Ghost's shadows and superluminal occultations

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integration *DID* demand that l_1 be small compared to an electron diffusion length.

The remaining integration across the SCL from c to b is readily carried out using a variable x' (see Fig. 1), and an equation for the reduced potential similar to (10). This time the equation for holes demands that l_2 be much less than a hole diffusion length, while integration of (4) for electrons satisfies the equivalent of (8b) from c to b without requiring that l_2 be much less than a electron diffusion length.

III. CONCLUSION

We have shown that the heart of the simple diode equation is the physical demand of very few collisions in the space-charge layer. This translates into a SCL width much smaller than a diffusion length, and the ability to neglect SCL recombination in the relative absence of SCL collisions.

The proof was done with an explicit integration of Eqs. (3) and (4) from point a to point b, demanding that Eqs. (5a) and (5b) apply. We found (5a) automatically satisfied for holes, but in order for (5b) to be valid, the distance from a to b had to be much smaller than an electron diffusion length. The integration from b to c was not shown in detail, but its results were given; (5b) applies automatically for electrons, while for (5a) to apply it must be true that the distance from b to c is much less than a hole diffusion length.

APPENDIX

The integrals to be evaluated are of the form

$$\int_0^1 \exp(sgy^2)dy,$$

where s is $+1$ or -1, and g is $v_b - v_a$. (For moderate forward bias, the numerical value of g will be in the neighborhood of 4 to 12.)

When $s = -1$, it is a very good approximation to set the upper limit to infinity, giving a value for the integral of $\sqrt{\pi/4g}$. (This is in error by 1.5% when $g = 3$, and by 0.3% when $g = 5$.)

When $s = +1$, one may integrate numerically and fit an expression to the results, or fit the tabulated results[11] of a related integral. It turns out that the function $\exp(g)/1.9g$ works fairly well, having an error of 11% when $g = 5$, and about 1% for g between 10 and 20.

[1]N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Holt, Rinehart, and Winston, New York, 1976).

[2]D. A. Fraser, *Physics of Semiconductor Devices* (Oxford University, New York, 1983), Chap. 3.

[3]J. Seymour, *Electronic Devices and Components* (Halsted, New York, 1981), Chap. 3.

[4]J. F. Gibbons, *Semiconductor Electronics* (McGraw-Hill, New York, 1966).

[5]G. W. Neudeck, *The PN Junction Diode* (Addison-Wesley, Reading, MA, 1983), Chap. 3.

[6]B. G. Streetman, *Solid State Electronic Devices*, 2nd ed. (Prentice-Hall, Englewood Cliffs, NJ, 1980), Chap. 5.

[7]Reference 4, Chap. 6, and Ref. 1, Chapter 29, problem 4.

[8]R. F. Pierret, *Semiconductor Fundamentals* (Addison-Wesley, Reading, MA, 1983), Sec. 3.2.3.

[9]Reference 6, Sec. 2.5.3.

[10]Reference 4, Sec. 3.3.1.

[11]M. Abramowitz and I. A. Stegun, Eds., *Handbook of Mathematical Functions* (National Bureau of Standards, AMS 55, 1964), p. 319.

Ghost's shadows and superluminal occultations

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The field intensity behind a moving (nonrelativistic) occulting object has a rather counterintuitive behavior at sufficiently large distance: finite transit time effects yield two simultaneous shadow paths on a plane, which separate from each other with superluminal velocity.

I. INTRODUCTION

In an eclipse situation, what happens if the respective distances of the source, occulting object, and observing plane are such that the shadow path moves with superluminal velocity? Then, the shadow path acquires a (nonidentical) twin, moving in the opposite direction, with arbitrary large velocity.

This example illustrates in a very simple way the counter intuitive results which are generally obtained when finite light transit time is taken into account.[1] Contrary to the classical problem of the apparent shape of a rapidly moving object,[2] which involves both the Lorentz contraction and the light transit time, this problem is not concerned with special relativity (except in the fact that relativity theory places an upper bound on possible signal velocities). The solution is easily calculated at the beginning student level.

II. THE PROBLEM

We define the problem in the simplest (two-dimensional) way (Fig. 1). Let a point source S at rest, occulted by an

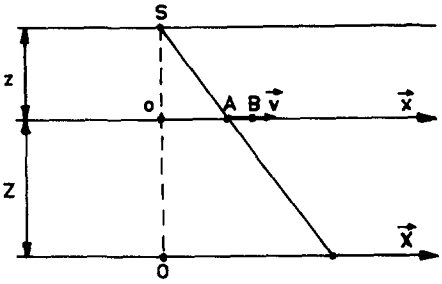

Fig. 1. Geometry of the problem.

object AB moving with nonrelativistic ($v/c \ll 1$, in order to neglect Lorentz contractions) velocity **v** on the ox axis. What is the intensity distribution in the geometrical optics limit at distance Z from the ox axis? Let z be the distance between the source and the ox axis and $M = Z/z$.

The intensity observed at time T, abscissa X, is zero if the point of abscissa x [satisfying $x/X = 1/(1+M)$] is at time $t = T - [Z^2 + (X-x)^2]^{1/2}/c$ included in the object. Taking the time origin such that $x_A = vt, x_B = vt + L$, one obtains trivially the occultation condition at time T, coordinates X,Z:

$$vt < X/(1+M) < vt + L,$$

$$t = T - [Z^2 + X^2(M/1+M)^2]^{1/2}/c. \quad (1)$$

This gives the abscissae of the shadow's limits

$$X_A = \frac{(1+M)\beta c}{1-\beta_M^2}\{T \pm [\beta_M^2 T^2 + Z^2(1-\beta_M^2)/c^2]^{1/2}\} = f(T),$$

$$X_B = f(T + L/v), \quad (2)$$

where $\beta = v/c$, $\beta_M = Mv/c$, and the sign $\pm$ is chosen by referring to Eq. (1).

If $\beta_M < 1$, the $+$ determination has no physical sense, one has one value of X_A and X_B, respectively, obtained by taking the $-$ determination, and the occultation has an usual behavior (Fig. 2). The shadow path velocity has the same direction as **v**; at large distances in the X direction, it takes the values

$$\frac{dX_{A,B}}{dT} \to \frac{v(1+M)}{1 \pm \beta_M} \; (X \to \pm \infty).$$

If $\beta_M > 1$, the behavior becomes qualitatively different (Fig. 3). One has two values for $X_{A,B}$, respectively, if $T > T_{OA,B}$, where

$$T_{OA} = Z(\beta_M^2 - 1)^{1/2}/\beta_M c, \quad T_{OB} = T_{OA} - L/v,$$

for which

$$X = X_o = -Z(1+M)/[M(\beta_M^2 - 1)^{1/2}],$$

and no values if $T < T_{OA,B}$. The branches $X_{A,B}(T)$ corresponding to the $+$ (or $-$) determination, respectively in Eq. (2) have $dX_{A,B}/dt$ negative (or positive) respectively; these velocities become infinite at X_o.

Thus, the shadow path at distance Z has the following behavior: at time T_{OB}, a shadow spot appears at X_o, which broadens quickly (with arbitrarily large velocity for $T = T_{OB}$); this path is entirely related to the B side of the object; it is the same as that given by a semi-infinite plane limited by B. Then, at T_{OA} a light spot appears at X_o inside

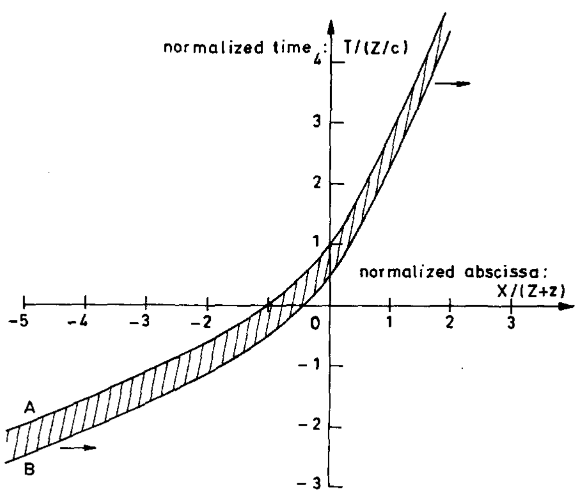

Fig. 2. Shadow patch (hatched area) as a function of time for $\beta_M = Mv/c = 1/\sqrt{2}$ and $L/\beta_M z = 0.5$. (The abscissa X is normalized to $Z + z$, the time T is normalized to Z/c.) The arrows indicate the shadow patch velocity direction.

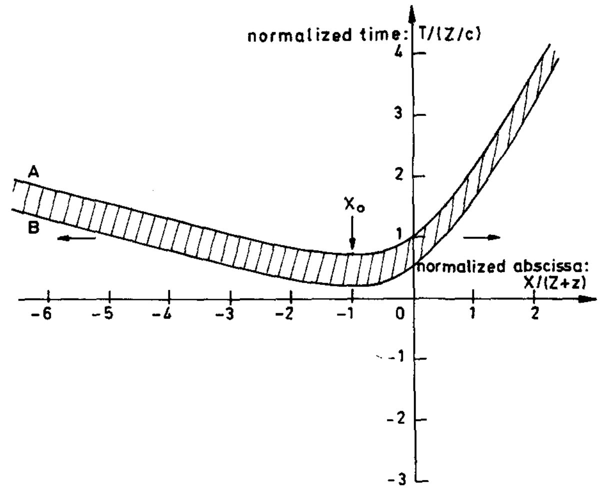

Fig. 3. Same as Fig. 2, but $\beta_M = \sqrt{2}$. For $T/(Z/c) > (\beta_M^2 - 1)^{1/2}/\beta_M$, one gets two shadow patches moving in opposite directions.

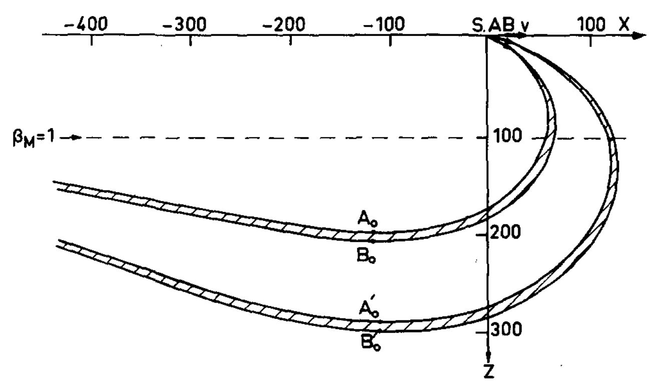

Fig. 4. Intensity distribution (the shadow is represented by a hatched area) in the X, Z plane at two different times T and T' for $\beta = 10^{-2}$; (distance source-screen $z = 1$, object abscissae $x_A(T) = 1.7$, $x_A(T') = 2.7$, $x_B - x_A = 0.1$). A, B, S nearly coincide at the scale of the figure: the arrows indicate the direction **SA** at T and T'.

the shadow patch; it quickly broadens, leaving finally two shadow patches, separating from each other with large velocity (arbitrarily large at T_{OA}).

Figure 4 shows the intensity distribution in the X,Z plane, at two different times T and T', for $\beta = 10^{-2}$. Note that the line A_oA is perpendicular to the direction AS. The interesting behavior appears on a plane parallel to OX, in the vicinity of A_o at T, or A'_o at T'.

III. DISCUSSION

When $\beta_M < 1$, the finite transit time effects do not change the qualitative behavior of the eclipse, except for the nonlinearity in the curve $X(T)$. But for $\beta_M > 1$, a second shadow patch appears, the behavior of which is dominated by light propagation delays. The fact that the shadow limits move with arbitrarily large (superluminal) velocity near X_o, involve no anticausal behavior. These superluminal "objects," that could be visualized by putting at distance Z a diffusing or fluorescent screen, are of the same nature as those produced for instance[1] by a rotating (angular velocity ω) source on a cylindrical screen at distance larger than the so-called light cylinder $R = c/\omega$. Though the superluminous velocity is real (and is not an apparent velocity seen by an observer), it cannot be used to transmit information.[1] One sees also that the time duration of the occulation, at a given location X,Z has the usual behavior.

These results arise in a Gedanken experiment. Could they be observed in practice? Similar behavior, obtained with an imaging system with high magnification (M) has been discussed[3] in the context of high-speed photography. Can these effects be observed in actual astronomical occultations? This would require both satisfying the peculiar geometrical conditions to observe near A_o or B_o, and measure the spatial and temporal intensity distribution in a portion of plane with the correct orientation. Furthermore, one would actually observe diffraction fringes instead of the geometrical optics shadow limit; since the latter satisfies $dX/dZ \to \infty$ near A_o or B_o, the spatial interfringe in a plane perpendicular to OZ is expected to be very large in the vicinity of these points. In any case, this is a simple instructive example where an apparently trivial situation ($v/c \ll 1$, though $Mv/c > 1$), yields surprising[4] results.

[1]See, for instance, V. L. Ginzburg, *Theoretical Physics and Astrophysics* (Pergamon, New York, 1979) and references therein.

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Undergraduate experiment: Determination of the band gap in germanium and silicon

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A method for determining the band gap in germanium and silicon at 0 K based upon the temperature dependence of the electrical conductivity of a *p-n* junction is described. Results are given for the band gaps that are in good agreement with the accepted values.

I. INTRODUCTION

A central feature in the explanation of the electrical characteristics of semiconductor devices is that of a forbidden range of energies between the valence and conduction bands. This paper describes a procedure for determining the band gap in germanium and silicon that may be performed as an undergraduate experiment using readily available *p-n* junction devices.

The approach adopted is similar to that described by Canivez.[1] Modifications have been made to the analysis of the data which has resulted in more accurate values for the band gaps being obtained.

II. THEORY

The current-voltage characteristics of a *p-n* junction may be represented by the junction equation found in many solid-state text books[2]:

$$I = I_0[\exp(eV/kT) - 1], \qquad (1)$$

where V is the potential difference across the junction, e is the charge on an electron, k is Boltzmann's constant, T is the temperature of the junction in Kelvins, and I_0 is the current that flows through the junction when it is reverse biased.

For an abrupt junction (where the doping profile across