Eur. J. Phys. **20** (1999) 167–176. Printed in the UK PII: S0143-0807(99)00564-4

How does the solar wind blow? A simple kinetic model

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Received 31 December 1998, in final form 22 February 1999

Abstract. A simple kinetic model of solar wind ejection is presented. It provides an alternative to the fluid derivation given in most space physics textbooks, and allows one to derive the wind speed from simple principles. This kind of description emphasizes the role of the electrostatic field which is implicit in the fluid point of view. It also illustrates the inadequacy of the classical heat conduction law in space plasmas, and allows one to deal with non-equilibrium velocity distributions, which are ubiquitous there.

1. Introduction

For science-fiction writers and some space engineers, the 'wind from the Sun' [7] is the thrust of the solar radiation pressure, which (in theory [4]) would allow solar sailing and might drive space windjammers through the inner solar system. Yet the Sun produces another kind of wind, made of particles. Although its pressure is only four orders of magnitude smaller than the photon pressure, this corpuscular wind interacts strongly with the gaseous environments of solar system bodies and shapes most of them. This is because all this dilute material is made of charged particles, which interact much more strongly with each other than with light. Indeed, their cross section for light scattering is the Thomson cross section, of the order of the square of the 'classical electron radius' $r_e = e^2/4\pi\epsilon_0 m_e c^2$. In contrast, the mutual interaction of charged particles is governed by the Coulomb potential, so that two particles of charge e interact strongly when they are closer than the distance r_C for which the Coulomb energy $e^2/4\pi\epsilon_0 r_C$ is of the order of their kinetic energy $k_B T$, i.e. $r_C \approx e^2/4\pi\epsilon_0 k_B T$. The cross section is thus greater than the Thomson cross section by a factor of about $(r_C/r_e)^2 \approx (m_e c^2/k_B T)^2$, which is greater than 10^9 for solar system environments. (Here, m_e is the electron mass, c the velocity of light and k_B Boltzmann's constant).

This wind of solar particles is unfamiliar to the layman, although it drives two bewildering sky displays: the blue straight tail of comets, which is due to the funnelling of cometary particles by the magnetic field of the solar wind draped around the comet as a wind sock [5], and, albeit in a less direct way, the auroral displays, whose changing patterns of light are caused by storms of particles impinging on the Earth's upper atmosphere and exciting its atoms [13]. But for the space physicist, the solar wind is ubiquitous. It bathes the whole solar system, shaping planetary and cometary environments, and pushes a bubble of flowing plasma through the interstellar medium up to a still unknown distance. This wind has been explored *in situ* by numerous space probes, from inside Mercury's orbit to beyond the distance of Neptune and Pluto, and, quite recently, at virtually all heliocentric latitudes [27]. At the Earth's orbit, it is made of a few protons and electrons per cm^3 at a temperature of about 10^5 K, as well as small quantities of heavier ions, flowing roughly radially away from the Sun at a supersonic velocity of several hundred km s^{-1} [2, 11, 24].

Most cosmic bodies, from comets to stars and galaxies, also eject matter into space. Depending on the object, the ejection varies from steady to chaotic, from symmetrical to jet-like, and involves different physical effects, ranging from thermal evaporation to explosive events, radiation pressure or centrifugal ejection. In most cases, the object is so distant that observation is not sufficient to constrain the proposed theoretical scenarios. This should not be the case for the solar wind, which was predicted a long time before being directly observed, and is now measured in almost embarrassing detail. Yet, from the beginning of modern physics to the present epoch, its physical explanation has stimulated (and continues to stimulate) much debate [9, 1].

The purpose of this paper is twofold. Firstly, I want to break away from the uniformity of the fluid calculations of the solar wind given in textbooks, and to use instead the alternative and complementary point of view of kinetic theory. Since this kind of derivation is known to specialists, although often misunderstood, I will avoid unnecessary detail in order to highlight the basic physics. Secondly, I will emphasize certain problems which the textbooks generally sweep under the carpet and try to illustrate some of the physics arising from the kinetic point of view.

2. Where is the problem?

How does the solar wind blow? The solar wind is in fact the outward extension of the million-degree hot upper atmosphere of the Sun, called the corona because of its crown-like shape which can be seen during eclipses. Close to the Sun, this atmosphere is strongly bound since the mean gravitational energy per ion is roughly ten times the thermal energy. However, because this medium is ionized and very hot, it conducts heat very efficiently; hence the temperature decreases very slowly with altitude so that the thermal energy becomes greater than the gravitational energy beyond about ten solar radii. In static fluid equilibrium, the pressure would not decrease very much beyond this point, and since it is many times higher than that of the tenuous interstellar medium, the corona expands away into space.

The first calculation yielding a supersonic wind was performed by Parker in 1958 [20], and was soon confirmed by *in situ* observations. This calculation, based on the hydrodynamic framework, is reported in virtually any space physics [13] or astrophysics [3] textbook and is still used with a number of additions and technical improvements in modern fluid wind theories.

So where is the problem? First of all, the flow energy must come ultimately from what is provided at the base of the wind, where the flow speed is very small. Hence the asymptotic flow speed $V_{\rm sw}$, at a very large distance where the flow kinetic energy dominates all other forms of energy, is constrained by the energy available as

$$\frac{V_{\rm sw}^2}{2} \approx \frac{5k_{\rm B}T_0}{m_{\rm p}} - \frac{M_\odot G}{r_0} + \frac{Q_0}{n_0 m_{\rm p} V_0} \tag{1}$$

where the index '0' refers to the base of the corona, r is the distance from the Sun so that r_0 is roughly equal to the solar radius $r_\odot$, T is the temperature, n the proton or electron number density, $m_{\rm p}$ and $M_\odot$ respectively the proton and solar mass, the electron mass has been neglected, and G is the gravitational constant. The terms on the right-hand side of (1) are respectively:

- the enthalpy per unit mass, due to both the protons and the electrons,
- the gravitational binding energy per unit mass,
- the heat flux per unit mass flux,

at the base of the wind; the initial bulk kinetic energy has been neglected, as well as the asymptotic enthalpy and heat flux terms. With a coronal temperature of 2×10^6 K, the radius

$r_0 \approx r_\odot \approx 7 \times 10^8$ m, and the solar mass $M_\odot \approx 2 \times 10^{30}$ kg, the enthalpy provides only 0.8×10^{11} J kg^{-1}, whereas the gravitational binding energy amounts to 2×10^{11} J kg^{-1}. Hence the available enthalpy is far from sufficient to lift the medium out of the Sun's gravitational well, so that the heat flux plays a key role.

How can one calculate this heat flux? A cornerstone of the fluid description is to assume that the medium is collisional, i.e. that the mean particle free path is infinitely small. Under this condition, the flux of thermal energy is related to the temperature gradient by the heat conduction equation:

$$Q = -\kappa_0 \frac{\mathrm{d}T}{\mathrm{d}r}. \tag{2}$$

The heat is transported by the electrons, since they have a much greater thermal speed than the protons. The heat capacity of one electron is $3k_\mathrm{B}/2$, which yields $3nk_\mathrm{B}/2$ per unit volume, so that the thermal conductivity is $\kappa_0 \approx 3nk_\mathrm{B} \times w_\mathrm{e} \times l$ where $w_\mathrm{e} = \sqrt{2k_\mathrm{B}T/m_\mathrm{e}}$ is the electron thermal speed and l is the mean-free path. To a first approximation, $l \sim 1/n\pi r_\mathrm{C}^2$ where πr_C^2 is the Coulomb cross section introduced above, but taking into account the numerous particle encounters at distances greater than r_C yields the more accurate value: $l \approx 3 \times 10^7 T^2/n$ [28]. To deduce Q, we must also estimate the temperature gradient at the base of the wind. Let us suppose that there are no losses so that in spherical geometry the heat balance equation yields

$$\frac{\mathrm{d}}{\mathrm{d}r}\left[r^2 \kappa_0 \frac{\mathrm{d}T}{\mathrm{d}r}\right] = 0. \tag{3}$$

With $\kappa_0 \propto T^{5/2}$ and assuming that $T \to 0$ at large distances, this yields $T \propto r^{-2/7}$, which from (2) gives the heat flux at the base of the wind: $Q_0 \approx 3.7 \times 10^7 k_\mathrm{B}^{3/2} m_\mathrm{e}^{-1/2} T_0^{7/2}/r_0$. Finally, we evaluate the initial proton (electron) flux from that measured at the Earth's orbit ($r \approx 214\, r_\odot$), which is about 2×10^{12} protons (electrons) m^{-2} s^{-1}; since particles are conserved, this yields $n_0 V_0 \approx 2 \times 10^{12} \times 214^2$.

Let us now substitute these numbers in (1). With a coronal temperature of 2×10^6 K, the heat flux at the base of the wind provides about 2×10^{11} J kg^{-1}, which just balances the binding gravitational energy. The remaining enthalpy term yields a terminal velocity of a few hundred km s^{-1}, so that enough energy seems available to drive the wind. This result, however, is very sensitive to the temperature since the heat flux varies as $T^{7/2}$: with a temperature only 15% smaller, the right-hand side of (1) becomes negative! The situation worsens when one comes to realize a now well-established fact: the wind which is the most stable, is the fastest and fills most of the heliosphere, comes from the coldest regions of the corona, where the electron thermal temperature (which determines the conductivity) is not significantly higher than 10^6 K. With such a temperature, the thermal conductivity falls short by roughly one order of magnitude of that required to drive even a low-speed wind.

How can one solve this problem? The usual way is to assume that some additional energy is injected, for example in the form of solar microflares, or of Alfvén waves; this argument, however, is the subject of some debate since, in spite of several decades of investigation, it is not yet understood how these perturbations could provide the right energy in the right place to accelerate the wind [22]. The same problem emerges in any fluid theory of the solar wind: to close the infinite hierarchy of hydrodynamic equations, one assumes a given temperature profile, or introduces an *ad hoc* addition of heat and/or momentum. In Parker's original derivation, T was taken as a constant, which yields an infinite asymptotic wind velocity since in this case an infinite amount of energy is available. More sophisticated derivations assume a more realistic temperature profile, but whatever the method, the fluid description is unable to give a quantitative estimate of the terminal velocity of a thermally driven wind from simple considerations alone.

3. The kinetic point of view

The starting point of the kinetic description is as follows: at the distance where the particles are free to escape since the gravitational binding energy there becomes smaller than the thermal energy, the mean-free path has already become greater than the scale height, so that the medium is no longer collisional. One might argue that the magnetic field comes to the rescue to ensure a fluid behaviour by forcing the particles to gyrate around the magnetic field lines. This argument, however, is dubious since in this case the wind blows along magnetic field lines, i.e. perpendicularly to the direction of particle gyration.

Having realized this, we no longer feel so comfortable with the fluid description, but we have a clue as to the solution of the energy problem: if the free path is not small enough at the base of the wind, then the classical expression (2) used for the heat flux is incorrect. Indeed, the classical heat law is invalid in plasmas that are not very strongly collision-dominated [25, 19]. The basic reason is, as already mentioned, that the charged particles interact through the Coulomb potential which varies inversely as the distance, making their cross section proportional to the inverse square of their energy. Hence the energetic particles, which contribute most to the heat flux, are virtually collisionless at the base of the corona, although the thermal ones are collision dominated.

Thus an alternative and simple way to describe the medium is to consider it as an escaping exosphere. Neglecting the collisions beyond a given radius called the 'exobase', the particle velocity distributions further out can be deduced directly from those at the exobase by applying Jeans' theorem, applying conservation of particle energy and magnetic moment.

There is, however, a basic departure from Jeans' [12] theory of neutral gas evaporation: because the protons carry an electric charge, their energy at the exobase includes an electrostatic contribution, which turns out to be greater than the gravitational term and of opposite sign, so that it can drive the wind. Early versions of such exospheric theories failed because they did not calculate this electrostatic term self-consistently [6]; however, this error has now been corrected, and the exospheric approach is indeed able to yield a supersonic wind, even with a moderate coronal temperature [14]. We present below a simplified version of this kind of calculation, and show why it may produce large wind speeds [16, 18].

4. The role of the electric field

Why does the particle energy include a large electrostatic contribution? Consider first the simple case of a static ionized atmosphere made of electrons and protons in thermal equilibrium. In the absence of an electric field, the proton pressure gradient should just balance the gravitational attraction, $F_g = m_p M_\odot G/r^2$ per proton; but in this case there would be nothing to balance the electron pressure gradient (equal to that of the protons) since the gravitational attraction on them is negligible, and so the electrons tend to be displaced outwards with respect to the protons. The corresponding space charge induces a radial electrostatic field $\boldsymbol{E}$ directed outwards, which adjusts itself so that the total attraction on a proton, $F_g - eE$, is equal to the attraction on an electron, eE. This yields $eE \approx F_g/2$, which halves the total attraction on the protons and allows the pressure gradients to be balanced, thereby preserving rough electrical neutrality.

The space charge needed to produce such an electric field is extremely small. Indeed, the difference Δn in the electron and proton number density is given from Poisson's equation by $|\boldsymbol{\nabla E}| = \Delta n \times e/\epsilon_0$. With the above electric field $E = m_p M_\odot G/(2er^2)$, this gives $\Delta n = \epsilon_0 m_p M_\odot G/(e^2 r^3)$. At a distance of, say, $3r_\odot$, the density difference is $\Delta n \approx 10^{-8}\ \mathrm{m}^{-3}$, which is only about 10^{-19} times the typical electron density there.

However, in an expanding collisionless medium, with no particles coming from infinite distance, the electrostatic field is much larger [14]. Otherwise, since the fluxes of electrons and protons escaping from the corona are proportional to their respective thermal velocities which vary as the inverse square root of the mass, the corona would eject roughly $(m_p/m_e)^{1/2}$ more electrons than protons, thereby charging the corona positively. At equilibrium the electric field adjusts itself in order to keep the escaping electron flux equal to that of the protons, so that there is no net electric current. This requires that the electron electrostatic energy at the exobase be several times its thermal energy in order to confine most of the electons in the potential well. The corresponding electric field pushes the protons in the opposite direction; since they carry most of the mass, a wind is produced.

In this context, it is easy to understand how a significant wind can be produced with even a moderate coronal temperature. This is because the electrostatic energy available to the protons is significantly greater than the gravitational binding energy at the exobase, which is located at several solar radii.

It may be worth noting that the positive charging of the corona, which is required to ensure zero net electric current from the Sun, is a phenomenon similar (albeit in reverse) to the charging of a space probe in the Earth's ionosphere. The space probe charges negatively because it would otherwise collect many more ambient electrons than ions, roughly in the ratio of their thermal velocities. At equilibrium, the negative electrostatic potential repels most of the electrons coming from the ambient plasma in order to ensure zero net electric current to the space probe. The corresponding particle electrostatic energy is several times the thermal energy, just as in the case of the solar corona, but of opposite sign [17].

Finally, as Fitzgerald [10] put it (albeit in a somewhat different context) as early as November 1892: '…the Sun is powerfully electrified, and repels similarly electrified molecules with a force of some moderate number of times the gravitation of the molecules to the Sun'.

5. Fluxes of escaping particles

5.1. The electrons

We neglect the collisions between particles beyond the exobase, which is defined as the location where the particle mean-free path becomes greater than the scale height. With typical coronal parameters, this occurs at a few solar radii, typically $r_0 \approx 6r_\odot$ [14]. We have seen that the electrons are subjected to an attractive electrostatic potential $\Phi(r)$ which is assumed to decrease monotonically, so that of those emerging from the exobase some are reflected, whereas those energetic enough can overcome the potential barrier and escape. At the exobase, where the potential energy of an electron is $-e\Phi(r_0)$, the velocities of the non-escaping electrons have their modulus v constrained by energy conservation to be smaller than

$$V_0 = [2e\Phi(r_0)/m_e]^{1/2} \tag{4}$$

and have any direction. In contrast, the escaping electrons have velocities $v > V_0$, and their angle θ to the outward radial direction lies between 0 and $\pi/2$. Let $f_{e0}(\boldsymbol{v})$ be the electron velocity distribution at the exobase. The simplest choice is a Maxwell–Boltzmann distribution

$$f_{e0}(v) = \frac{n_{e0}}{\pi^{3/2} w_e^3} \exp\left(-\frac{v^2}{w_e^2}\right) \tag{5}$$

of thermal velocity $w_e = \sqrt{2k_B T_0/m_e}$, truncated according to the above conditions on v and θ for each class of particles. Note that the total electron density $n_e(r_0)$ at the exobase is not exactly n_{e0}. Indeed, n_{e0} is the integral of the distribution (5) over the whole velocity space, whereas the total electron density $n_e(r_0)$ does not include the velocity domain defined by $v > V_0$ and $\pi/2 < \theta < \pi$, since no particles are coming from infinity. Because in general $V_0 \gg w_e$, the proportion of particles in this domain is very small, so that $n_e(r_0) \approx n_{e0}$.

The escaping flux of electrons is given by integrating the radial velocity $v_r = v\cos\theta$ over the velocity distribution of the escaping electrons:

$$F_e(r_0) = \int d^3v \, v_r f_{e0}(\boldsymbol{v}) \tag{6}$$

$$= 2\pi \int_{V_0}^{\infty} \mathrm{d}v\, v^3 f_{e0}(v) \int_0^{\pi/2} \mathrm{d}\theta\ \sin\theta\cos\theta \tag{7}$$

where we have substituted $\mathrm{d}^3 v = 2\pi v^2 \sin\theta\, \mathrm{d}\theta\, \mathrm{d}v$ (in spherical coordinates), and the ranges of integration in v and θ determined above. With the distribution (5) and the change of variable $u = v^2$, we obtain

$$F_e(r_0) = \frac{n_e(r_0)}{2\sqrt{\pi}} w_e (1 + U_0) \mathrm{e}^{-U_0} \tag{8}$$

where

$$U_0 = \frac{V_0^2}{w_e^2} = \frac{e\Phi(r_0)}{k_B T_0}. \tag{9}$$

Since the escaping electrons must overcome the electrostatic potential, their flux decreases strongly as the potential increases, as expected. Because this flux consists of high-speed particles ($v > V_0$), it is very sensitive to the shape of the velocity distribution at high energies. We will see that this has an important consequence.

5.2. The protons

The electric field repels the positively charged protons, and their potential energy at distance r is $-m_p M_\odot G/r + e\Phi(r)$ which is assumed positive and monotonically decreasing. Hence the proton speeds increase with distance, and they all escape. For their velocity distribution at the exobase, $f_{p0}(\boldsymbol{v})$, we take a Maxwell distribution

$$f_{p0}(v) = \frac{n_{p0}}{\pi^{3/2} w_p^3} \exp\left(-\frac{v^2}{w_p^2}\right) \tag{10}$$

of thermal velocity $w_p = \sqrt{2k_B T_0/m_p}$. The velocity direction is constrained by the inequality $0 < \theta < \pi/2$ since no particles are coming from infinity, but, in contrast to the electrons, there is no constraint on the modulus since all protons moving outwards can escape, whatever their speed. Hence the proton number density $n_p(r_0)$ at the exobase is just half the integral of the distribution (10) over the whole velocity space, i.e. $n_p(r_0) = n_{p0}/2$ since there are no particles with $\pi/2 < \theta < \pi$.

The escaping proton flux is calculated in the same way as the electron flux, just replacing the minimal speed V_0 by zero, so that instead of (8) we have

$$F_p(r_0) = \frac{n_p(r_0)}{\sqrt{\pi}} w_p \tag{11}$$

using the equality $n_p(r_0) = n_{p0}/2$. This value is independent of the potential, which is not surprising since all protons escape. It depends on the bulk of the velocity distribution, in contrast to the electron flux which only depends on the high-speed particles.

6. Electrostatic potential and terminal velocity

The electrostatic potential at the exobase is deduced from the condition that the plasma is neutral with zero net flux of charge, i.e. $n_e(r_0) = n_p(r_0)$ and $F_e(r_0) = F_p(r_0)$. With the above (truncated) Maxwell velocity distributions, from (8) and (11) this yields

$$\begin{aligned}(1 + U_0)\mathrm{e}^{-U_0} &= 2w_p/w_e \\ &= 2(m_e/m_p)^{1/2}\end{aligned} \tag{12}$$

With $m_e/m_p \approx 5.4 \times 10^{-4}$, we find $U_0 \approx 5$, i.e. $e\Phi(r_0) \approx 5k_B T_0$.

We can now calculate the terminal wind velocity from the proton energy balance, taking into account the electrostatic energy. We neglect the initial bulk kinetic energy and the proton heat flux term, which are small compared to the terminal bulk kinetic energy. Some care is needed to calculate the proton enthalpy since the distribution is not a *bona fide* Maxwellian because of the hole in velocity space due to the absence of protons going inwards. Since, however, the proton enthalpy is significantly smaller than the other contributions, we may approximate it by the equilibrium value: $5k_BT_0/2$ (per proton) without changing the final result very much. This finally yields

$$\frac{V_{\rm sw}^2}{2} \approx \frac{5k_{\rm B}T_0}{2m_{\rm p}} + \frac{e\Phi_0}{m_{\rm p}} - \frac{M_\odot G}{r_0} \qquad (13)$$

where the index '0' refers to the exobase radius. With the moderate temperature $T_0 \approx 10^6$ K and $r_0 \approx 6r_\odot$, we find $V_{\rm sw} \approx 250$ km s^{-1}. This value is not extremely sensitive to the position of the exobase since the gravitational term is not dominant, and is far less sensitive to the temperature than the determination made in section 2. A more accurate result can be obtained by calculating the asymptotic flow parameters from Jeans' theorem [14] or from the total (electron plus proton) wind energy equation: this latter equation has no electrostatic term since the wind is electrically neutral, but the heat flux (roughly the electron one) is no longer given by (2); instead, in the spirit of the kinetic calculation, it is given by the thermal energy flux of the escaping electrons, which can be calculated in a straightforward way from the velocity distribution, and turns out to be proportional to the electrostatic potential at the exobase [18].

This simple calculation yields a wind in spite of a moderate coronal temperature, and it furnishes a simple analytical approximation to the terminal velocity which is not a sensitive function of the coronal parameters. It is not, however, completely satisfactory since the most conspicuous wind, which fills most of the heliosphere and comes from coronal regions of moderate electron temperature, travels more than twice as fast as that found above [21]. Nevertheless, this calculation does give a clue to a possible solution: the electron velocity distribution might have an excess of high-energy particles. Indeed, space physicists have come to realize that the Maxwell distribution, although it furnishes a convenient theoretical framework, is the exception rather than the rule in natural plasmas. The fundamental reason is again the strong increase of the free path with energy, which makes these particles collisionless, even at low altitudes, so that given the non-equilibrium processes at work there is no reason why their distribution should be Maxwellian. In particular, turbulent waves in the solar atmosphere can produce high-energy tails in the electron velocity distribution, although these waves are unable to drive the wind directly [23].

It is easy to predict that such particles can increase the wind speed significantly, even though they contribute negligibly to the number density and do not change the mean electron temperature very much [19, 16, 18]. This is because the Maxwell distribution (5) has very few electrons of speed greater than V_0, so that a small increase in the number of these electrons can increase the electron flux considerably. In order to keep this flux equal to the proton flux, the electrostatic potential must increase, thereby accelerating the protons and increasing the terminal speed. A similar excess of high-energy protons does not have much effect because the proton flux only depends on their bulk distribution.

7. Pushing the wind with a suprathermal tail

7.1. The kappa distribution

As noted above, the particle velocity distributions observed in space are not in equilibrium. They are generally nearly Maxwellian at low energies, but they decrease much more slowly at high energies, having a suprathermal tail which varies as an inverse power law. Such distributions have been found in virtually all space plasmas and their ubiquity suggests that a fundamental mechanism is at work [8, 29]. Be that as it may, such behaviour is not surprising

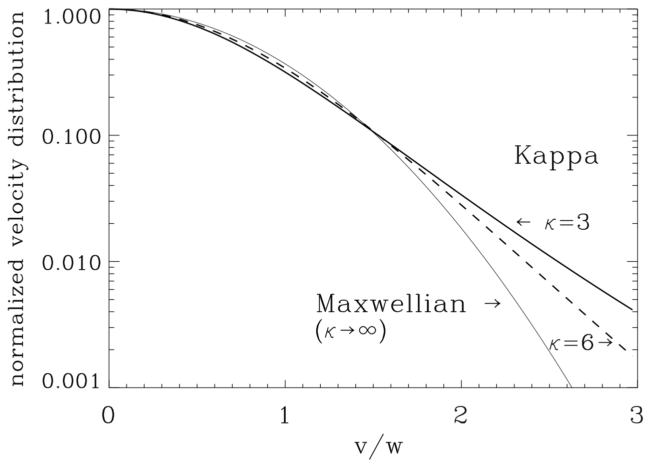

Figure 1. Kappa velocity distributions (equation (14)) for $\kappa = 3$ (solid line) and $\kappa = 6$ (dashed line), respectively, plotted versus the speed normalized to the most probable speed. We have superimposed the limiting case $\kappa \to \infty$, which is simply the Maxwell distribution $\exp(-v^2/w^2)$ and which has the same most probable speed. The distributions are similar for $v \leqslant w$, whereas the kappa distributions have an excess of high-speed particles.

given the strong decrease of the collisional cross section of charged particles with increasing energy, which drives the high-energy particles very easily out of equilibrium, and enables them to escape to large distances.

The simplest and most commonly used function having these properties is the generalized Lorenzian

$$f_{e0}(v) = \frac{n_{e0}}{2\pi}\frac{A_\kappa}{(\kappa w^2)^{3/2}}\left[1+\frac{v^2}{\kappa w^2}\right]^{-(\kappa+1)} \tag{14}$$

$$A_\kappa = \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)\Gamma(3/2)} \tag{15}$$

where the Gamma (factorial) function Γ arises as a result of the normalization. This so-called 'kappa' distribution is close to a Maxwellian of temperature $T_0 = m_e w^2/2k_B$ at speeds $v \leqslant w$, and has a suprathermal inverse power-law tail, in agreement with observation (see figure 1). The contribution of the suprathermal particles decreases as κ increases, and the distribution approaches the Maxwellian (5) as $\kappa \to \infty$, so that it has the agreeable property of including the Maxwell distribution as a limiting case. The most probable speed is w, as in the Maxwellian limit, but the temperature, defined as usual by $m_e\langle v^2\rangle/3k_B$ (where the angular brackets denote a mean over the distribution) is equal to $m_e w^2/2k_B \times \kappa/(\kappa - 3/2)$, due to the contribution of the suprathermal tail. Hence κ is constrained by the inequality $\kappa > 3/2$ for the temperature to remain finite. In practice, κ is generally observed to lie in the range 2–6 (see, for example, [15]). Despite its apparent agreement with observations, some care is needed in handling this function because it has only a limited number of finite moments.

7.2. Increasing the wind speed

With this distribution, the escaping electron flux is easily calculated when $V_0^2 \gg \kappa w^2$ (which can be verified *a posteriori*). We substitute in (7) the distribution (14) where we approximate the bracket by its leading term $v^2/\kappa w^2$; since the integral over θ in (7) equals $1/2$, this gives

$$F_e(r_0) \approx \frac{n_{e0}}{2}\frac{A_\kappa}{(\kappa w^2)^{3/2}}\int_{V_0}^{\infty} dv\, \frac{v^3}{(v^2/\kappa w^2)^{\kappa+1}}$$

$$\approx \frac{A_\kappa \kappa^{1/2}}{4(\kappa - 1)} n_{e0} w \left(\frac{\kappa}{U_0}\right)^{\kappa-1}. \tag{16}$$

To calculate the proton flux, we use expression (11) obtained with a Maxwell distribution since, as already noted, this flux is not significantly changed by the presence of suprathermal particles. Equalizing the electron and proton fluxes and using $n_{e0} \approx n_e(r_0) = n_p(r_0)$, since the medium is neutral, we deduce the normalized potential

$$U_0 \equiv \frac{V_0^2}{w_e^2} \equiv \frac{e\Phi(r_0)}{k_B T_0} \approx \left[\frac{\sqrt{\pi}}{4} \frac{\kappa^{\kappa-1/2} A_\kappa}{\kappa - 1} \sqrt{\frac{m_p}{m_e}}\right]^{1/(\kappa-1)}. \tag{17}$$

As already noted, it is bounded by the inequality $\kappa > 3/2$. With $\kappa = 3$, for example, which is a typical value in a number of space media, $A_\kappa = 2^4/\pi$, which yields $U_0 \approx 28$ so that $e\Phi(r_0) \approx 28 k_B T_0$.

With such a large potential, the electrostatic energy dominates in the proton energy equation (13), so that the wind terminal velocity is roughly

$$V_{sw} \approx \left[2e\Phi(r_0)/m_p\right]^{1/2} \tag{18}$$

With $T_0 \approx 10^6$ K, this yields the wind terminal velocity $V_{sw} \approx 700$ km s^{-1}.

This shows that the presence of a suprathermal tail in the electron velocity distribution can considerably increase the flow speed. This enhancement is much greater than might be naively expected from the contribution of these electrons to the mean electron temperature. A similar result can be obtained from energy balance, with the heat flux calculated from the thermal energy of the escaping electrons [18].

The terminal wind velocity increases as κ decreases, i.e. as there are more suprathermal electrons. This result is generic in that any distribution having a suprathermal tail will produce a velocity greater than with a Maxwellian.

8. Final remarks

A very simple kinetic collisionless model can explain solar wind ejection, even with a moderate coronal temperature, without the need of an *ad hoc* addition of energy, and can also furnish an analytical approximation to the terminal velocity. From this point of view, the wind is driven by the electrons through the electrostatic field set up by the large ion-to-electron mass ratio, so that the energy available to counteract the gravitational attraction is significantly greater than the thermal value.

With Maxwell velocity distributions at the base of the wind, one finds a moderate terminal velocity. However, a small suprathermal tail in the electron distribution can drastically increase the electrostatic potential, and thus the flow velocity. Suprathermal tails are a privilege of plasmas, due to the strong increase of the particle free path with energy. This suggests that ionized winds can achieve greater terminal speeds than predicted from thermal energy alone, as in the case of neutral winds, for example in comets [5].

A related outstanding problem in space physics is how a million-degree corona can be produced just above the solar 'surface', whose temperature is around 6000 K. Just as for solar wind acceleration, one has a challenging energy problem, which has not yet been solved despite several decades of research. Most tentative calculations use the magnetohydrodynamic framework, but it has been recently suggested [26] that suprathermal tails in the distribution of particles below the corona might play an important role. The basic reason is that close to the Sun, the particles are confined by the gravitational (and electrostatic) potential. This potential filters the particles by letting only the most energetic ones escape, so that the mean particle energy, i.e. the temperature, should increase with altitude. This does not happen with a Maxwellian, whose temperature does not change with altitude because all the particles are filtered in the same way.

Nevertheless, the kinetic collisionless description is not fully satisfying. This is because the particle mean-free path in the solar wind is of the same order of magnitude as the scale height. Hence, just as the fluid description is not justified because the mean-free path of the particles is not sufficiently small, so the collisionless description is not fully justified because the mean-free path is not large enough. This means that a complete solution requires some kind of compromise between both theories, i.e. a kinetic model taking collisions into account. However, the particles that contribute most to wind acceleration in the kinetic description are the suprathermal electrons, which are roughly collisionless even at low altitudes. This gives some confidence that the collisionless calculation, which has the immense advantage of simplicity, may correctly describe a large part of the physics involved. But as we noted, this is not the whole story: it will probably turn out, as usual, that Nature is a little more subtle that we had imagined.

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