Fourier radio interferometric imaging : a super-brief summary

Blue = definition          Red = important

Basic expressions for 2 antennas separated by d :

Phase shift between 2 antennas separated by d : $\psi = 2\pi d \sin\theta / \lambda \approx 2\pi d\theta/\lambda \approx 2\pi u\theta$

- Point source

$P_\oplus = 2\, E_o^2\, (1+\cos\psi)$ ⇐ just for info, not used actually

$P_\otimes = E_1.E_2^* = E_o^2 \exp(i\psi)$    $Re(P_\otimes) = E_o^2 \cos\psi$    $V(d) = \exp(i\psi)$

- Extended source

$P_\oplus = \int_{source} 2\, E(\theta)^2\, d\theta + \int_{source} 2\, E(\theta)^2 \cos\psi\, d\theta$

$P_\oplus = 2 <T_A>|_{source} + 2Re( \int_{source} T_A(\theta) \exp(i\psi)\, d\theta )$

With $T_A(\theta) \approx E(\theta).E(\theta)^* \approx |E(\theta)|^2$

By extension : $V(d) = ( \int_{source} T_A(\theta) \exp(i\psi)\, d\theta ) / ( \int_{source} T_A(\theta)\, d\theta )$

$= ( \int_{source} T_A(\theta) \exp(i\psi)\, d\theta ) / <T_A>|_{source}$

$\Rightarrow P_\oplus = 2 <T_A>|_{source} [ 1 + Re(V(d)) ]$

$P_\otimes = \int_{source} E(\theta)^2 \exp(i\psi)\, d\theta = ( \int_{source} T_A(\theta) \exp(i\psi)\, d\theta ) = V(d) <T_A>|_{source}$

Generalisation at 2D (with $u=x/\lambda$ & $v=y/\lambda$) :

$V(u,v) = ( \int_{source} T_A(\theta,\phi) \exp[i2\pi(u\theta+v\phi)]\, d\theta d\phi ) / ( \int_{source} T_A(\theta,\phi)\, d\theta d\phi )$

$V(u,v) = t_A(u,v) / <T_A>|_{source}$    ⇒ ZVC theorem

Also

$V(u,v) = P_\otimes / <T_A>|_{source} = < E(0,0).E(u,v)^* > / <T_A>|_{source}$

= measured correlations (to a constant value)

with

$E(\theta,\phi) = T.F. [E(u,v)]$ ⇔ $E(u,v) = T.F.^{-1} [E(\theta,\phi)]$

- Imaging an extended source

$T_A(\theta,\phi) = 1/4\pi \times [ g(\theta,\phi) \otimes T(\theta,\phi) ]$

$\Rightarrow t_A(u,v) = G(u,v).t(u,v)$

with

$T_A(\theta,\phi) = T.F. [t_A(u,v)]$ ⇔ $t_A(u,v) = T.F.^{-1} [T_A(\theta,\phi)]$

$T(\theta,\phi) = T.F. [t(u,v)]$ ⇔ $t(u,v) = T.F.^{-1} [T(\theta,\phi)]$

$G(u,v) = 1/4\pi \times TF[g(\theta,\phi)]$

- How to compute G(u,v) ?

Apply to a Point source :

$T(\theta,\phi) = \delta \Rightarrow t(u,v)=1 \Rightarrow t_A(u,v) = G(u,v)$

$T_A(\theta,\phi) = 1/4\pi \times g(\theta,\phi) = E(\theta,\phi).E(\theta,\phi)^* = |E(\theta,\phi)|^2$

$\Rightarrow t_A(u,v) = G(u,v) = E(u,v) \otimes E^*(u,v)$