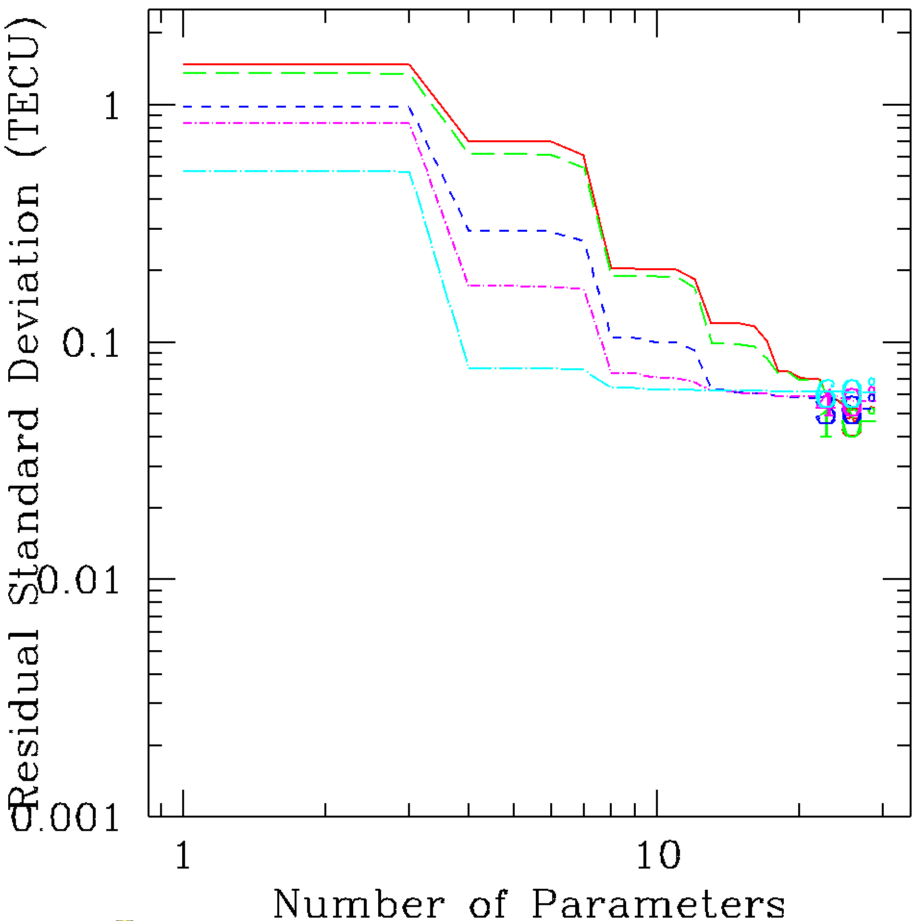
2D Absolute Residuals

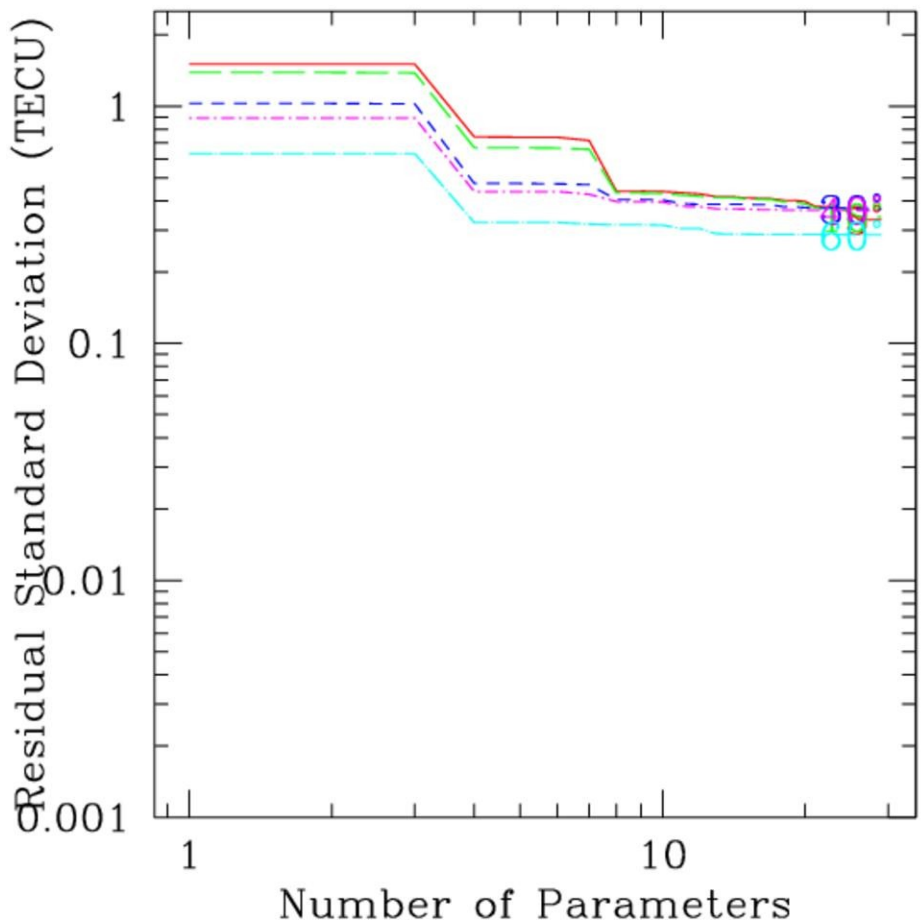


- Comparison against full 3D integrated density (static ionosphere)
- Fit to random directions on sky above specified elevation limit
- Stations within 1 km
- Works well for small areas of the sky

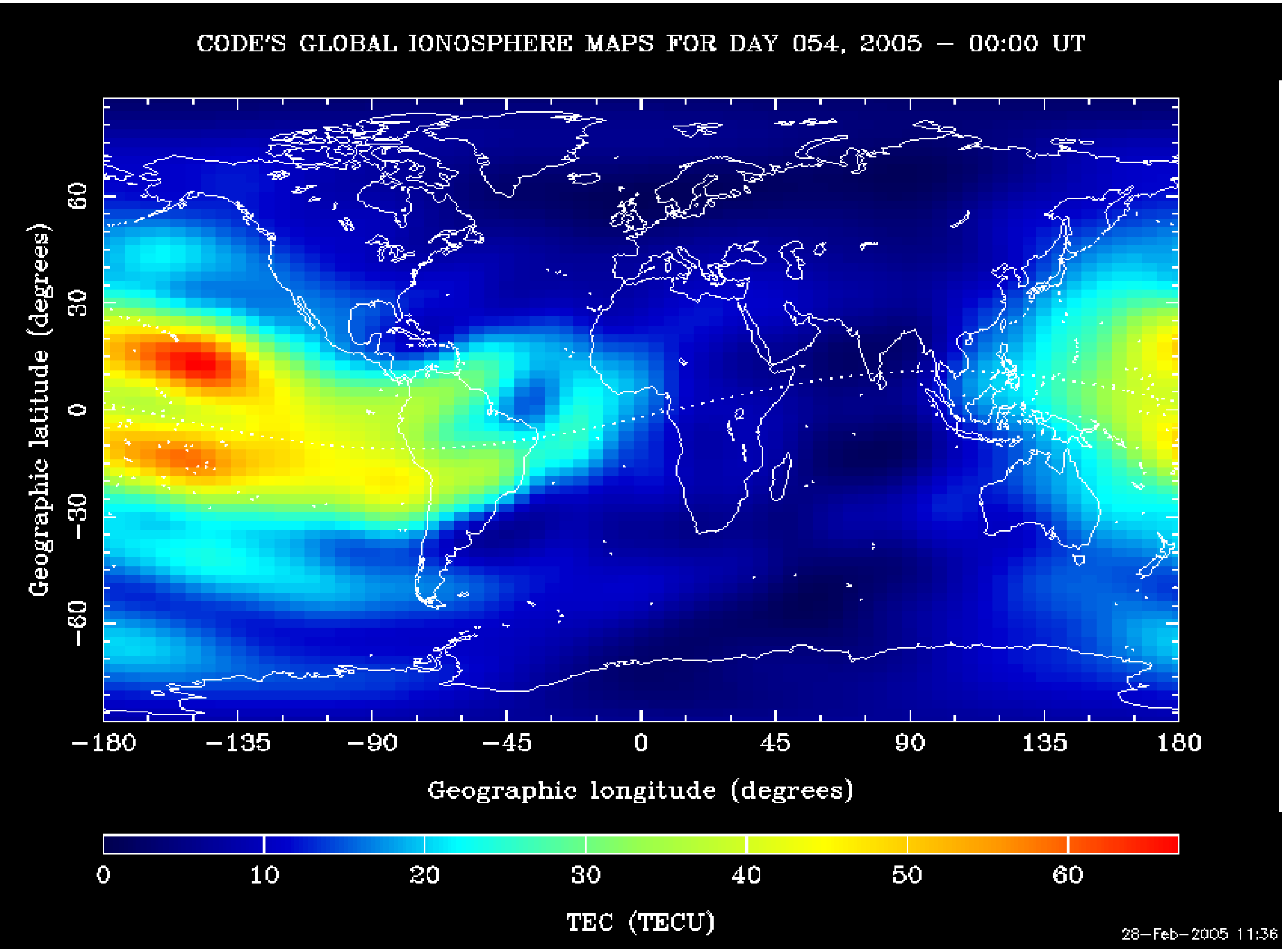
2D Residuals: Poor for Long Baselines

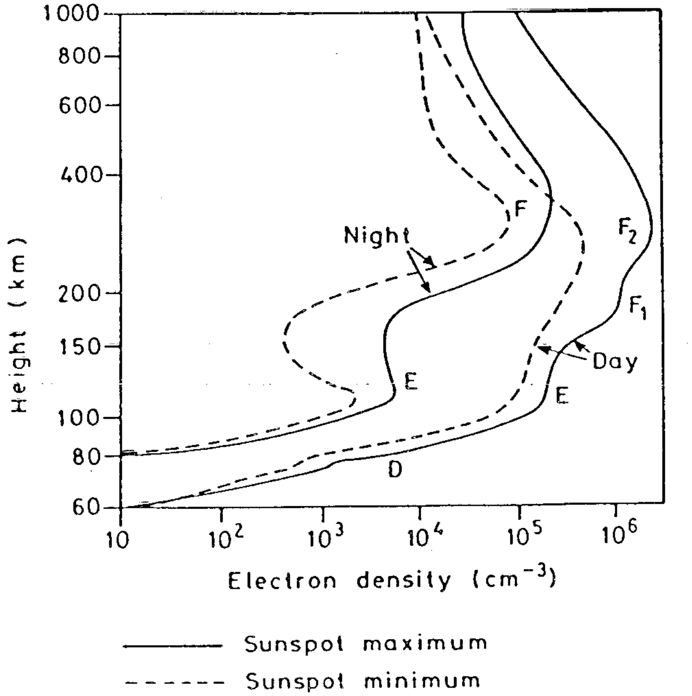
50 km Maximum Baseline 1000 km



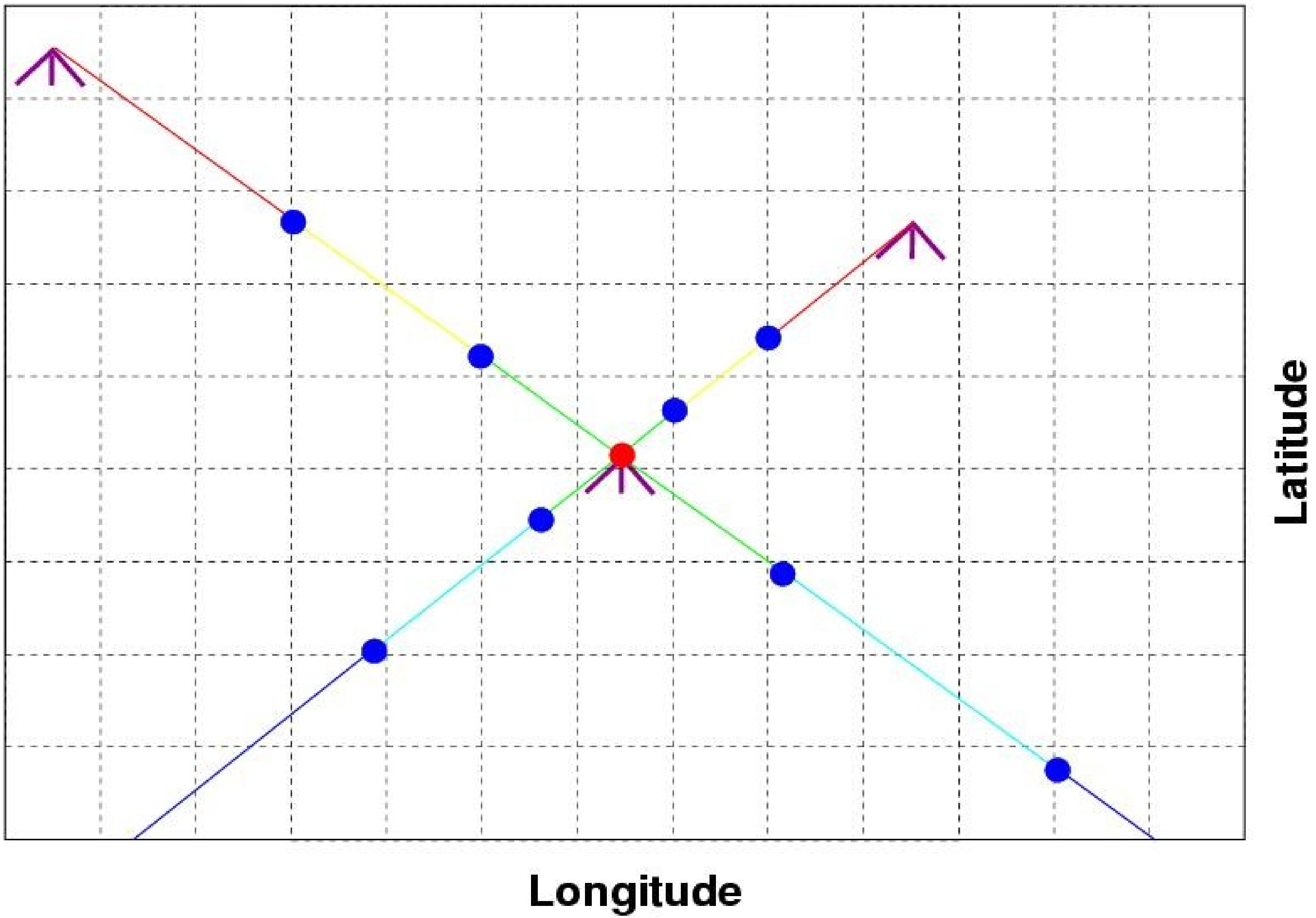


Ionosphere Varies with Latitude, Longitude, and HEIGHT

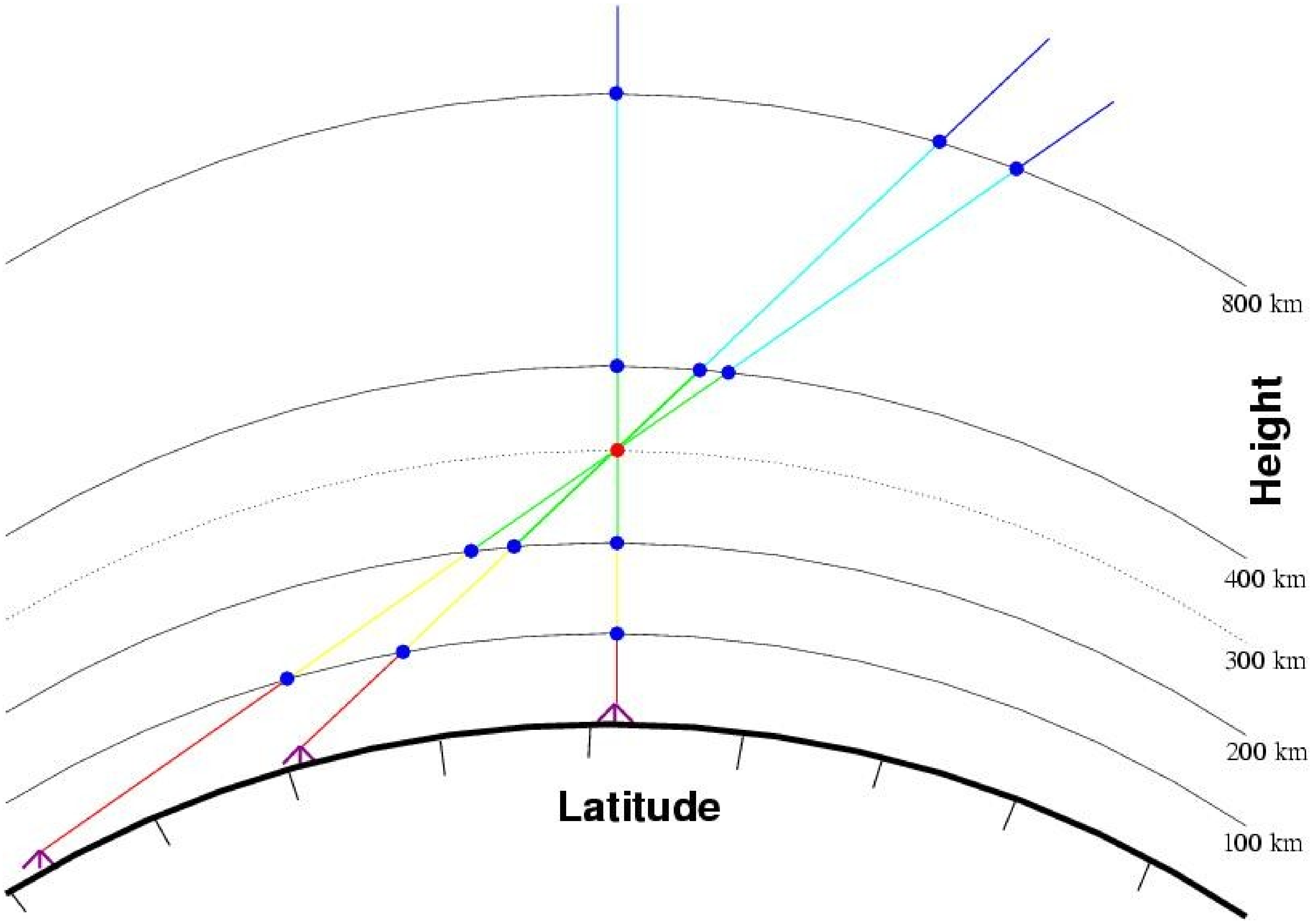




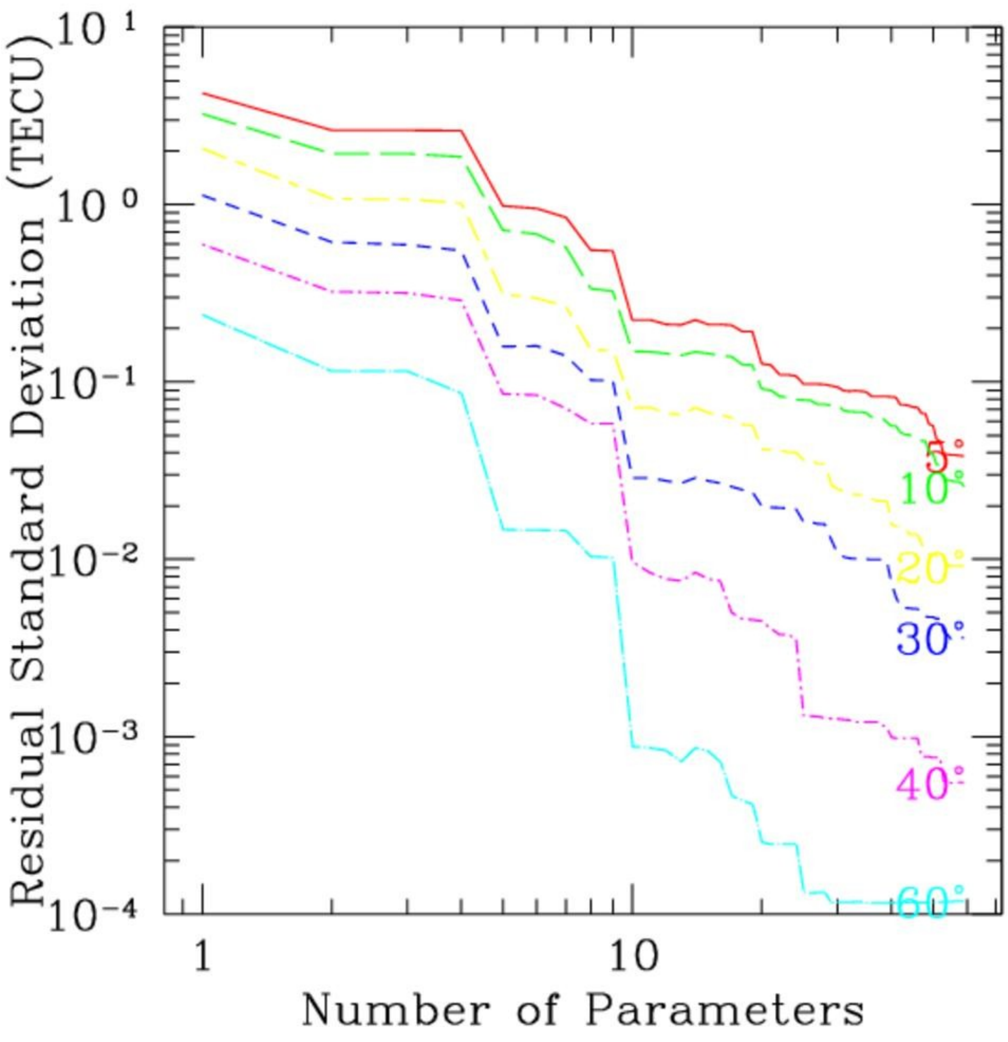
3D MIM --- Lat,Lon,Height



3D MIM --- Height

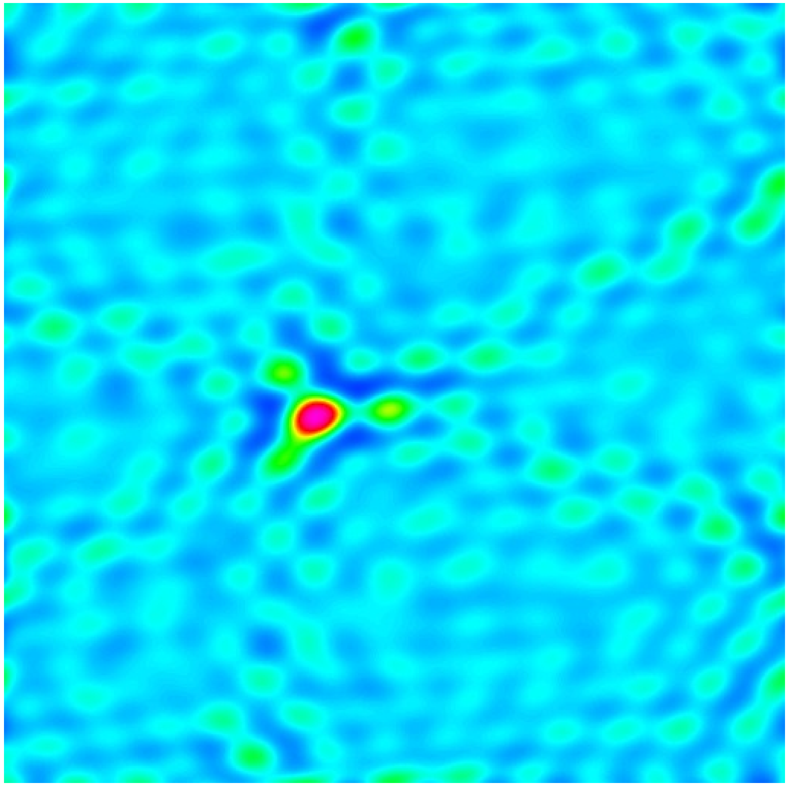


3D Absolute Residuals



- Residuals reduced more than an order of magnitude
- Works well with baselines out to at least 400 km
- More improvement possible
- Relative residuals (interferometry) are even smaller

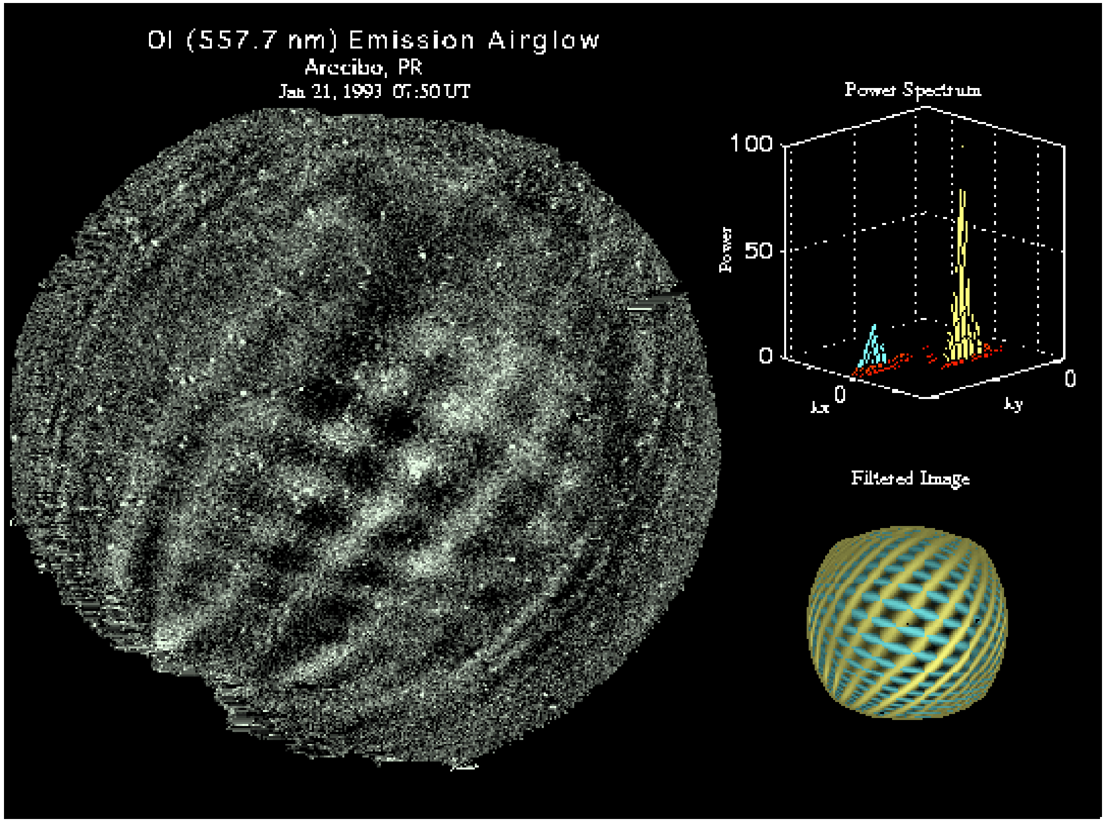
Dealing With a Variable Ionosphere: Waves



- Simulated VLA observation with sinusoidal ionospheric wave
- Large position motions replicated
- Beam shape changes replicated
- 2 sinusoidal waves in different directions reproduce the complex behavior of actual observations

MeqTree simulation by O. Smirnov

MIM Design Conclusions

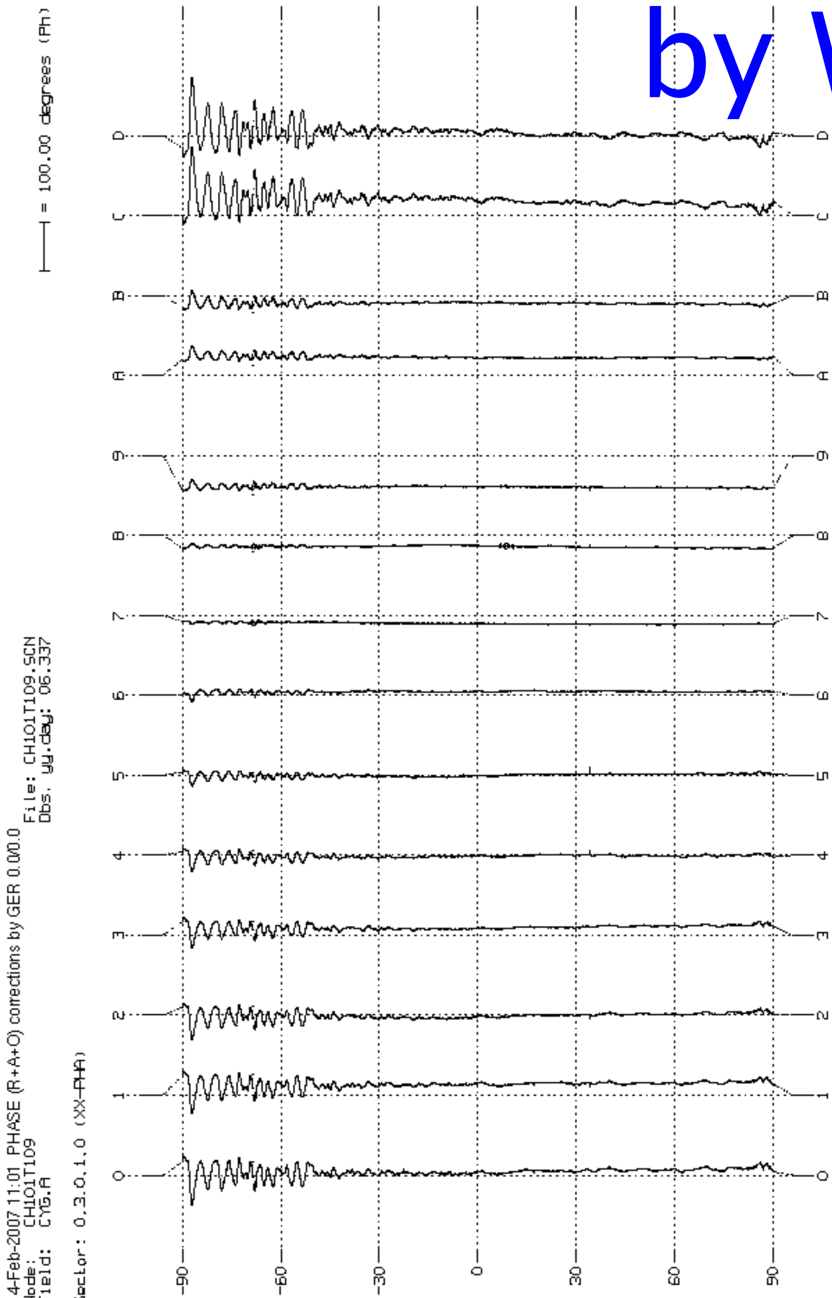


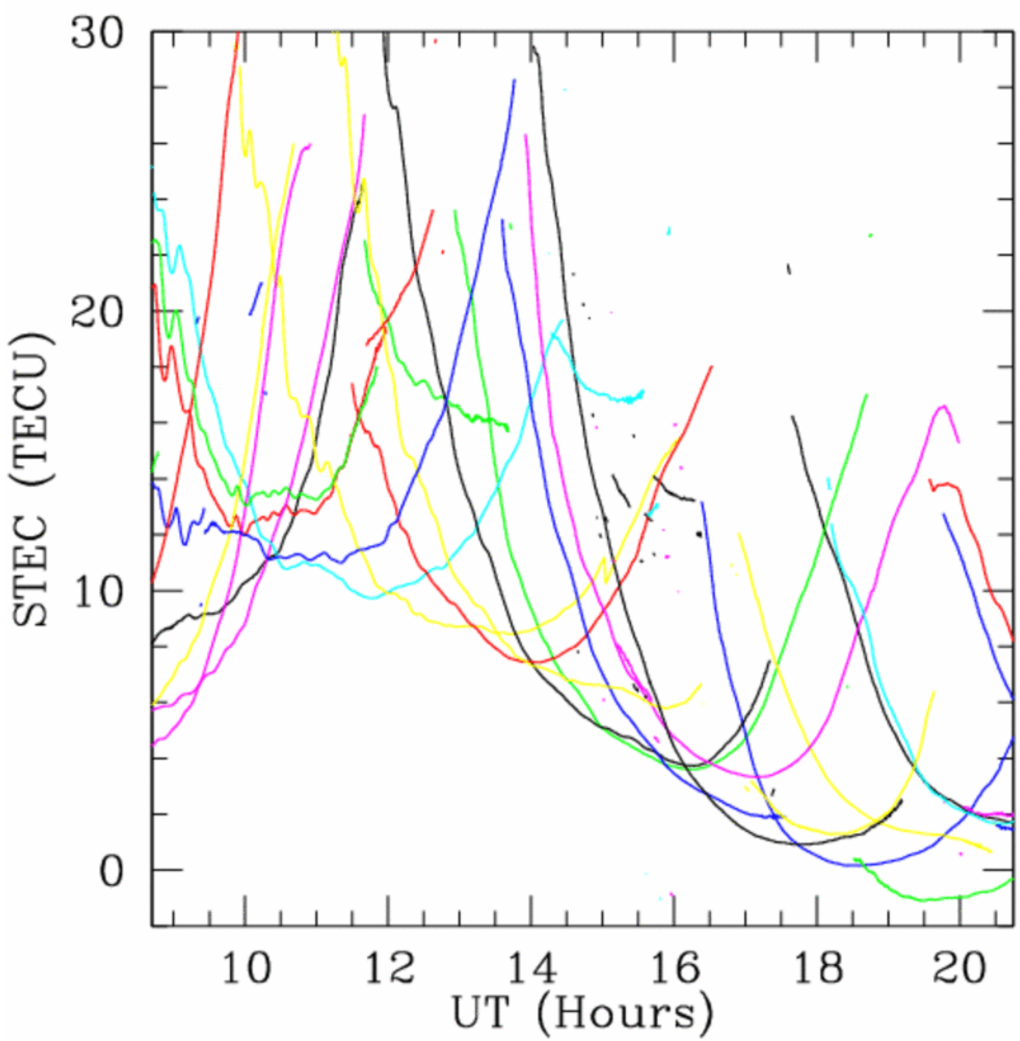
- Need 10—30 parameters to model static ionosphere for entire Dutch LOFAR array
- Ionospheric waves require 6—8 parameters each
- Probably need 10—20 extra parameters
- Need 20 to 60 total parameters
- Dutch LOFAR should have 32--77 stations * several beams, so should have sufficient measurements
- Extended LOFAR calibration requires more study

Testing MIM With GPS Data

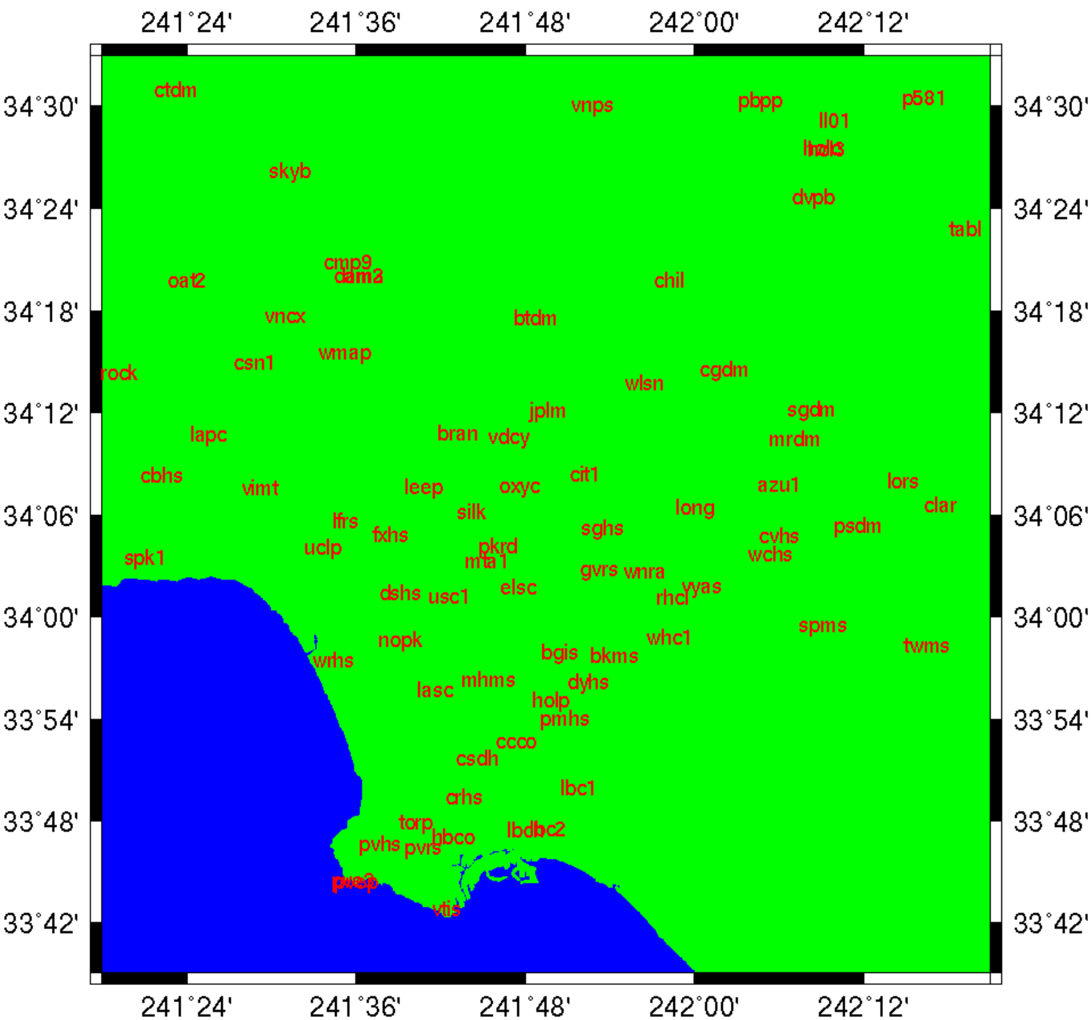
- Work in collaboration with Jan Noordam and Maaike Meevus
- Developing MIM model in MeqTrees for eventual LOFAR calibration
- Testing over Los Angeles
  - Dense GPS network, ~10 km between stations
  - Data freely available through anonymous FTP
- GPS data theoretically can achieve 0.01 TECU, enough to get LOFAR calibration started

GPS Data Show TIDs Measured by Westerbork



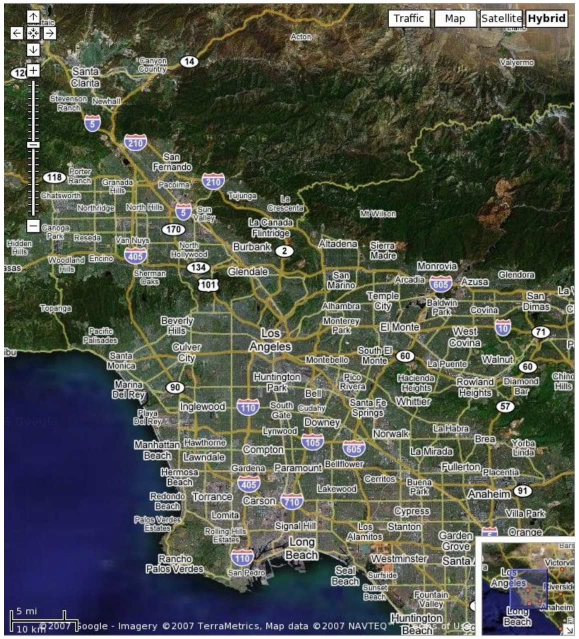


LA GPS Stations

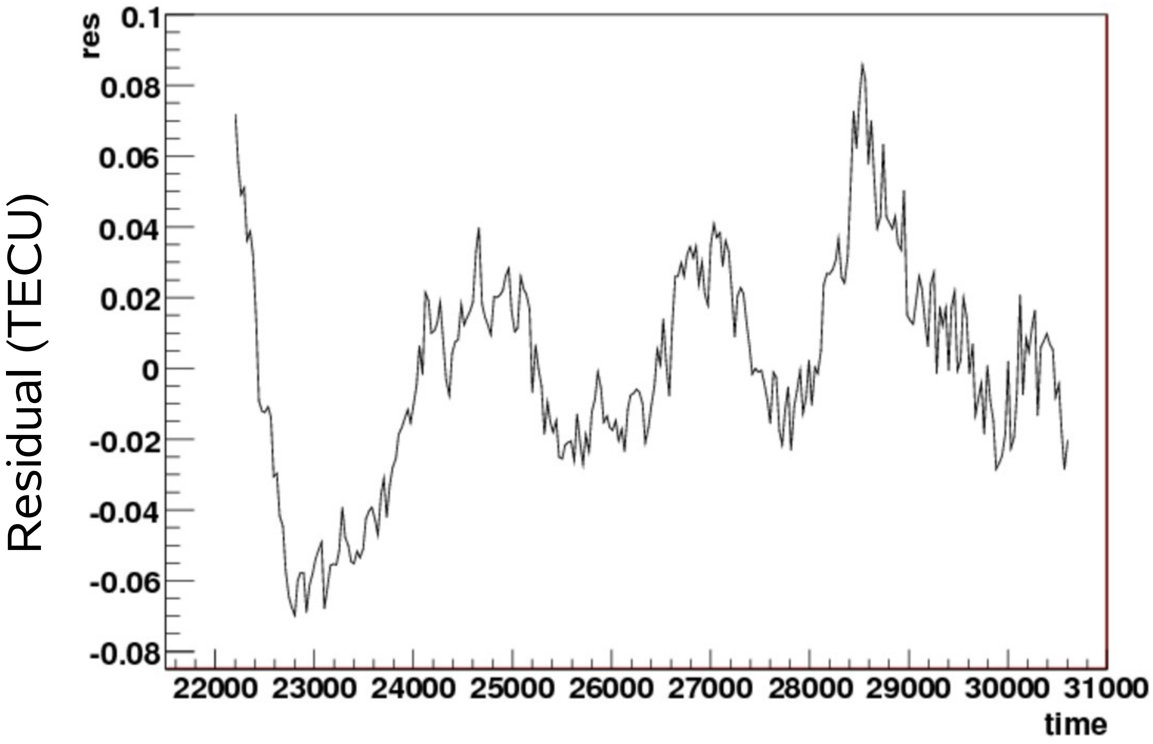


- Public GPS network
- Designed for tectonic plate motion study
- ~10 km between stations

Google map

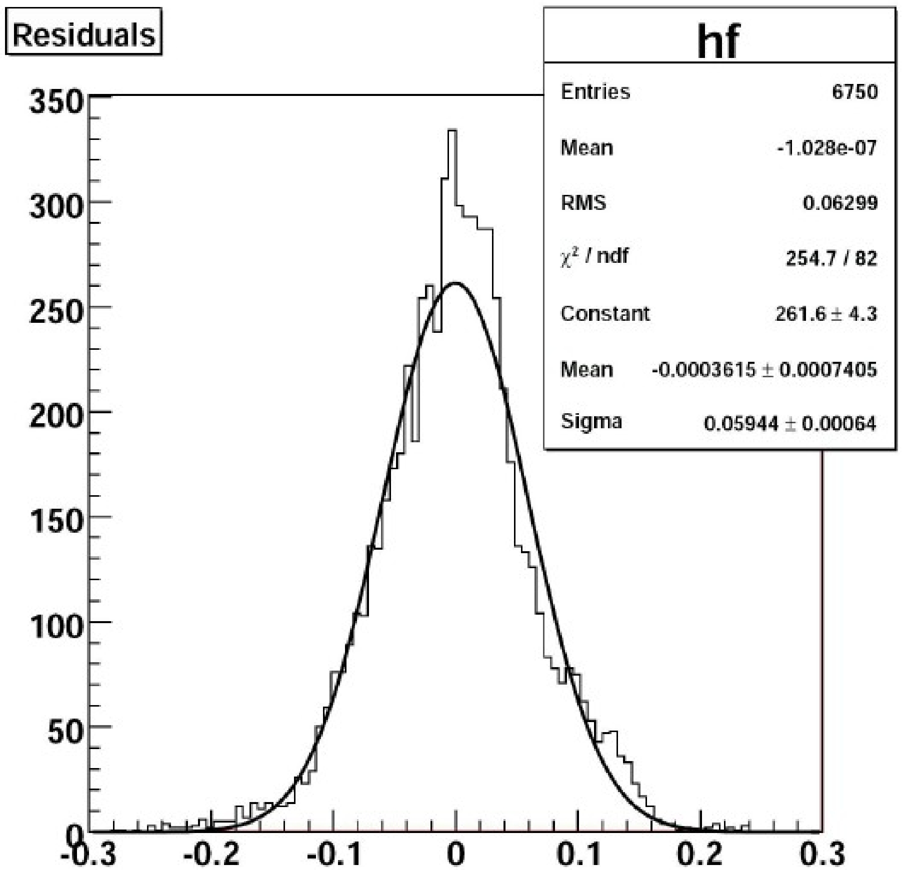


Current Results

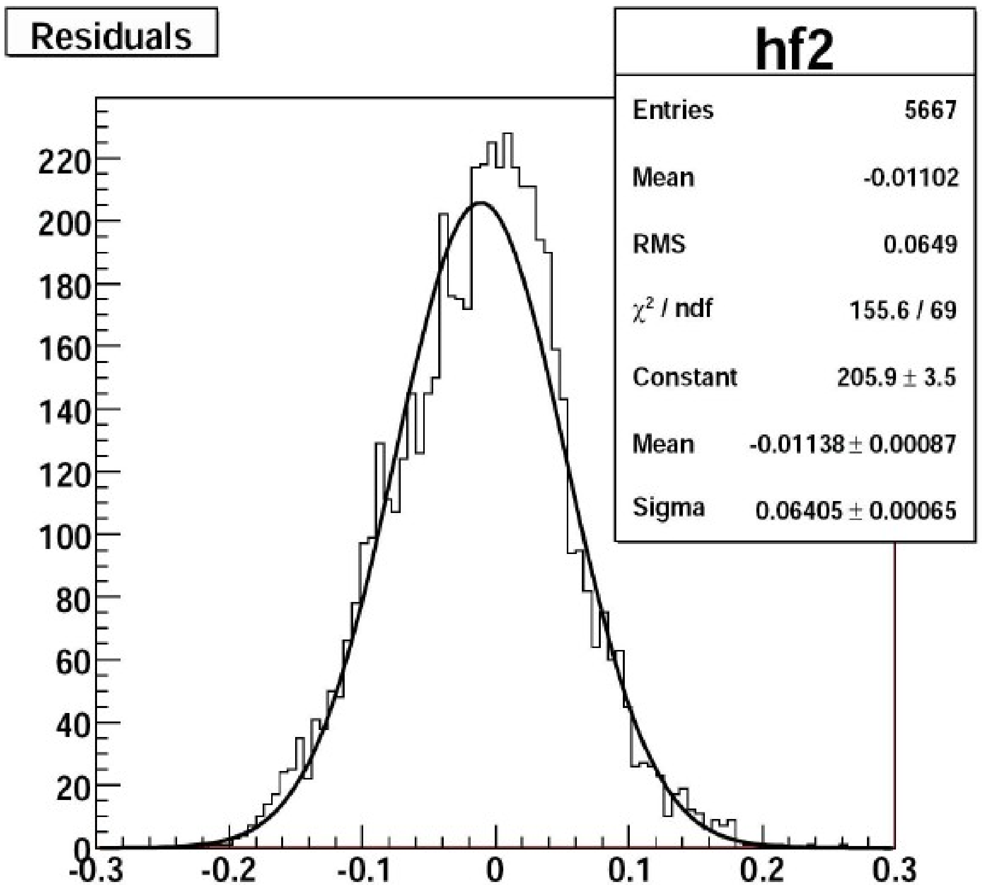


- Residual ionosphere after modeling ~0.05 TECU
- 0.01 TECU scatter in plot is noise level of GPS receiver
- Ionospheric wave clearly visible in residuals

Model Valid for Internal Stations



5 selected stations



5 stations NOT used for MIM fit

MIM GPS Conclusions

- GPS data already can achieve 0.05 TECU prediction level
- 0.01 TECU prediction level seems likely
- GPS can provide valuable calibration information for regions with dense GPS networks
  - LA region ok
  - Probably need more stations for Europe
    - Purchase commercial GPS survey network data?

Wide-Field Imaging

- Introduction
- Facet imaging
- w-projection

More Caveats

- Wide-field imaging here is defined as imaging the entire primary beam (and beyond) at the full resolution of the interferometer
- Does not cover mosaicking --- the joining of observations made at different pointings
- I am again biased by my VLBI experiences
- Many slides taken directly from 2006 talk by R. Perley

LOFAR Wide Field

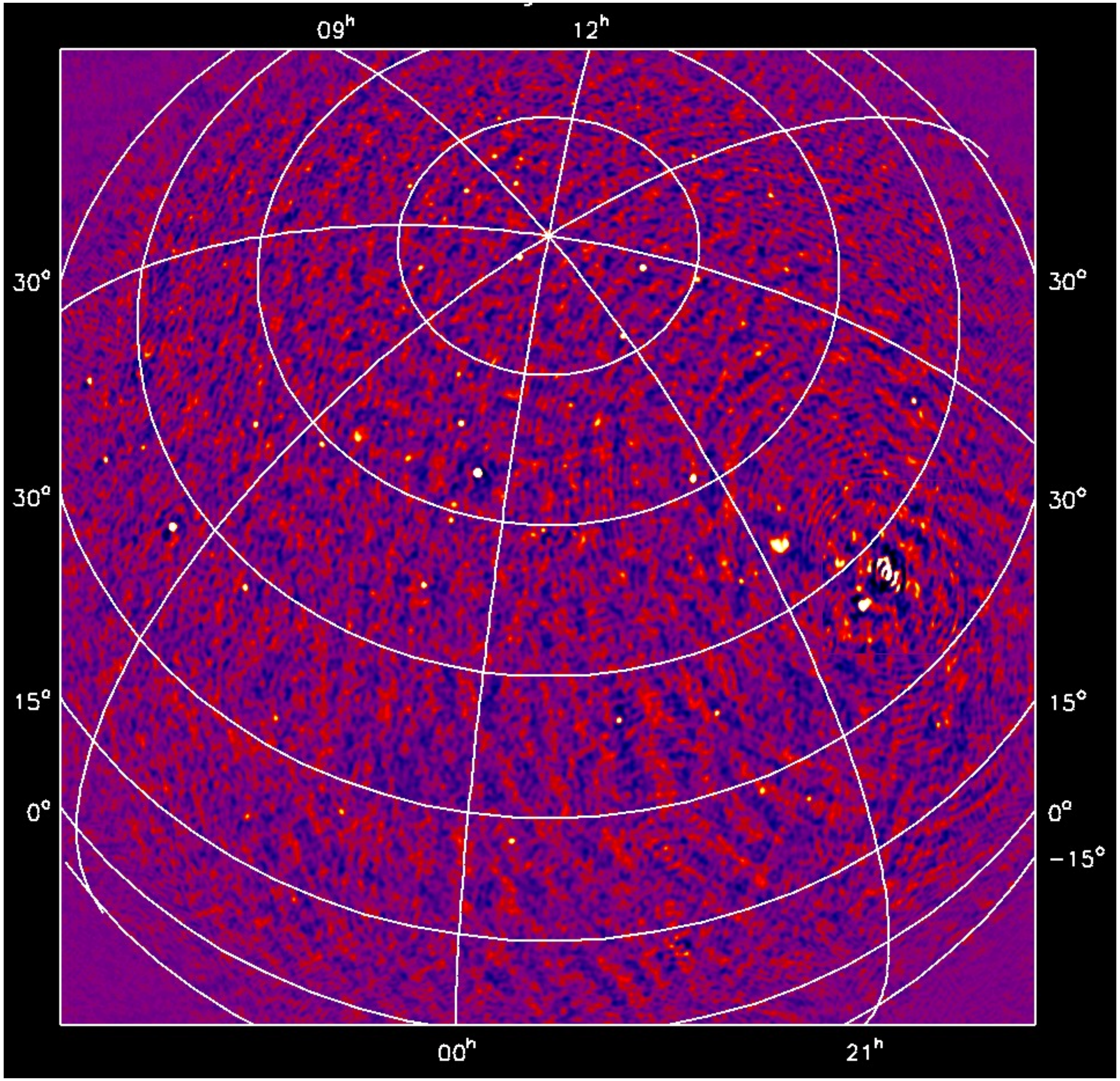


image by Sarod Yatawatta

- LOFAR CS1 all-sky image
- Made at ~50 MHz
- Cyg A self-calibrated and removed
- Dynamic range > 2000
- > 50 3C sources visible

Wide-Field Imaging Challenges

- Want to observe to full extent of sensitivity of primary (station) beam
- But the approximations which made the 2-D Fourier Transform relationship between visibilities and the sky brightness fail
- Must also allow for calibration to vary across field of view

Visibility Equation

- From earlier, we have a general relation between the complex visibility V(u,v,w), and the sky intensity *I*(*l*,*m*):

$$V(u,v,w)=\iint I(l,m)\exp\{-2\pi i[ul+vm+w(n-1)]\}\,dl\,dm/n$$

where $$n=\sqrt{1-l^2-m^2}$$

This equation is valid for:

spatially incoherent radiation from the far field,

phase-tracking interferometer

narrow bandwidth

What is 'narrow bandwidth'?

$$\Delta\upsilon << \frac{\theta_s}{\theta}\upsilon_0=\frac{\lambda}{D}\frac{d}{\lambda}\upsilon_0=\frac{d}{D}\upsilon_0$$

D is the baseline length, d is the station diameter

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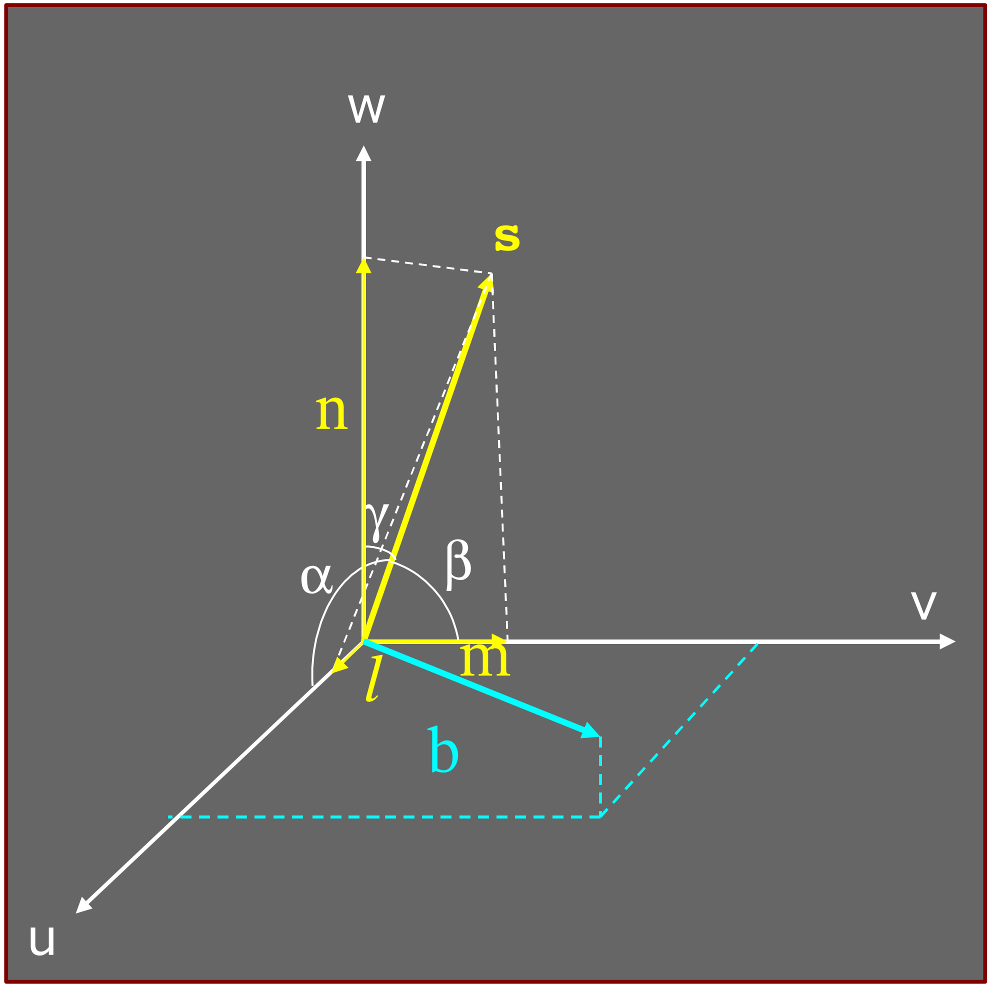
Review: Coordinate Frame

The unit direction vector **s** is defined by its projections on the (u,v,w) axes. These components are called the **Direction Cosines**, (l,m,n)

$$l = \cos(\alpha)$$
$$m = \cos(\beta)$$
$$n = \cos(\gamma) = \sqrt{1 - l^2 - m^2}$$



The baseline vector D is specified by its coordinates (u,v,w) (measured in wavelengths).

$$D = (\lambda u, \lambda v, \lambda w)$$

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VLA Approximation Breakdown

Under certain conditions, this integral relation can be reduced to a 2-dimensional Fourier transform.

This occurs when one of two conditions are met:

1. All the measures of the visibility are taken on a plane, or
2. The field of view is 'sufficiently small', given by:

$$\theta_{\max} < \sqrt{\frac{1}{w}} \leq \sqrt{\frac{\lambda}{D}} \sim \sqrt{\theta_s}$$

Table showing the VLA's distortion free imaging range (green), marginal zone (yellow), and danger zone (red)

λ	θ_{ant}	A	B	C	D
6 cm	9'	6'	10'	17'	31'
20 cm	30'	10'	18'	32'	56'
90 cm	135'	21'	37'	66'	118'
400 cm	600'	45'	80'	142'	253'

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Not a 3-D F.T. – But Close

- If your source, or your field of view, is larger than the 'distortion-free' imaging diameter, then the 2-d approximation employed in routine imagine are not valid, and you will get a crappy image.

- In this case, we must return to the general integral relation between the image intensity and the measured visibilities.

- The general relationship is not a Fourier transform. It thus doesn't have an immediate inversion.

- But, we can consider the 3-D Fourier transform of V(u,v,w), giving a 3-D 'image volume' F(l,m,n), and try relate this to the desired intensity, I(l,m),

- The mathematical details are straightforward, but tedious, and are given in detail on pp 384-385 in Synthesis Imaging in Radio Astronomy II.

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The 3-D Image Volume

- We find that:

$$F(l,m,n) = \iiint V_0(\mathrm{u,v,w}) \exp\left[2\pi \mathrm{i}(ul + vm + wn)\right] dudvdw$$

where

$$V_0(\mathrm{u,v,w}) = \exp(-2\pi \mathrm{iw}) V(\mathrm{u,v,w})$$

F(l,m,n) is related to the desired intensity, I(*l*,*m*),

$$F(l,m,n) = \frac{I(l,m)}{\sqrt{1-l^2-m^2}} \delta\left(\sqrt{1-l^2-m^2}-1\right)$$

This relation looks daunting, but in fact has a lovely geometric interpretation.

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Interpretation

- The modified visibility $V_0(u,v,w)$ is simply the observed visibility with no 'fringe tracking'.
- It's what we would measure if the fringes were held fixed, and the sky moves through them.
- The bottom equation states that the image volume is everywhere empty ($F(l,m,n)=0$), except on a spherical surface of unit radius where

$$l^2 + m^2 + n^2 = 1$$

- The correct sky image, $I(l,m)/n$, is the value of $F(l,m,n)$ on this unit surface
- Note: The image volume is not a physical space. It is a mathematical construct.

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Benefits of a 3-D Fourier Relation

- The identification of a 3-D Fourier relation means that all the relationships and theorems mentioned for 2-D imaging in earlier lectures carry over directly.
- These include:
  - Effects of finite sampling of V(u,v,w).
  - Effects of maximum and minimum baselines.
  - The 'dirty beam' (now a 'beam ball'), sidelobes, etc.
  - Deconvolution, 'clean beams', self-calibration.
- All these are, in principle, carried over unchanged, with the addition of the third dimension.
- But the real world makes this straightforward approach unattractive (but not impossible).

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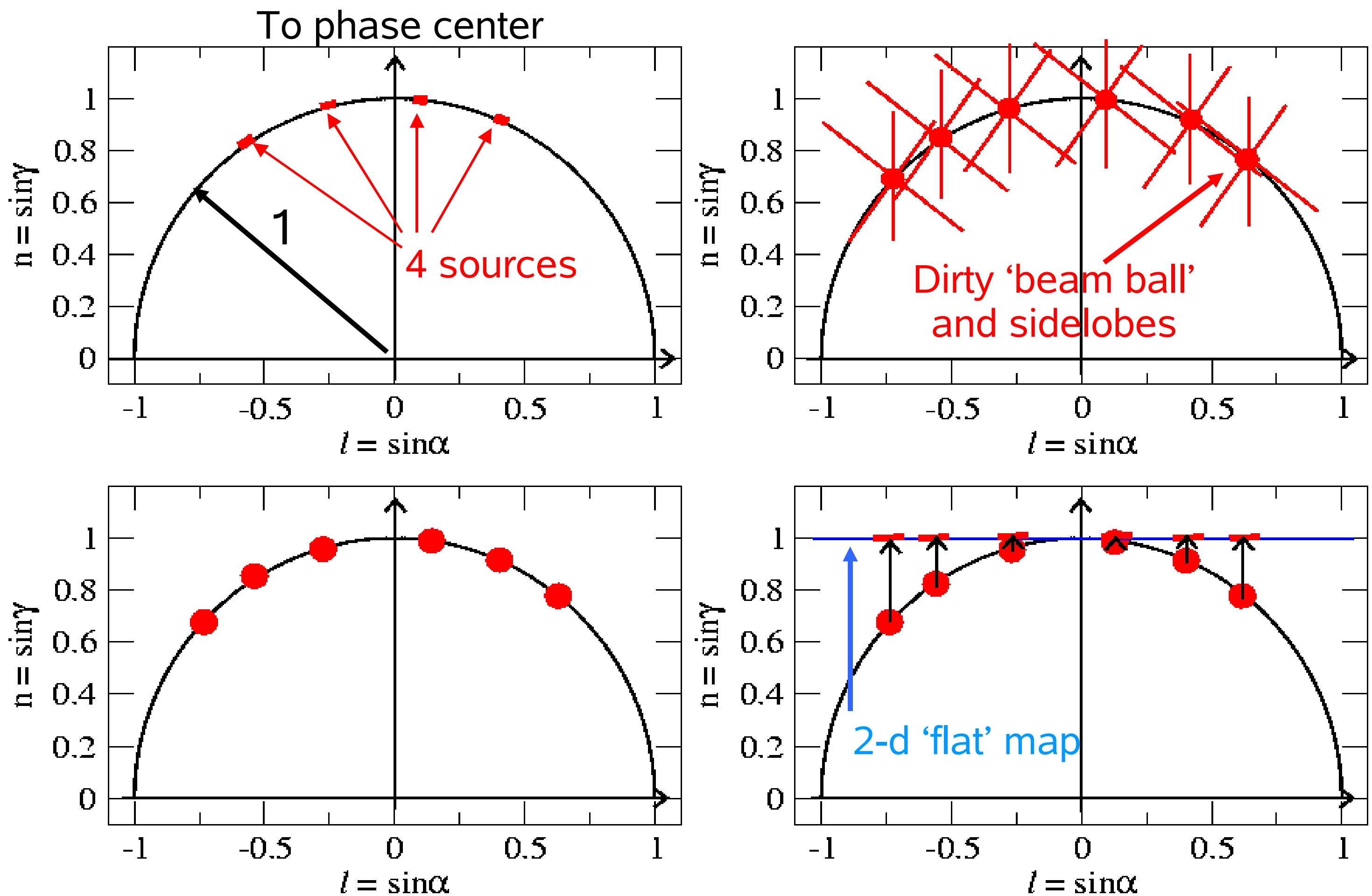
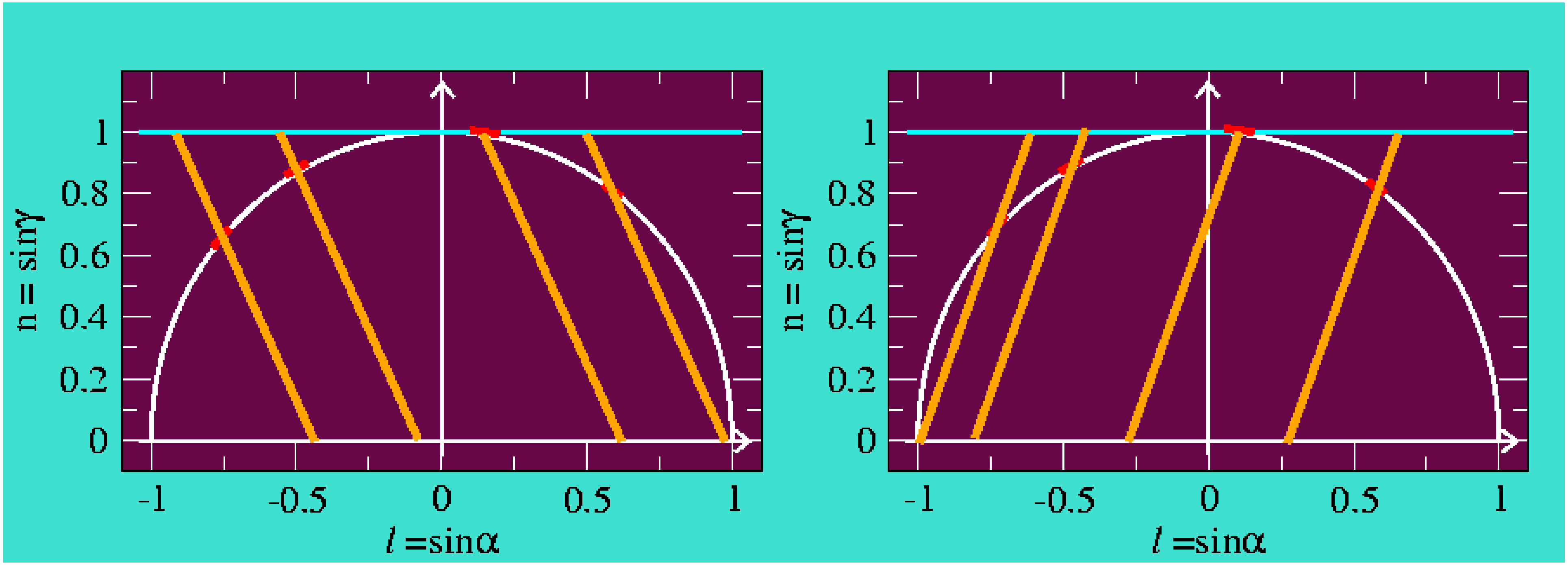
Illustrative Example:
A Slice Through the m = 0 Plane

Upper Left: True Image. Upper right: Dirty Image.
Lower Left: After deconvolution. Lower right: After projection



Snapshots in 3D Imaging

- A snapshot VLA observation, seen in '3D', creates 'line beams' (orange lines) , which uniquely project the sources (red bars) to the image plane (blue).
- Except for the tangent point, the apparent locations of the sources move in time.



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Apparent Source Movement

- As seen from the sky, the plane containing the VLA rotates through the day.
- This causes the 'line-beams' associated with the snapshot images to rotate.
- The apparent source position in a 2-D image thus rotates, following a conic section. The loci of the path is:

$$l' = l - \left(1 - \sqrt{1 - l^2 - m^2}\right) \tan Z \sin \Psi_P$$

$$m' = m + \left(1 - \sqrt{1 - l^2 - m^2}\right) \tan Z \cos \Psi_P$$

where Z = the zenith distance, and Ψ_P = parallactic angle,
And (l,m) are the correct angular coordinates of the source.

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Wandering Sources

- The apparent source motion is a function of zenith distance and parallactic angle, given by:

$$\tan \chi = \frac{\cos \varphi \sin H}{\sin \varphi \cos \delta - \cos \varphi \sin \delta \cos H}$$

$$\cos Z = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos H$$

where
H = hour angle
δ = declination
ϕ = antenna latitude

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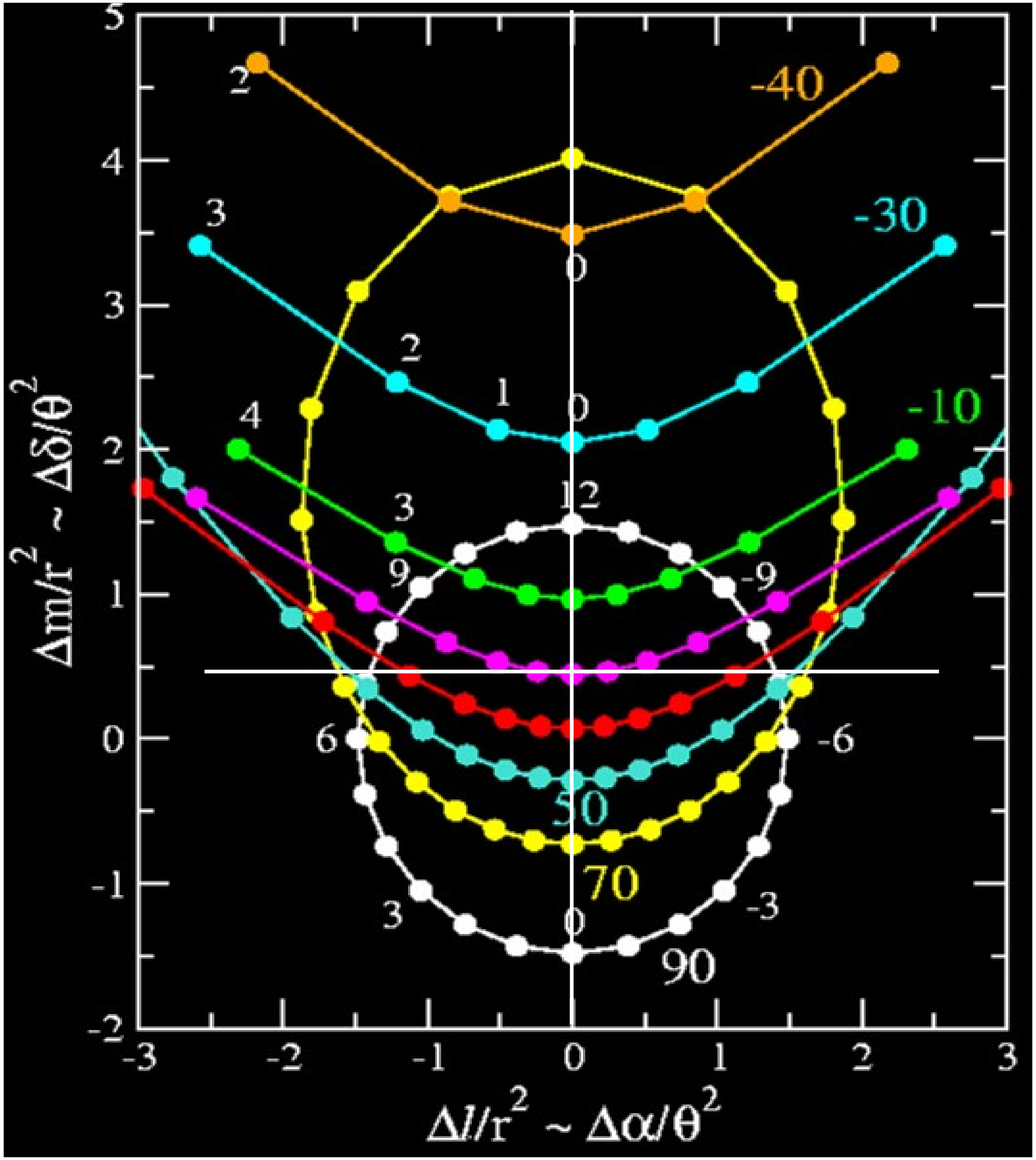
And around they go …

- On the 2-d (tangent) image plane, source positions follow conic sections.
- The plots show the loci for declinations 90, 70, 50, 30, 10, −10, −30, and −40.
- Each dot represents the location at integer HA.
- The path is a circle at declination 90.
- The only observation with no error is at HA=0, δ =34. (for the VLA)
- The error scales quadratically with source offset from the phase center.



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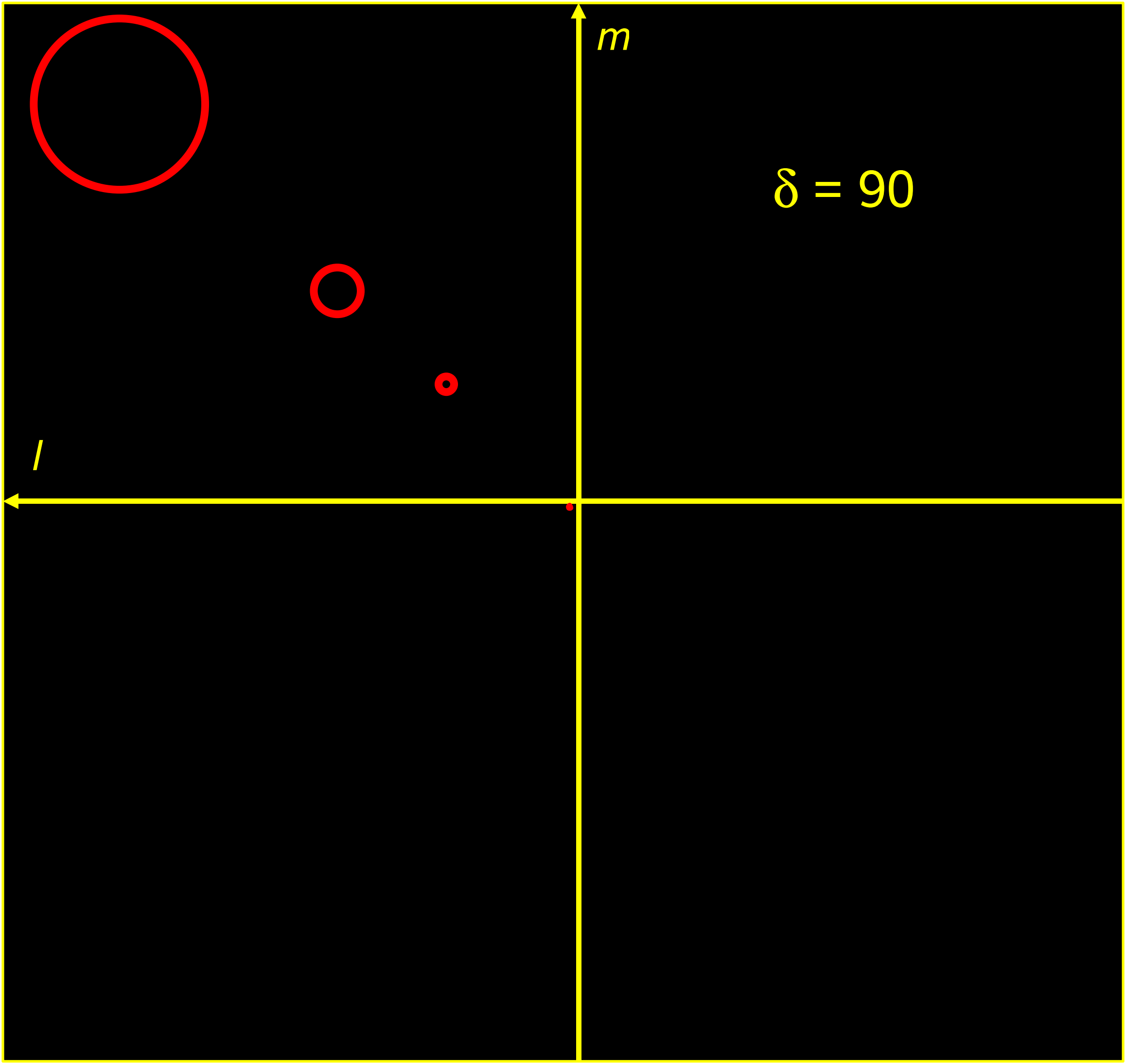
Schematic Example

- Imagine a 24-hour observation of the north pole. The `simple' 2-d output map will look something like this.
- The red circles represent the apparent source structures.
- Each doubling of distance from the phase center quadruples the extent of the distorted image.



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How bad is it?

- In practical terms …
- The offset is $(1 - \cos \gamma) \tan Z \sim (\gamma^2 \tan Z)/2$ radians
- For a source at the antenna beam half-power, $\gamma \sim \lambda/2d$
- So the offset, ε, measured in synthesized beamwidths, (λ/D) at the half-power of the antenna beam can be written as

$$\varepsilon = \frac{\lambda D}{8d^2} \tan Z$$

D = maximum baseline
d = antenna diameter
Z = zenith distance
λ = wavelength

- For the VLA's A-configuration, this offset error, at the antenna FWHM, can be written:

$$\varepsilon \sim \lambda_{cm} (\tan Z)/20 \qquad \text{(in beamwidths)}$$

This is very significant at meter wavelengths, and at high zenith angles (low elevations).

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So, What Can We Do?

There are a number of ways to deal with this problem.

2. Compute the entire 3-d image volume.

The most straightforward approach, but hugely wasteful in computing resources!

The minimum number of 'vertical planes' needed is:

$N_n \sim D\theta^2/\lambda \sim \lambda D/d^2$

The number of volume pixels to be calculated is:

$N_{pix} \sim 4D^3\theta^4/\lambda^3 \sim 4\lambda D^3/d^4$

But the number of pixels actually needed is: $4D^2/d^2$

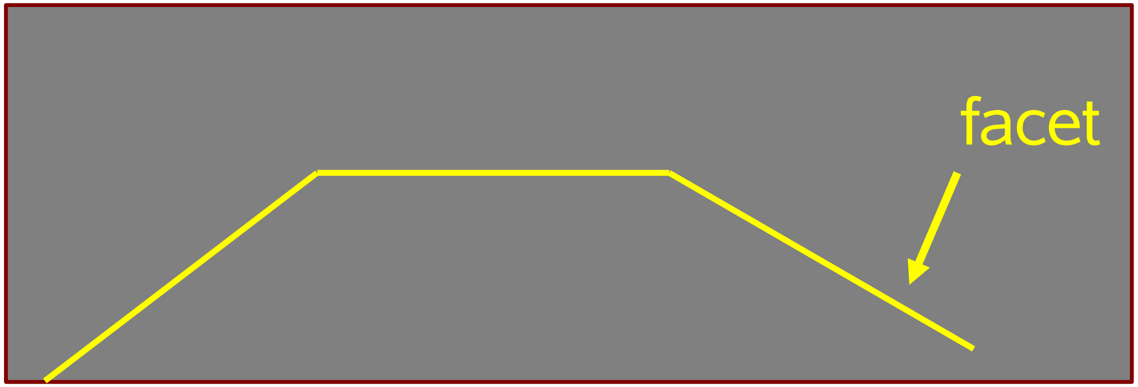
So the fraction of the pixels in the final output map actually used is: $d^2/\lambda D$. (~ 2% at λ = 1 meter in A-configuration!)

VLBI Context

- For global VLBI at 20 cm, one needs
  - 3200 planes
  - 2×10^{15} 3-D pixels
  - 6.4×10^{11} final pixels
  - Many Moore's Law times from now
- LOFAR 1000 km baselines at 5 m
  - 900 planes
  - 8.4×10^{9} 3-D pixels
  - 7.1×10^{8} final pixels
  - Possibly doable
- But this is for each frequency channel!

2. Polyhedron Imaging

- The wasted effort is in computing pixels we don’t need.
- The polyhedron approach approximates the unit sphere with small flat planes, each of which stays close to the sphere’s surface.



For each subimage, the entire dataset must be phase-shifted, and the (u,v,w) recomputed for the new plane

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Polyhedron Approach, (cont.)

- How many facets are needed?
- If we want to minimize distortions, the plane mustn't depart from the unit sphere by more than the synthesized beam, λ/B. Simple analysis (see the book) shows the number of facets will be:

$$N_f \sim 2\lambda D/d^2$$

or twice the number needed for 3-D imaging.

- But the size of each image is much smaller, so the total number of cells computed is much smaller.
- The extra effort in phase computation and (u,v,w) rotation is more than made up by the reduction in the number of cells computed.
- This approach is the current standard in AIPS.

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Polyhedron Imaging

- Procedure is then:
  - Determine number of facets, and the size of each.
  - Generate each facet image, rotating the (u,v,w) and phase-shifting the phase center for each.
  - Jointly deconvolve the set. The Clark/Cotton/Schwab major/minor cycle system is well suited for this.
  - Project the finished images onto a 2-d surface.
- Added benefit of this approach:
  - As each facet is independently generated, one can imagine a separate antenna-based calibration for each.
  - Useful if calibration is a function of direction as well as time.
  - This is needed for meter-wavelength imaging.

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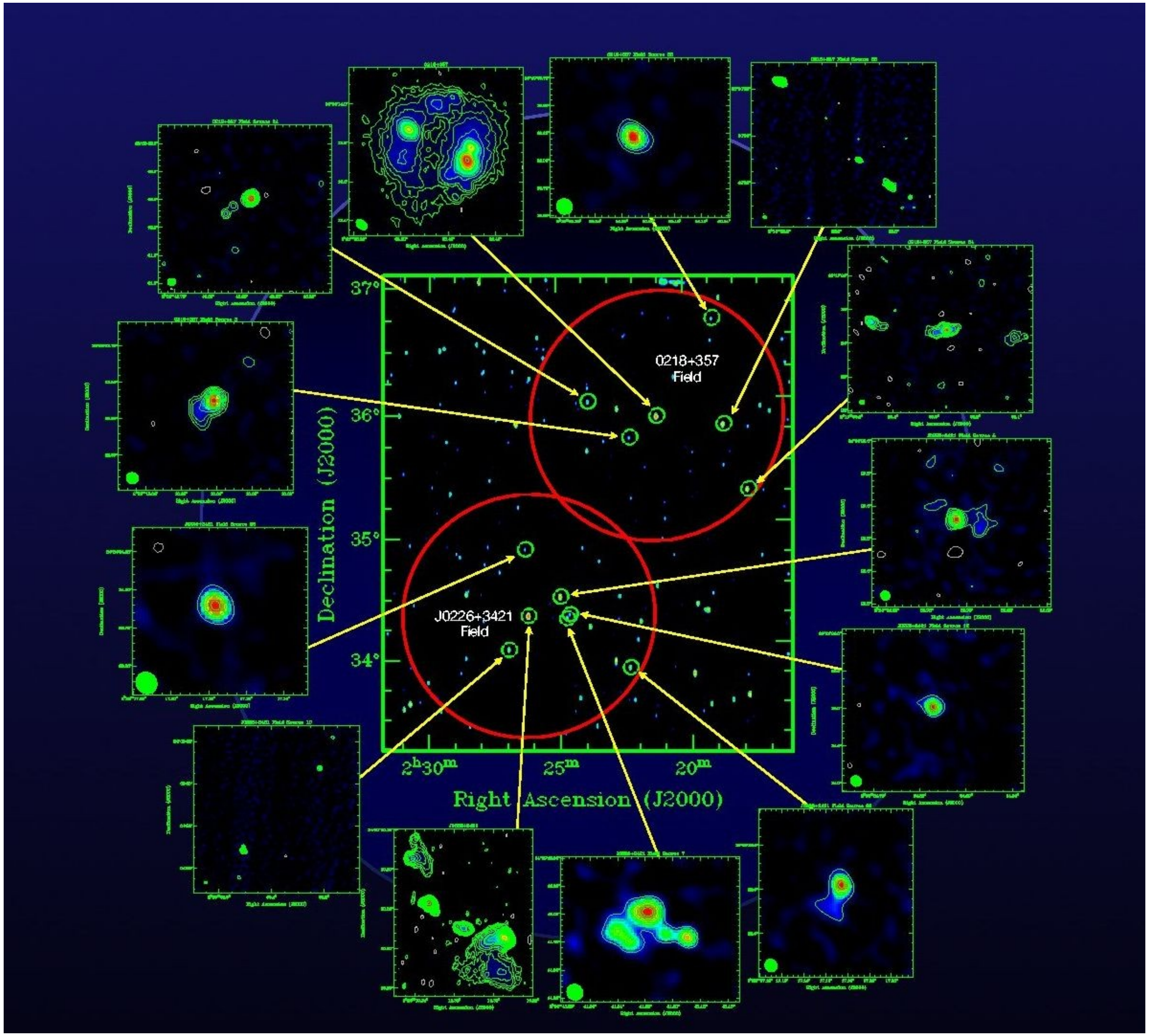
VLBI Perspective

- Not all facets need to be computed
  - Radio sky is mostly empty (uniform) at high resolution
  - Can just image in direction of known sources
- This can dramatically reduce the computational costs
- Current software development at JIVE (ALBUS) to develop parallelized software for cluster environment

VLBI Example: 320 MHz



- ParselTongue (python) scripts used to automate ionospheric calibration and imaging of individual sources for 320 MHz VLBI experiment

W-Projection

- Although the polyhedron approach works well, it is expensive, and there are annoying boundary issues – where the facets overlap.
- Is it possible to project the data onto a single (u,v) plane, accounting for all the necessary phase shifts?
- Answer is YES! Tim Cornwell has developed a new algorithm, termed ‘w-projection’, to do this.
- Available only in (what used to be known as) CASA (formerly known as AIPS++), this approach permits a single 2-D image and deconvolution, and eliminates the annoying edge effects which accompany re-projection.

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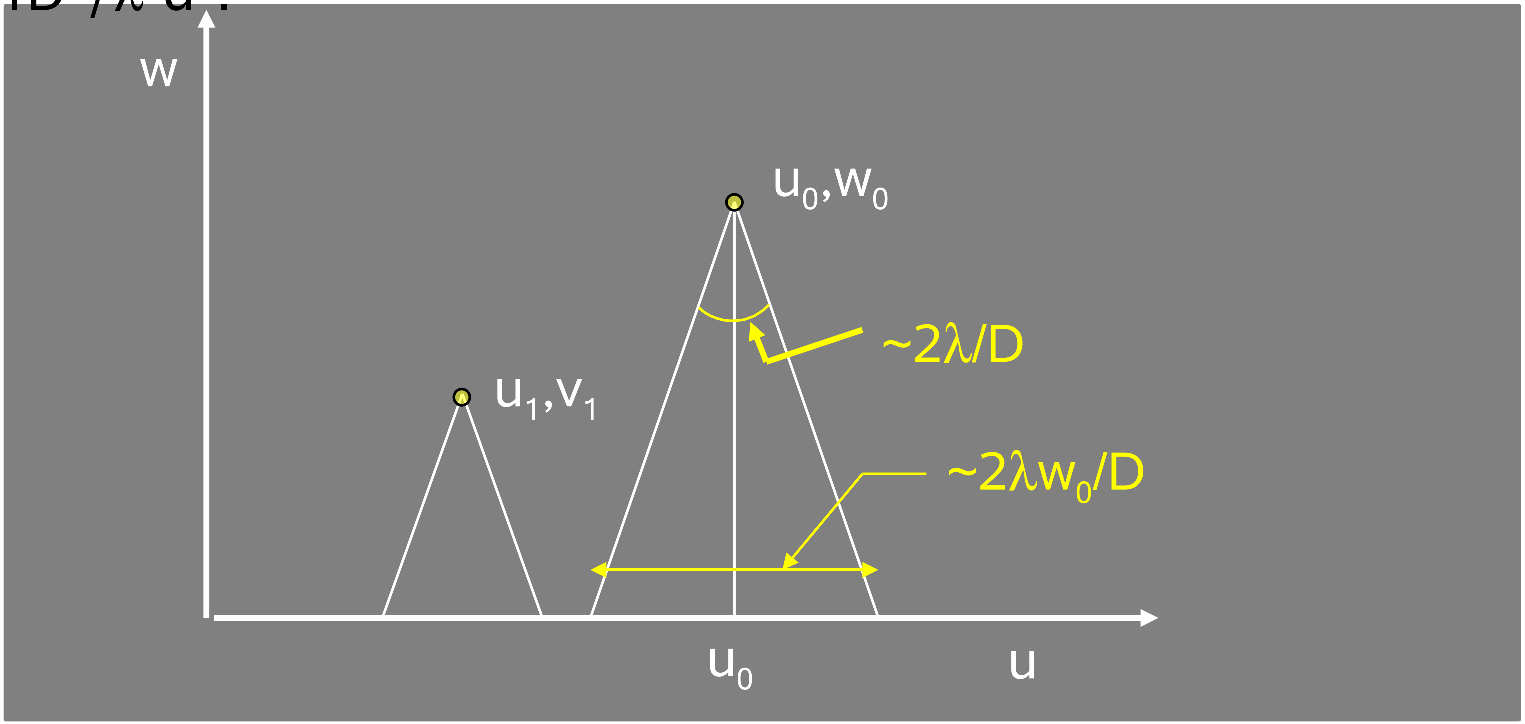
W-Projection

- Each visibility, at location (u,v,w) is mapped to the w=0 plane, with a phase shift proportional to the distance.
- Each visibility is mapped to ALL the points lying within a cone whose full angle is the same as the field of view of the desired map – $\sim 2\lambda/d$ for a full-field image.
- Area in the base of the cone is $\sim 4\lambda^2 w^2/d^2 < 4D^2/d^2$. Number of cells on the base which 'receive' this visibility is $\sim 4w_0^2 D^2/d^2 < 4D^4/\lambda^2 d^2$.



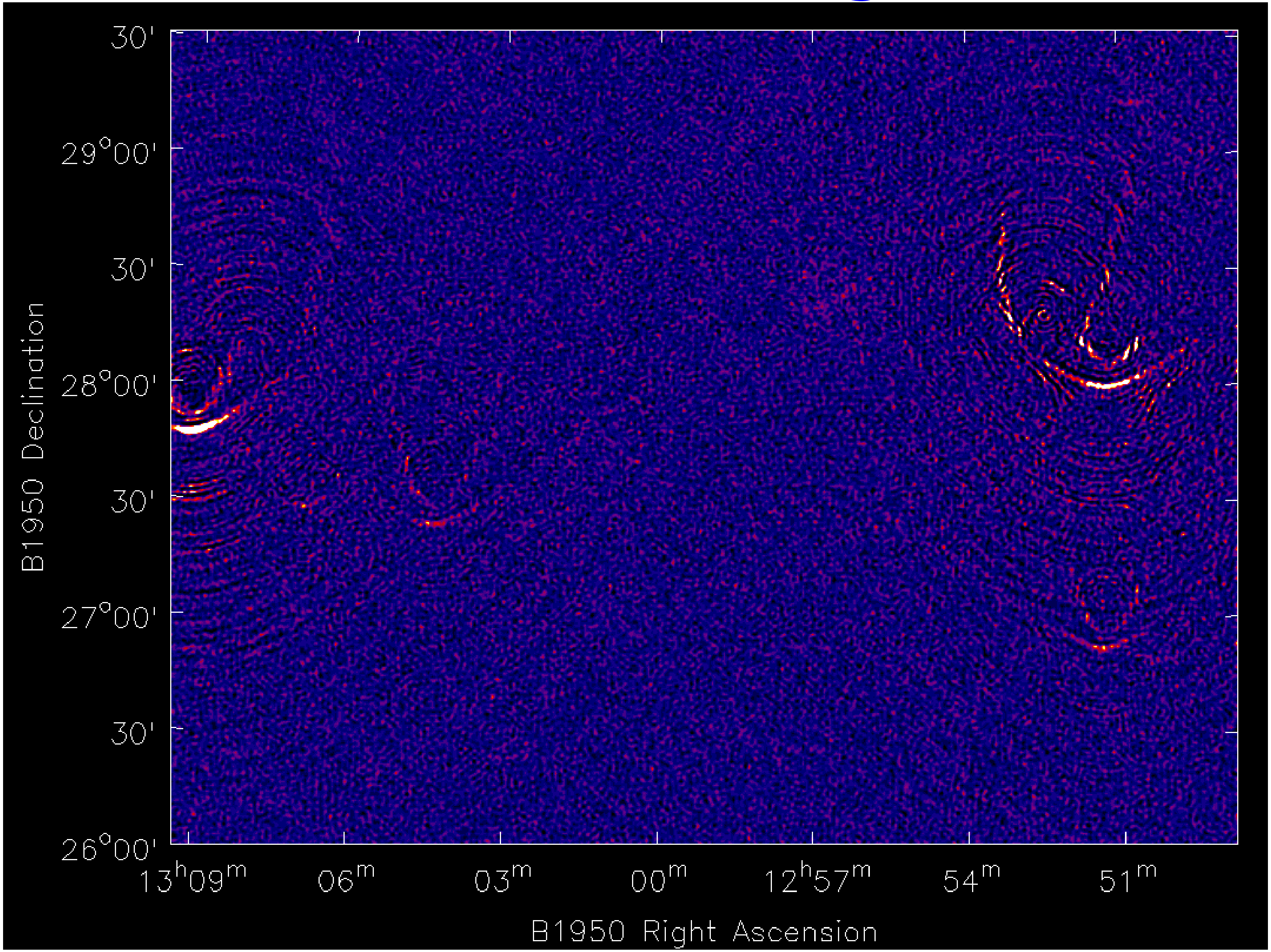
Perley 2006

W–Projection

- The phase shift for each visibility onto the w=0 plane is in fact a Fresnel diffraction function.
- Each 2–D cell receives a value for each observed visibility within an (upward/downwards) cone of full angle $\theta < \lambda/d$ (the antenna's field of view).
- In practice, the data are non–uniformly vertically gridded – speeds up the projection.
- There are a lot of computations, but they are done only once.
- Spatially–variant self–cal can be accommodated (but hasn't yet, as far as I know).

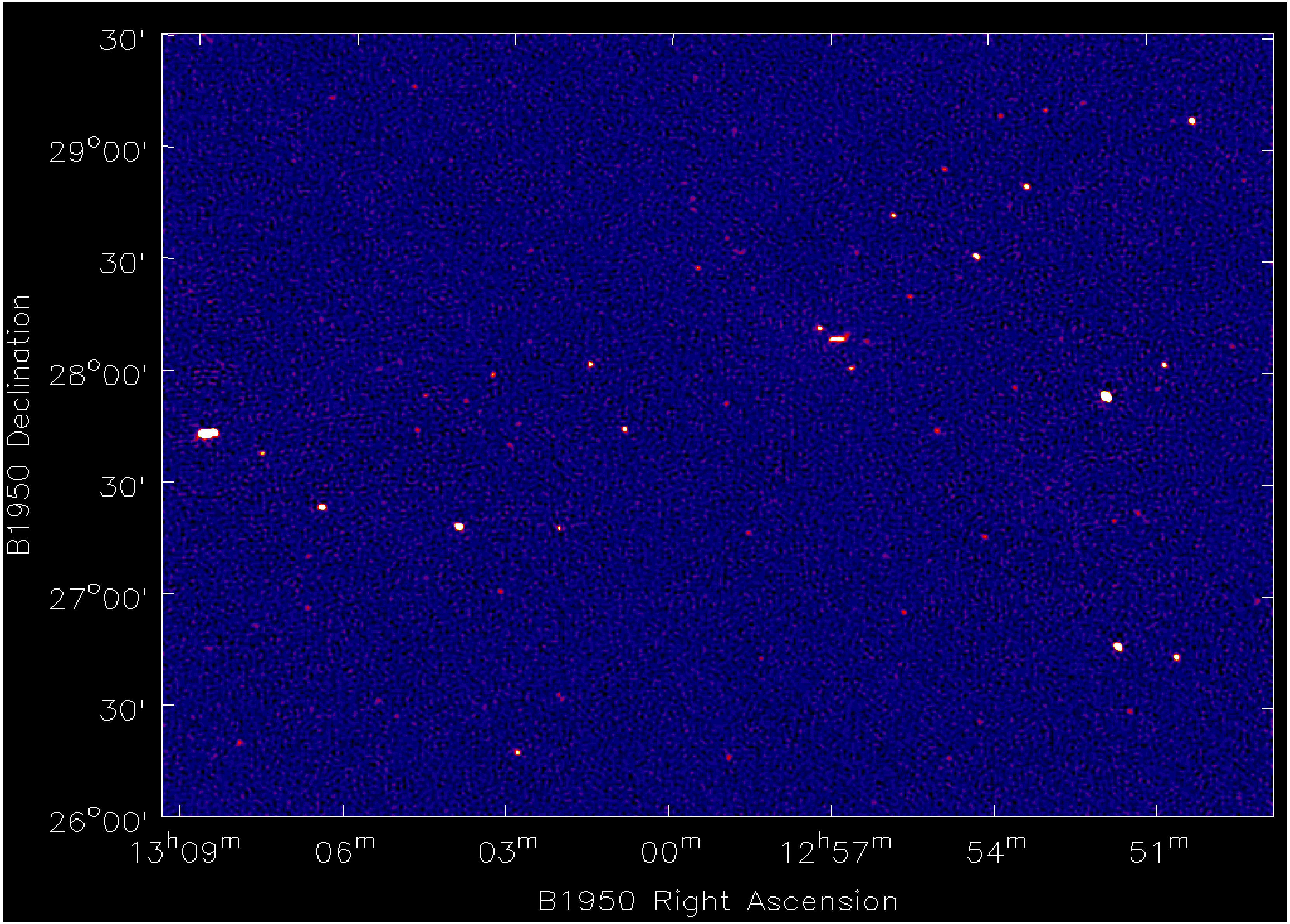
Perley 2006

An Example – Without '3-D' Procesesing



Perley 2006

Example – With 3D Processing



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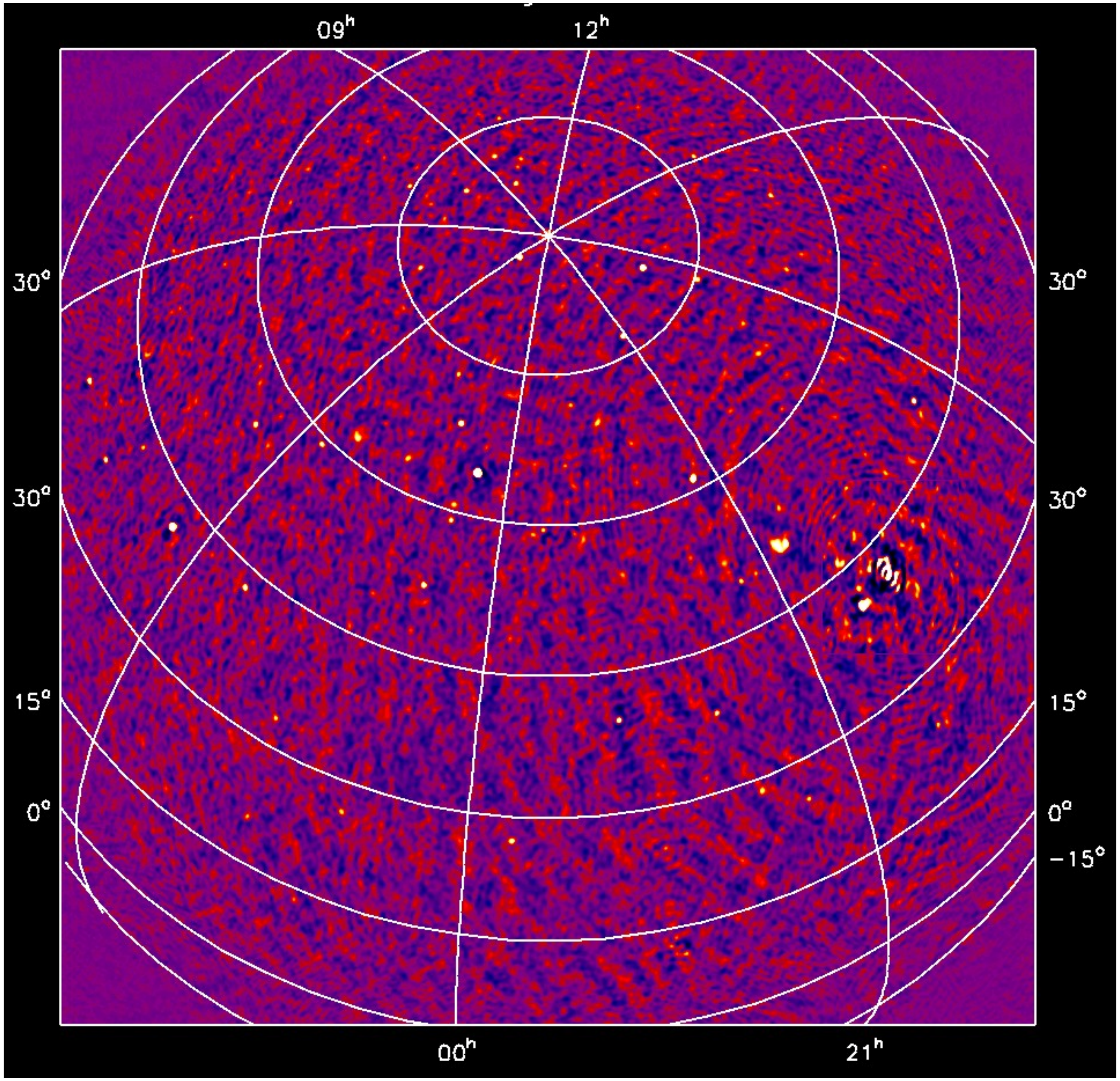
LOFAR Wide Field



- LOFAR CS1 all-sky image
- Made using MeqTrees software, using CASA w-projection

image by Sarod Yatawatta

Wide-Field Imaging Conclusions

- Arrays which measure visibilities within a 3-dimensional (u,v,w) volume, such as the VLA, LOFAR, VLBI, cannot use a 2-D FFT for wide-field and/or low-frequency imaging.
- The distortions in 2-D imaging are large, growing quadratically with distance, and linearly with wavelength.
- In general, a 3-D imaging methodology is necessary.
- Recent research shows a Fresnel-diffraction projection method is the most efficient, although the older polyhedron method is better known.
- Undoubtedly, better ways can yet be found.

Perley 2006

The End