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Surprises in classical physics: radiation problems in stable and linear plasmas

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Abstract The discussion of erroneous common-sense ideas is useful for developing the critical spirit of students and improving the understanding of the physics involved. We give here a few examples, connected with radiation in stable and linear plasmas. Though this subject is now fairly well-known by physicists, it is often applied incorrectly in astrophysical and geophysical contexts, where—as it should be—thinking more deeply about problems yields a lot of surprises.

Résumé La discussion des idées fausses joue un rôle intéressant pour développer l'esprit critique des étudiants et améliorer la compréhension de la physique. On donne ici quelques exemples, rencontrés dans des problèmes de rayonnement en plasmas linéaires et stables. Ce sujet, qui est maintenant bien connu des physiciens, est souvent, appliqué de manière incorrecte en astronomie et en géophysique, et, comme il est normal, une étude non superficielle des problèmes apporte quelques surprises.

1. Introduction

Levy-Leblond (1980) has recently emphasised the role of wrong theories in developing the critical spirit of science students. In addition to wrong theories, the present scientific literature also contains and uses 'well-known' though often incorrect ideas: the error lies in applying correct concepts out of their domain of validity, or equivalently the use of implicit (or forgotten) hypotheses.

This situation, which has been discussed recently in genetics (Jacquard 1982), is particularly widespread in those sciences such as astrophysics or geophysics, that use a lot of physics, but employ few physicists.

Some examples connected with radiation problems have been discussed by Ginzburg (1979). We study below a few others, involving radiation in stable and linear plasmas.

Though the selection of examples is very subjective, they have been chosen in order both to illustrate classical methods of calculations in the field (source harmonic (sections 2 and 4) or not (sections (2 and 3), plasma 'cold' (section 2), 'hydrodynamic' (section 3) or 'Vlasov's' (section 4)), and to be of interest in other contexts.

Section 2 recalls the so-called Herlofson paradox, i.e. how a source loses energy in a 'lossless' system. This introduces the collisionless losses in a plasma and the getting round poles which is then used in the subsequent sections. Section 3 deals with the (un)shielding of a moving charge, and section 4 recalls the importance of plasma effects in the losses of a (harmonic) hertzian dipole, even far from the plasma frequency.

2. Losses in a system without losses

The common-sense idea that a harmonic source cannot lose energy in a system which neither radiates nor has any collisions or other loss mechanisms, has led to a lot of apparent paradoxes and irrelevant scientific disputes. Here, the error lies in considering as stationary a problem which is in fact transient.

We recall below a simple example occurring in plasma physics (for a detailed account, see Crawford and Harker 1972) which is representative of many equivalent problems in different contexts. Take a plane (infinite in y and z directions) capacitor filled with an inhomogeneous electron plasma. Assume the plasma to be cold and without collisions ('no losses'). Feed the capacitor with an oscillating current. The question is: calculate the

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losses of the source, if there are any losses.

Let $\omega_p(x)(\omega_p = (ne^2/\varepsilon_0 m)^{1/2})$ be the (angular) plasma frequency spatial variation, and the plates located at $x = 0$, $x = X$. The cold plasma is only temporally dispersive; at frequency ω, the local dielectric permittivity is $\varepsilon(x) = 1 - \omega_p^2(x)/\omega^2$. We assume that $\varepsilon(x)$ has one zero at $x = x_0 \in]0, X[$.

The naïve calculation proceeds as follows. Let the driving current density be J (variation in $e^{-i\omega t}$). The field in the plasma is

$$E(x) = -J/[i\omega\varepsilon_0\varepsilon(x)]. \quad (1)$$

Hence the resistance per surface unit

$$R(\omega) = \mathrm{Re}(V/J) = \frac{1}{\omega\varepsilon_0}\,\mathrm{Im}\int_0^X \frac{dx}{\varepsilon(x)}. \quad (2)$$

The integral is undefined since there is a pole on the integration axis; interpreting it as principal value plus residue yields

$$R(\omega) = \pm\frac{\pi}{2\varepsilon_0\,|\omega_p'(x_0)|}. \quad (3)$$

This resistance poses two problems: first, its sign is undefined; second, if it represents losses, where does the energy go? This is sometimes called the 'Herlofson paradox'.

The elementary way of solving the first problem is to assume that the plasma has small losses. Introducing an effective collision frequency ν, the equation of motion for electrons becomes $m\,dv/dt = -eE - \nu m v$; hence the permittivity (for $e^{-i\omega t}$ variation) $\varepsilon_\nu(x) = 1 - \omega_p^2(x)/\omega(\omega + i\nu)$. Thus equation 2 becomes

$$R(\omega) = \frac{1}{\omega\varepsilon_0}\int_0^X dx\,\frac{z(1-\varepsilon(x))}{\varepsilon^2(x) + z^2} \quad (4)$$

where $z = \nu/\omega$ and $\varepsilon(x)$ is the permittivity for $\nu = 0$. When $z \to 0$ the result is independent on z and equation (4) reduces to equation (3) with the + determination.

However, this does not solve the second difficulty: where does the energy go if we insist on suppressing the collisional losses?

The solution is to recognise that, contrary to what was implicitly assumed when taking a single Fourier component ($e^{-i\omega t}$), the problem is not stationary in the resonant region $x \simeq x_0$: the field $E(x_0)$ increases, even in the limit $\omega t \to \infty$. Thus, the energy supplied to the capacitor serves to increase its electrostatic energy.

Therefore, one must take into account the initial (or boundary) conditions, and the causality, i.e. the fact that the circuit has to be switched-on at some time, and that the perturbed quantities are zero before switch-on. The usual expeditive method is to add to ω a small positive imaginary part, in order that $e^{-i\omega t}$ represents a slow temporal growth (see Lighthill 1978). This displaces the pole from the integration axis and specifies the sign of the residue. As is well-known, a similar procedure can also be used in radiation problems, to yield the retarded Green functions.

Here, one can calculate the actual time variation by using a Laplace transformation. Let the source current be $J(t) = J\sin\omega t H(t)$, where $H(t)$ denotes the unit-step function, and take all the perturbed quantities to be zero for $t \leq 0$. Define, for Re $(s) > 0$

$$J(s) = \int_0^\infty J(t)e^{-st}\,dt = \omega J/(s^2 + \omega^2).$$

Laplace transforming Maxwell equations and the linearised electrons' equations of motion yields, instead of equation (1)

$$E(s, x) = \frac{J(s)}{\varepsilon_0 s(1 + \omega_p^2(x)/s^2)}. \quad (5)$$

Thus, for $t \geq 0$

$$E(t, x) = \frac{1}{2i\pi}\int_{A-i\infty}^{A+i\infty} E(s, x)e^{st}\,ds$$
$$= \frac{\omega J}{\varepsilon_0}\left(\frac{\cos(\omega_p(x)t) - \cos(\omega t)}{\omega^2 - \omega_p^2(x)}\right). \quad (6)$$

In equation (6), the integration contour had to be closed in the left-hand side complex plane, giving the contribution of the residues $s = \pm i\omega$, $s = \pm i\omega_p(x)$. Equation (6) yields $E(t, x_0) = Jt\sin(\omega t)/2\varepsilon_0$, showing linear temporal growth in the resonant region.

From equation (6), the terminal voltage is easily calculated in the limit $\omega t \to \infty$. The losses come from its $\sin(\omega t)$ component due to the contribution of the resonant region. This yields equation (3) with the + determination.

Two remarks are in order. First, the main practical effect of introducing small collisions (or another loss mechanism) into the problem, is to ensure saturation of the growing quantities: the key point is that collisions remove energy at exactly the same rate as it is provided by the source in the collisionless case. Second, in the lossless case, the result is not valid in practice for long times since the temporal growth can invalidate the linearisation of the equations.

This example is representative of many less simple radiation problems in plasmas (Crawford and Harker 1972). The most familiar one is the classic Landau damping (see for instance Stix 1962), which occurs when instead of assuming the plasma to be cold, one takes into account the particles velocities in the absence of perturbation. Then, the plasma becomes also spatially dispersive, i.e. the current at a given point does not depend only on the field at that point. The calculation of a (small-amplitude) field component (wave number $\boldsymbol{k}$, frequency ω) in a homogeneous plasma, involves an integration over the particles velocities $\boldsymbol{v}$: there is a pole at $\omega - \boldsymbol{k} \cdot \boldsymbol{v} = 0$, which is a Cerenkov condition ensuring that the particle is at rest in the wave frame and thus interacts resonantly. The integration contour is prescribed as previously (in this context, this is called Landau prescription) yielding wave damping in a maxwellian plasma. As previously, the solution to the apparent paradox ('losses without losses') is that the problem is nonstationary. Actually, in the absence of collisions, the damping reverses and exhibits damped oscillations (see Davidson 1972) when the temporal growth yields linearisation breakdown.

The two cases present also an interesting historical similarity. Tonks (1931), studying the inhomogeneous capacitor problem, discarded the residue and missed the losses. Vlasov (1945), studying the homogeneous warm plasma problem, similarly discarded the residue, but was promptly and vigorously criticised by Landau (1946). Both cases have led to irrelevant disputes about the physical nature of the losses.

The form of equation (2) or equation (4), where the losses stem from an integration over a continuous network of undamped oscillators including a resonant one, appears in many different contexts. It has been used recently (Greenberg 1983) to obtain the confinement of ringlets (playing the role of the continuous set of oscillators) by a 'shepherding' satellite (the harmonic source) in planetary rings.

3. Fooling the Debye shielding

It is well-known that the perturbation due to an electric charge in a plasma is generally shielded at a distance called the Debye length. This feature corresponds to the static solution of a Klein–Gordon equation, and occurs in many other contexts. The shielding requires several conditions which are not always completely mentioned in the textbooks (see for instance Stix 1962, Landau and Lifshitz 1981) and are sometimes forgotten in astrophysics (some examples of misapplications are quoted in Opik 1964).

In particular, the shielded field which is a solution of a *static* equation, is not expected to be valid for moving charges, and in fact it is not at all for suprathermal ones. To illustrate this feature, which does not appear to be widely known even today (for instance Morfill *et al* 1983), we shall calculate the force between two charges moving in an electron plasma. The result is rather basic, though it cannot—as far as I know—be found as such in the textbooks, and the calculation exhibits some interesting features typical of radiation in plasmas.

In its simplest form, the problem is the following. Let two charges q_1, q_2 located at $z_1 = -\delta L + vt$, $z_2 = vt (\delta = \pm 1)$ on the $0z$ axis, i.e. moving together with velocity $\boldsymbol{V} = v\boldsymbol{0z}$ in the frame of an equilibrium electron plasma. In the linear description, the force on (say) particle 1 consists of two components. The first one is the familiar drag due to its interaction with the plasma charges (Chandrasekar 1943, Spitzer 1962, Cohen 1961, Sitenko 1967). The other one, which we shall calculate, is due to particle 2 in the presence of the plasma.

Let us try to further simplify the problem, while keeping the essentials of the physics. We cannot, as in section 2, assume the plasma to be cold, since the temperature plays a key role in the shielding. However, we shall make two main approximations. First, we shall neglect the transverse electromagnetic part of the field: this could be justified *a posteriori* but we shall only say here that $v/c \ll 1$, and also that the unshielding is due to Cerenkov radiation, which does not exist for EM waves in a plasma (without a static magnetic field), since their phase velocity is greater than c. Thus, the plasma is defined only by its longitudinal permittivity $\varepsilon_L(k, \omega)$. In the collisionless Vlasov description (i.e., broadly speaking, if there are many particles in a cubic Debye-length $L_D = v_T/\omega_p (v_T = (\chi T/m)^{1/2})$, the classic and nonrelativistic expression of ε_L is (see for instance Sitenko 1967):

$$\begin{aligned} \varepsilon_L(k, \omega) &= 1 + \frac{\omega_p^2}{k^2 v_T^2}(1 - \phi(z) + \mathrm{i}\pi^{1/2} z \mathrm{e}^{-z^2}) \\ z &= \omega/\sqrt{2}\, k v_T \\ \phi(z) &= 2z\mathrm{e}^{-z^2}\int_0^z \mathrm{e}^{x^2}\,\mathrm{d}x. \end{aligned} \tag{7}$$

The imaginary part represents losses due to the resonant interaction (Landau damping) mentioned in section 2. Our second important approximation will be to assume $z \gg 1$ in ε_L (note that $z \to \infty$ yields the cold plasma used in section 2). In other words, we shall take dispersion into account in the simplest (first order) way. Since only waves $\omega = \boldsymbol{k} \cdot \boldsymbol{v}$ will contribute to the calculation, we expect the result to be correct if $v/v_T \gg 1$. Developing $\phi(z)$ up to second order in $1/z^2$, neglecting the small term in the numerator, and setting $\omega \sim \omega_p$, in the small term in the denominator, yields

$$(1 - \varepsilon_L)/\varepsilon_L \simeq \omega_p^2/[\omega^2 - \omega_p^2 - 3k^2 v_T^2 + \mathrm{i}o\,\mathrm{sgn}(\omega)]. \tag{8}$$

At this order of the development, other expressions could have been chosen; the above choice permits easy calculations and is identical to that obtained by using a set of hydrodynamic equations (with an adiabatic hypothesis). (See for instance Clemmow and Dougherty 1969.)

The calculation proceeds straightforwardly as follows. The charge q_2 produces the density distribution

$$\rho_2(\boldsymbol{r}, t) = q_2\,\delta(x)\,\delta(y)\,\delta(z - vt), \tag{9}$$

where $\delta(x)$ denotes the Dirac distribution.

This induces a longitudinal field in the plasma, which derives from a scalar potential, and thus satisfies, in Fourier space

$$\mathbf{E}_2(\boldsymbol{k}, \omega) = -\mathrm{i}\boldsymbol{k}\Phi_2(\boldsymbol{k}, \omega) = -\mathrm{i}\boldsymbol{k}\frac{\rho_2(\boldsymbol{k}, \omega)}{\varepsilon_0 k^2 \varepsilon_{\mathrm{L}}(k, \omega)}. \tag{10}$$

The corresponding force on particle 1 is thus readily obtained as

$$\boldsymbol{F}_2(\mathbf{r}_1, t) = q_1\mathbf{E}_2(\mathbf{r}_1, t) = \frac{-\mathrm{i}\mathbf{0z}\, q_1 q_2}{(2\pi)^3\varepsilon_0} \times \int \mathrm{d}^3k \frac{k_z \mathrm{e}^{-\mathrm{i}\delta k_z L}}{k^2 \varepsilon_{\mathrm{L}}(k, k_z v)}. \tag{11}$$

Inserting equation (8) yields (with $x = k_z L$, $y = k_\perp L$, $M = v/\sqrt{3}\, v_{\mathrm{T}}$, $\alpha = \omega_{\mathrm{p}} L/\sqrt{3}\, v_{\mathrm{T}} = L/\sqrt{3}\, L_{\mathrm{D}}$)

$$\boldsymbol{F}_{21} = \mathbf{0z}\frac{q_1 q_2}{4\pi^2\varepsilon_0 L^2}\,\mathrm{Im}\int_0^\infty \mathrm{d}y\, y \int_{-\infty}^{+\infty} \mathrm{d}x\, x \times \frac{\mathrm{e}^{-\mathrm{i}x\delta}[(M^2-1)x^2 - y^2]}{(x^2+y^2)[(x^2-x_0^2)(M^2-1) + \mathrm{i}o\,\mathrm{sgn}\, x]} \tag{12}$$

$$x_0^2 = (y^2+\alpha^2)/(M^2-1).$$

The small imaginary term in the denominator, which stems from the imaginary part of ε_{L}, ensures, as in section 2, that the integral be always well-defined since the poles $\pm\mathrm{i}y$, $\pm x_0 - \mathrm{i}o$ are complex. It can be integrated by residues by closing the contour in the lower (or upper) half complex planes if δ is positive (or negative). Assume $M > 1$ (recall the approximation on ε_{L}). Then x_0 is real and the poles $\pm x_0 - \mathrm{i}o$ are located both inside the contour (if $\delta > 0$) or outside (if $\delta < 0$). One obtains

$$\boldsymbol{F}_{21} = -\mathbf{0z}\frac{q_1 q_2\, \delta}{4\pi\varepsilon_0 L^2}\int_0^\infty \mathrm{d}y\, y \times \left(\frac{y^2 M^2}{y^2 M^2 + \alpha^2}\mathrm{e}^{-y} + H(\delta)\frac{2\alpha^2 \cos x_0}{y^2 M^2 + \alpha^2}\right) \tag{13}$$

(H denotes the unit-step function).

At small distances or dilute plasma ($\omega_{\mathrm{p}} L/v \ll 1$) this reduces approximately to the Coulomb force. At large distances ($\omega_{\mathrm{p}} L/v \gg 1$), the first term disappears but the second term, which exists only if $\delta > 0$ i.e. on the trailing charge (and $M > 1$) yields a force decreasing slowly with distance. The first integral can be found in (Abramowitz and Stegun 1965); the second one is calculated by transforming it into (let $\beta = \alpha/(M^2-1)^{1/2}$, $\gamma = \alpha/M$)

$$\int_\beta^\infty \mathrm{d}y\, y\frac{\cos y}{y^2 - \gamma^2} = \frac{1}{2}\left(\int_{\beta-\gamma}^\infty \mathrm{d}u \frac{\cos(u+\gamma)}{u} + \int_{\beta+\gamma}^\infty \mathrm{d}u\frac{\cos(u-\gamma)}{u}\right).$$

One finds

$$\boldsymbol{F}_{21} = -\mathbf{0z}\frac{q_1 q_2\, \delta}{4\pi\varepsilon_0 L^2}(1 + \gamma^2[\cos\gamma\, \mathrm{Ci}(\gamma) - \sin\gamma(\pi/2 - \mathrm{Si}(\gamma))] - \gamma^2 H(\delta)\{[\cos\gamma\, \mathrm{Ci}(\beta-\gamma) + \sin\gamma(\pi/2 - \mathrm{Si}(\beta-\gamma))] + [\ ]_{\gamma\to-\gamma}\}). \tag{14}$$

If $\gamma = \omega_{\mathrm{p}} L/v \gg 1$, equation (14) reduces to

$$\boldsymbol{F}_{21} \simeq \mathbf{0z}H(\delta)\frac{q_1 q_2 (M^2-1)^{1/2}}{2\sqrt{3}\,\pi\varepsilon_0 L L_{\mathrm{D}}} \times \sin[\omega_{\mathrm{p}} L/(\sqrt{3}\, v_{\mathrm{T}}(M^2-1)^{1/2})]. \tag{15}$$

Thus, at large distances, the Coulomb field of the suprathermal moving charge q_2 is shielded only at the front; at the back, it is replaced by an oscillating field varying inversely with distance and generally larger than the Coulomb field. This field corresponds to the Cerenkov emission of plasma waves in a cone of half-angle $\sin^{-1}(1/M)$ trailing the particle (see for instance Cohen 1961, who uses a different and instructive method, but does not calculate explicitly the electric field).

Note that, owing to the term $H(\delta)$, one has $F_{21} \neq -F_{12}$, which may seem surprising at first sight; actually this is not a surprise since the interaction involves not only particles 1 and 2 but also the plasma.

The same method can be used (see Sitenko 1967) to calculate the well-known drag acting on (say), particle 1, in the plasma, independently of particle 2, (set $q_2 = q_1$ and $L = 0$ in equation (11)). In this case, the integral diverges (logarithmically) for large k; the convergence is restored by taking account of a finite size R of the charge and/or noting that the Vlasov framework neglects the close interactions and thus becomes invalid for distances smaller than $b \sim q^2/4\pi\varepsilon_0 KT$; practically this cuts the integral at $k_{\max} \sim \mathrm{Min}(1/R, 1/b)$.

Finally, we note that in practical applications, the charges often move suprathermally with respect to the ions rather than the electrons. Then the contribution of the former to the dielectric permittivity must be taken into account, yielding similar effects. One should also be careful to ensure that the linear approximation holds, and that the nonelectromagnetic interactions between the moving body and the plasma (for instance binary collisions) can be neglected: this limits in particular the body's size and charge. Thus it is not too surprising that the above results cannot be applied to spacecraft (see Laframboise 1966).

4. Plasma effects on antenna losses far from the plasma frequency

That plasma effects can be neglected when calculating the high-frequency losses of a source, is a widespread idea which has led occasionally to erroneous results in astrophysics and geophysics (some examples are quoted in Meyer-Vernet 1981). The simple-minded justification is that only electromagnetic waves matter, since plasma waves are heavily Landau damped far from the plasma frequency.

However, in many cases the source loses much more energy in the form of (damped) plasma waves or fluctuations than in electromagnetic waves. Though the former energy is absorbed in the plasma and thus not observed at a distance, it is important in the source energy balance.

We study below such a simple situation. Consider a small hertzian dipole antenna, at rest in an isotropic homogeneous equilibrium electron plasma, assumed to be described by the (collisionless) linearised Vlasov framework. The problem is defined by the following quantities: L = dipole half-length, $\pm q$ = dipole oscillating charges, ω = angular frequency, ω_p and v_T as in previous sections. We assume $\omega \gg \omega_p$ and $\omega L/v_T \ll 1$. The latter inequality ensures that the antenna length is small with respect to all the relevant wavelengths (the so-called dipole approximation). By electron plasma, we mean, as in previous sections that the ions act as a uniform static background. (It is important to note that, contrary to a common belief, this hypothesis can be invalid even when $\omega \gg \omega_p \gg \omega_{pi} = (m/m_i)^{1/2}\omega_p$, where m_i is the ion mass, as discussed below).

The power radiated by the antenna consists of two components. One is the well-known contribution of electromagnetic waves

$$\bar{P}_{EM} = \frac{q^2\omega^4 L^2}{3\pi\varepsilon_0 c^3}[1 + O(\omega_p^2/\omega^2)] \tag{16}$$

where plasma effects are negligible at the order ω_p^2/ω^2. Note that the electrons' temperature does not enter in this expression; this is no surprise since the phase velocity of electromagnetic waves is larger than c, so that there is no plasma particle with a similar velocity which could (Landau) damp the waves.

The other contribution comes from (Landau damped) plasma waves or fluctuations. Since it cannot be found in the textbooks, and since some well-known papers (see for instance Birmingham *et al* 1965) seem to suggest that it is negligible except for $\omega \sim \omega_p$, it is useful to recall its derivation (see Meyer-Vernet 1983). The (harmonic) source charge distribution is

$$\rho(\boldsymbol{r}) = q\,\delta(y)\,\delta(z)[\delta(x-L) - \delta(x+L)](e^{-i\omega t}). \tag{17}$$

It induces a (harmonic) longitudinal field in the plasma $E_L(\boldsymbol{r})$ given as in equation (10) of section 3.

The corresponding time-averaged power-loss is

$$\bar{P}_L = -\frac{1}{2}\int d^3r \boldsymbol{E}_L(\boldsymbol{r}) \cdot \boldsymbol{J}(\boldsymbol{r})$$
$$= -\frac{1}{2(2\pi)^3}\,\mathrm{Re}\int d^3k \boldsymbol{E}_L(\boldsymbol{k}) \cdot \boldsymbol{J}(\boldsymbol{k})^* \tag{18}$$

where $\boldsymbol{J}(\boldsymbol{r})$ denotes the source current distribution (which satisfies, in Fourier space $\boldsymbol{k} \cdot \boldsymbol{J}(\boldsymbol{k}) = \omega\rho(\boldsymbol{k})$). This yields:

$$\bar{P}_L = -\frac{\omega}{2(2\pi)^3\varepsilon_0}\,\mathrm{Im}\int d^3k \frac{|\rho(\boldsymbol{k})|^2}{k^2\varepsilon_L(k,\omega)} \tag{19}$$

$\varepsilon_L(k,\omega)$ is written in equation (7), and in the present section, it is neither useful nor *a priori* justified to approximate it as in section 3. Indeed, we remark that, since $\omega/\omega_p \gg 1$, $|\varepsilon_L| \sim 1$ in the relevant part of the integral (19) (since whenever $|\varepsilon_L| \neq 1$, the integrand is exponentially small). The remaining terms can be easily integrated, yielding:

$$\bar{P}_L = \frac{4q^2L^2\omega^2\omega_p^2}{(2\pi)^{3/2}\varepsilon_0 v_T^3} G(\sqrt{2}\,\omega L/v_T)$$
$$G(x) = \frac{1}{x^2}\int_0^\infty dz\, z\left(1 - \frac{\sin(x/z)}{x/z}\right)e^{-z^2}. \tag{20}$$

For $x \ll 1$, $G(x) \sim -(\mathrm{Ln}\, x)/6 + 0.17$; thus

$$\bar{P}_L \simeq \frac{q^2L^2\omega^2\omega_p^2}{3\sqrt{2}\,\pi^{3/2}\varepsilon_0 v_T^3}[\mathrm{Ln}/(v_T\sqrt{2}\,\omega L) + 1] \tag{21}$$

($\omega L/v_T \ll 1$.) Hence, the contribution of plasma effects satisfies approximately:

$$\bar{P}_L/P_{EM} \sim (2\pi)^{-1/2}(\omega_p/\omega)^2(c/v_T)^3. \tag{22}$$

Therefore, in order to neglect plasma effects on the high-frequency dipolar power-loss ($\omega L/v_T \ll 1$) one should have $\omega/\omega_p \gg (c/v_T)^{3/2}$. (It is worth noting that, if one has not only $\omega L/v_T \ll 1$, but also $\omega L/v_{Ti} = (m_i/m)^{1/2}\omega L/v_T < 1$, then the ion contribution cannot be neglected and P_L is multiplied by $(m_i/m)^{1/2} \gg 1$ (see Meyer-Vernet 1983); though not widely known, this fact is not surprising, since the inequality ensures that the ions can have the same velocity as the bulk of the emitted waves, which satisfy mostly $\omega/k > \omega L$).

Remembering our high-frequency hypothesis (hence our approximation for $|\varepsilon_L|$), it comes rather as a surprise that equation (21) is also approximately correct below (and even near) the plasma frequency, provided the dipole approximation still holds. Let us prove this result for $\omega/\omega_p \ll 1$ (assuming that the conditions for neglecting the ion motion are yet satisfied). Then, equation (7) can be approximated by its low-frequency limit:

$$\mathrm{Re}\,\varepsilon_L(k,\omega) \sim 1 + 1/k^2L_D^2. \tag{23}$$

(As is well-known, Fourier transforming equation (10) with this expression for ε_L gives the Debye shielding.) The power-loss is now given by an expression similar to equation (20) where G is replaced by:

$$F(x) = \frac{1}{x^2}\int_0^\infty dz\, z\left(1 - \frac{\sin(x/z)}{x/z}\right)\frac{e^{-z^2}}{(1 + 2\omega_p^2 z^2/\omega^2)^2}. \tag{24}$$

The dipole approximation below ω_p requires $x\omega_p/\omega \ll 1$ (thus $L/L_D \ll 1$). This ensures $F(x) \sim G(x)$, so that equation (21) still approximately holds.

It is useful to comment on the significance of the length L. First, if L is smaller than the critical impact parameter defined in section 3, the integral in equation (20) should be cut at $k_{max} \sim 1/b$; practically, this changes only logarithmically the final result. In the limiting case of an infinitesimal dipole, equation (19) yields integrals analogous to those appearing for plasma bremsstrahlung calculations (Dawson and Oberman 1962).

Second, what happens if the antenna size is such that the dipole approximation does not hold? This is not an academic question since practical macroscopic antennas, though often smaller than the wavelengths in vacuum for the radio-frequency range, are seldom small at the scale of the Debye length in usual plasmas. Then, the problem becomes more difficult since the actual geometry of the antenna must be taken into account, but a preliminary idea can be obtained by looking at equation (20). Let us assume for instance $\omega L/v_T \gg 1$; using $G(x) \sim 1/(2x^2)$, we obtain a very different value for the high-frequency power-loss

$$\bar{P}_L \simeq \frac{q^2\omega_p^2}{(2\pi)^{3/2}\varepsilon_0 v_T} \tag{25}$$

for $\omega/\omega_p \gg 1$ and $\omega L/v_T \gg 1$. Similarly, equation (24) yields a very different value for the low-frequency power-loss. This is not surprising, since large L means that small k plays a dominant role (see equation (19)), which is the range where the plasma temporal dispersion is most important. This explains why the behaviour of long antennas in plasmas is much more frequency sensitive than that of short ones. The difference is most important at $\omega \geqslant \omega_p$. There, ε_L can be approximated by equation (8) and allowing for large L (small k) yields a spike in the losses, so much sharper as L is larger (see for instance Kuehl 1967).

Finally, we note that the above features have useful practical applications when the antenna is used as a passive 'thermometer', instead of an emitter. Knowing the radiation resistance (which stems from the above values), the noise measured by a passive antenna can be used to deduce the plasma temperature from Nyquist theorem (see for instance Sitenko 1967). Then, one must be careful in remembering that the resistance depends itself on the temperature, contrary to what happens when there are only electromagnetic waves. In particular, the noise may be a decreasing function of the temperature (deduce the resistance from equation (21) and use Nyquist theorem). So, the physical intuition acquired with antennas in vacuum must not be used in plasmas.

Here comes the real surprise of this section, and this is a historical one. The main relevant results on antennas in isotropic plasmas were obtained nearly twenty years ago (see the pioneering works by Cohen 1962, Fejer 1964, Balmain 1965 and Kuehl 1966). It was subsequently suggested (Andronov 1966, De Pazzis 1969) to use these results to measure plasma parameters and to this aim they were extended to nonthermal stable plasmas (Fejer and Kan 1969). Nevertheless, when this stable plasma noise was recently measured in the solar wind and planetary magnetospheres, it was not always recognised as such and attributed instead to instabilities, which were indeed more exciting. So, the surprise is not to discover that antennas in plasma behave differently than when in a vacuum; it is that it was so difficult to convince some well-known geophysicists of this fact (see the geophysical papers quoted in Meyer-Vernet 1979 and Couturier *et al* 1981).

5. Conclusion

We have given a few examples where the lack of care in reasoning or out-of-context 'well-known' ideas yield incorrect results for radiation in stable plasmas. Some (in sections 3 or 4) have led (and continue to lead) to erroneous and irrelevant papers in reputable scientific journals. Others are harmless and appear rather as surprises in the results of calculations, due to insufficient thinking in advance, as often occurs in physics (see Peierls 1979).

Both cases are of pedagogical value. They show the dangers of using improper generalisations or blindly trusting the textbooks, and ilustrate the difficult art of choosing a correct approximation. Returning to plasma physics, it is hoped that the present paper may convince students to ensure that they have properly understood stable and linear plasmas, before attacking the more fashionable subject of plasma instabilities.

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I have borrowed the title of this paper from a very nice and stimulating book by R Peierls.

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