because there are an infinite number of additional states with almost the same energy for the successively larger values of n_r, n_θ, and $|m|$. The limit of the one-dimensional hydrogen atom $\theta_0=0$ is expected to be valid only for the $m=0$ states, since according to Eq. (12) the probability amplitude for the other m states vanishes.

The discussion of the previous paragraph has been restricted to the low-lying states of the hydrogen atom, i.e., finite values of its quantum numbers. Going back to the paragraph of Eq. (14) we can consider some complementary and alternative situations in which the atom can be ionized for finite positions of the conical boundary. First, $\nu\to\infty$ can be obtained for finite values of λ and $n_r\to\infty$. Second, within the original assumption of $\lambda\to\infty$ and finite n_r, the energy threshold is reached by taking the states with high polar excitation n_θ equal to the least integer that is greater than or equal to $\lambda-|m|-1$, $\lambda-|m|-2$, ..., for which the conical boundaries are far from $\theta_0=0$. In any case, the general conclusion is that the presence of the conical boundary cannot by itself ionize the hydrogen atom in low-lying states; it can produce ionization of states that are highly excited radially or polarly.

We close this discussion by pointing out that the model studied in this paper is the limiting situation of the model of the hydrogen atom in a semi-infinite space limited by a hyperboloidal boundary[9] when the focal distance of the latter tends to vanish. Both models share the dynamical and geometrical properties studied in Sec. II and discussed in this section. The model with the conical boundary is obviously much simpler and its study may open a door for the interested reader to some of the physics of surface effects.[1–9]

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Aspects of Debye shielding

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The Debye shielding is derived in a simple way without assuming Boltzmann's equilibrium. The conditions under which it applies and some of its consequences are discussed at the elementary level.

I. INTRODUCTION

One of the most basic ideas of plasma physics is Debye shielding, first recognized[1] when the plasma did not even have a name.[2] Yet, elementary textbooks discuss it rather briefly and in virtually the same way, and one is accustomed to take it for granted. Thinking more deeply about it, however, raises some questions and reveals a few surprises.

At first sight, the concept seems rather trivial. Since electric charges attract oppositely charged particles and repel the others, ionized matter tends to maintain electrical neutrality; but the thermal agitation counteracts this tendency. Loosely speaking, the Coulomb attraction keeps opposite charges together, whereas the particle agitation tends to separate them; the balance allows the existence of nonneutral regions whose scale—the so-called Debye length, increases with the thermal agitation—i.e., the temperature, and is an inverse function of the density (since increasing the density of charge carriers favors the shielding). Neutrality is not perfect at the Debye scale but is effective farther out, so that any charge has a dress of size the Debye length which makes it "invisible" from larger distances.

This loose argument, however, as the classical Debye–Hückel derivation that can be found in virtually any textbook, is based on thermal equilibrium. But many plasmas, and in particular most space plasmas, are collision-free, and not in equilibrium. Does the Debye shielding still take place in this case, and what is the shielding length? The usual derivation also assumes that the problem is linear, which is seldom true in practice; how are real objects dressed? And how is a particle shielded when it is moving, which is after all the usual situation? These questions are not trivial, and their complete solution requires concepts, mathematics, and even numerical computations which are difficult to grasp at the elementary level. The present paper is an attempt to discuss these points and some of their consequences in a simple way.

I shall assume that the collisions and the ambient magnetic field are both negligible; this requires that the particle-free paths between collisions and their radii of gyration in the magnetic field be large compared to the other scales.

II. DEBYE SHIELDING WITHOUT BOLTZMANN'S EQUILIBRIUM

Consider a medium made up of n-free electrons of mass m_e, charge $-e$, and n positive ions of mass m_i, charge $+e$ per unit volume. Consider two such particles approaching each other with relative velocity v. They interact via their Coulomb field and when they come close enough that their potential energy becomes larger than the kinetic one, they undergo a strong perturbation not unlike a collision in an ordinary gas. This happens when they come closer than the distance r_0 where $e^2/4\pi\epsilon_0 r_0 \sim mv^2$ (m being roughly the mass of the lighter particle). For such events to be rare, this "close encounter" radius r_0 must be much smaller than the mean distance between one particle and its nearest neighbor, which is of order $n^{-1/3}$. With $mv^2 \sim m_e v_e^2$ this gives the approximate condition

$$r_0 \sim e^2/4\pi\epsilon_0 m_e v_e^2 \ll n^{-1/3}. \quad (1)$$

In this case,[3] a given plasma particle is mostly driven by the combined small effects of distant Coulomb encounters; these can be described by a mean electric field. Conversely, each particle produces a field which is modified by the presence of other particles: In this sense, it is dressed.

We also assume that the plasma is collision-free, i.e., that the particle-free paths between collisions are large enough. An upper limit to the free path can be obtained by assuming the particle effective interaction distance to be r_0, which gives a collision cross section $\sim \pi r_0^2$. In this case, a particle traveling a unit distance crosses a volume equal to πr_0^2 and thus encounters $n\pi r_0^2$ electrons and ions. Hence, the mean-free path between such encounters is equal to $1/n\pi r_0^2$. This figures does not take into account the numerous encounters at a distance larger than r_0, and is thus an overestimate.

Since the collision-free plasma is not in equilibrium, the particle velocity distributions need not be Maxwellian, but I shall assume, for simplicity, that they are sufficiently well behaved and *isotropic*, i.e., the number of particles having a certain velocity $\mathbf{v}$ depends only on the modulus v.

A. Shielding of a point charge at rest

Let us put a pointlike charge q at the origin in such a plasma. It attracts charges of opposite sign and repels the others, so that the density of particles changes around it and the electric potential $\Phi(r)$ will no longer be $q/4\pi\epsilon_0 r$ as *in vacuo*. Suppose that it perturbs *slightly* the particles, i.e., that the magnitude of their potential energy $|e\Phi(r)|$ is much smaller than their kinetic energy.

1. Boltzmann's equilibrium

Let us recall what would happen in thermal equilibrium at temperature T.[4] The particles would obey Boltzmann's law, namely would be distributed in proportion to e^{-E/k_BT}, E being their energy (kinetic plus potential). Thus at point r their volume density within a velocity interval dv would be $n(r)f(v)d^3v$ with $d^3v = 4\pi v^2 dv$ and

$$f(v) \propto \exp(-mv^2/2k_BT)$$

$$\text{(Maxwellian velocity distribution)}, \quad (2)$$

$$n(r)/n = \exp(-W/k_BT), \quad (3)$$

with $m = m_e$ or m_i, and the potential energy $W = -e\Phi(r)$ or $+e\Phi(r)$ for electrons and ions, respectively [with the normalization $\int d^3v f(v) = 1$].

If $|W| \ll k_BT$, the exponential in (3) can be expanded in a series, giving the density at point r: $n(r) \approx n + \delta n(r)$ with

$$\delta n_e(r)/n \approx +e\Phi(r)/k_BT,$$

$$\delta n_i(r)/n \approx -e\Phi(r)/k_BT, \quad (4)$$

for electrons and ions, respectively.

The potential is then deduced from Poisson's equation, with the charge volume density $e(n_i - n_e) = e(\delta n_i - \delta n_e)$:

$$\nabla^2\Phi = -e(\delta n_i - \delta n_e)/\epsilon_0. \quad (5)$$

Substituting the electron and ion density perturbations (4), one obtains

$$\nabla^2\Phi - \Phi/L_D^2 = 0, \quad (6)$$

where the Debye length L_D is defined by

$$1/L_D^2 = 1/L_{De}^2 + 1/L_{Di}^2, \quad (7)$$

with

$$L_{De}^2 = L_{Di}^2 = \epsilon_0 k_B T/ne^2. \quad (8)$$

The solution of Eq. (6) with the charge q at the origin is the classical Debye potential $\Phi(r) = qe^{-r/L_D}/4\pi\epsilon_0 r$. The Coulomb field is exponentially screened by a charge layer of density $e(\delta n_i - \delta n_e) \approx -2ne^2\Phi(r)/k_BT$ [from (4)], of scale length L_D and whose volume integral is $-q$.

Note that this reasoning still holds if the charge q has a different symmetry, being for example an infinite plane or wire. The potential has then a different form, but keeps the same exponential shielding.

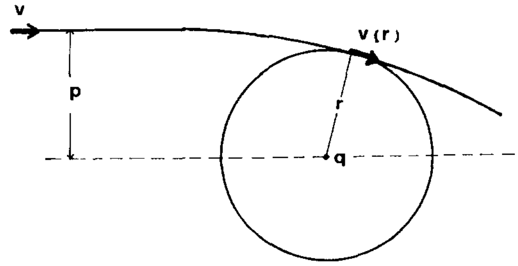

Fig. 1. Sketch of the path of a charged particle showing the impact parameter p and the distance of closest approach r.

2. *Without Boltzmann's equilibrium*

Now what happens when the medium is collision-free? Consider electrons arriving (isotropically, i.e., without preferred direction) at velocity v from the unperturbed plasma at "large distance," where the potential is zero and the density n, and assume that they are slightly perturbed by the charge's potential. In practice, this "large distance" must still be smaller than the free paths, in order that the particles do not suffer any collisions during their trajectory. In this case their total energy (kinetic plus potential) is conserved, so that the modulus of their velocity at point r is $v(r)=v+\delta v$ with

$$\delta(m_e v^2/2)\approx m_e v\delta v=e\Phi(r),$$

or

$$\delta v/v=e\Phi(r)/m_e v^2. \tag{9}$$

This velocity perturbation is associated with a density perturbation. If, for example, the electrons are attracted (i.e., if $\Phi>0$), their trajectories are bent toward the charge, which tends to increase the density. But since their velocity increases, they spend less time within a given region, which tends on the contrary to decrease the density. The resulting effect is easily deduced if the particles are conserved between large distance and position r.

Imagine a fictitious spherical collector of radius r which would collect electrons arriving on its surface [with density $n_e(r)$ and (isotropic) velocity $v(r)$]. Since for a given infinitesimal surface element, $n_e(r)/2$ particles per unit volume are incident from one side and their average *perpendicular* velocity is $v(r)/2$, the number of particles collected per second on the surface $4\pi r^2$ is

$$N(r)=n_e(r)v(r)\pi r^2. \tag{10}$$

Now consider those hitting the collector at grazing incidence as in Fig. 1. Their impact parameter p is deduced by noting that their angular momentum is just $rm_e v(r)$ and is conserved along the trajectory so that

$$pv=rv(r). \tag{11}$$

In order that a particle reach the collector (at any incidence), its impact parameter must be less[5] than p, so that the number of particles collected per second corresponds to those arriving from large distances in a cross section πp^2, i.e.,

$$N(r)=nv\pi p^2. \tag{12}$$

Comparing with (10), we have $nvp^2=n_e(r)v(r)r^2$; with the aid of (11) we deduce $n_e(r)/v(r)=n/v$, whence

$$\delta n_e/n=\delta v/v. \tag{13}$$

Substituting the velocity perturbation (9), we obtain

$$\delta n_e/n=e\Phi(r)/m_e v^2. \tag{14}$$

In general, the electrons do not have the same initial velocity v and we have to average over their initial velocity distribution. The ion density perturbation is similar (changing $-e$ to e and m_e to m_i). Finally,

$$\frac{\delta n_e}{n}=\frac{e\Phi(r)}{m_e}\langle v^{-2}\rangle_e,\quad \frac{\delta n_i}{n}=\frac{-e\Phi(r)}{m_i}\langle v^{-2}\rangle_i, \tag{15}$$

where the brackets denote a mean over the electron and ion velocity distributions in the unperturbed plasma:[6]

$$\langle v^{-2}\rangle=\int d^3v\,\frac{f(v)}{v^2}.$$

The potential is deduced as previously from Poisson's equation. With the density perturbations (15) instead of (4), we find the same result as with Boltzmann's equilibrium, except that the quantity $m/\langle v^{-2}\rangle$ replaces the thermal energy k_BT, so that the Debye length is defined from[7] now with

$$L_{De}^2=(\epsilon_0 m_e/ne^2)/\langle v^{-2}\rangle_e,\quad L_{Di}^2=(\epsilon_0 m_i/ne^2)/\langle v^{-2}\rangle_i. \tag{16}$$

Let us see what happens in the particular case where the distributions *in the unperturbed plasma* are Maxwellian at temperature T. In this case, we have

$$\langle v^{-2}\rangle=m/k_BT,$$

so that Eqs. (15) reduce to the result with thermal equilibrium and yield the classic Debye length (although we have not assumed thermal equilibrium near the charge). With a Maxwellian distribution, one can also find the density perturbations by using linearized hydrodynamic equations (balancing the electric force by a pressure gradient), and assuming that the pressure and density variations are related by an equation of state. The pressure, defined as the force exerted on a wall per unit area — is $P=nm\langle v^2\rangle/3$. With a Maxwellian distribution, the mean $\langle v^2\rangle=3k_BT/m$ has the same order of magnitude as the quantity $1/\langle v^{-2}\rangle$, so that in that case the pressure and the Debye length are closely related.

But imagine a rather odd distribution consisting of two Maxwellians with half of the particles at temperature T_{cold}, the others being much hotter (at T_{hot}). The particles of low velocity bring the dominant contribution to $\langle v^{-2}\rangle$ which is thus roughly equal to half the value corresponding to a single Maxwellian at T_{cold}, so that (15) gives $\delta n_e/n\sim e\Phi/2k_BT_{\text{cold}}$. Let us now calculate the pressure. Since the particle of large velocity bring the dominant contribution to $\langle v^2\rangle$, it is roughly equal to half the value corresponding to a single Maxwellian at T_{hot}, so that $P\sim nk_BT_{\text{hot}}/2$. We see that the cold particles determine the density perturbations and the Debye length, whereas the hot ones determine the pressure. The pressure is just the momentum flux of particles, but there is no collisional coupling so that if for some reason the velocity of a particle increases, the energy will not be shared with the neighboring particles, and in general there is no equation of state.

We have seen that the concept of Debye shielding does not require thermal equilibrium near the charge, nor even Maxwellian distributions at large distance. In the general case, the temperature is replaced by a quantity which involves the mean *inverse* squared velocity. This point is worth emphasizing since elementary derivations of the Debye shielding are based on Boltzmann's equilibrium.

However, our derivation is based on the relation (13) between the density and velocity perturbations, which depends on the symmetry of the problem. What happens if one replaces the point charge by an infinite plane or wire, the undisturbed plasma still being isotropic in three dimensions? We have also assumed the perturbations to be small; what happens if this is not so?

B. One- or two-dimensional charges

Consider a wire of infinite length charged uniformly. The density perturbation is given by a similar reasoning as in three dimensions, just replacing the spherical collector of radius r by a cylindrical one. Putting v for the velocity component in a plane perpendicular to the wire axis, and r and p for the radius and impact parameter in this plane, we now have $pv = rv(r)$, but $nvp = n_e(r)v(r)r$ (since the surfaces now vary as r and p instead of r^2 and p^2). This gives $n_e(r) = n$ and also holds for ions so that

$$\delta n_e = \delta n_i = 0.$$

Thus the plasma is not perturbed at all. This surprising result comes about because in two dimensions, the density change due to the deflection of the particles just balances that produced by their acceleration (see Ref. 10). With $\delta n = 0$ for both species, there should be no shielding at all. Actually this is not necessarily so, because in this case the potential would be the same as *in vacuo* and would vary logarithmically with distance to the wire axis; it would thus increase indefinitely with distance, so that the derivation, which assumes the potential to be small, is inconsistent.

Let us see whether the same difficulty arises in one dimension. Consider a charged plane, so that the potential depends only on the distance z to the plane, and only the component v_z of the particle velocity does change as they approach the plane. The corresponding density perturbation is given by writing particle flux conservation in one dimension: $n(z)v_z(z) = nv_z$ or

$$\delta n/n = -\delta v_z/v_z,$$

for both electrons and ions, which is just the opposite of the point-charge result. This comes about because, considering for example the attracted particles, the density decreases as they speed up, and this is not counterbalanced by a concentration of particles bent toward the charge as in two or three dimensions. Since $\delta(v^2) = 2v_z\delta v_z$, we have instead of (9),

$$\delta v_z/v_z = e\Phi(z)/m_e v_z^2, \tag{17}$$

for electrons (and the same expression, substituting $-e \to e$, $m_e \to m_i$, for ions). Thus we now find a density *decrease* if $\Phi > 0$, i.e., for *attracted* particles, and an *increase* if $\Phi < 0$, i.e., for the *repelled* ones, which is the opposite of the three-dimensional result. However this does not shield the potential; thus it does not tend to zero at large distance nor is a constant, so that the derivation is not fully consistent.

Such problems are not uncommon with charges at infinite distance, and anyway, with infinite wires or planes, the imposed perturbation has a different symmetry from that of the unperturbed plasma, which we have assumed to be isotropic in three dimensions.[7]

This behavior contrasts with the classical case of Boltzmann's equilibrium, for which one obtains the usual Debye shielding in the linear approximation whatever the dimensions of the problem.[8] However, it is known (see for example Ref. 9) that, even at equilibrium, the nonlinearity reveals some surprises. What happens in the present case?

C. Nonlinear shielding

Assume that the particle potential energy is not small compared to their initial kinetic energy. Particles arriving at velocity v from the unperturbed plasma (where $\Phi = 0$) have an energy equal to $mv^2/2$, which is conserved along their trajectory, i.e.,

$$mv^2(r)/2 + e|\Phi(r)| = mv^2/2 > 0$$

$$\text{(for repelled particles)}, \tag{18}$$

$$mv^2(r)/2 - e|\Phi(r)| = mv^2/2 > 0$$

$$\text{(for attracted particles)}. \tag{19}$$

The *repelled* particles are turned back before reaching position $\mathbf{r}$ if they have an initial kinetic energy $mv^2/2 < e|\Phi(\mathbf{r})|$ (or a similar inequality with the relevant velocity component for a wire or a plane). Thus the conservation relations written above do not hold for them.

In this nonlinear case, the density of *repelled* particles strongly *decreases* near the charge because some of them have not enough kinetic energy to overcome the potential. This happens whatever the geometry. In particular, with a Maxwellian distribution at large distance, one can show that the density of repelled particles just follows the Boltzmann's exponential decrease whatever the dimensions of the problem (although we have not assumed thermal equilibrium near the charge).

This is not so for the *attracted* particles, and for them the conservation relations previously derived still hold, namely we have for particles of velocity v: $n/v =$ constant, $n =$ constant or $nv_z =$ constant with distance, in, respectively, three, two, or one dimension. Thus the density of attracted particles *increases* (as does the velocity) near a point charge; however, with for example a Maxwellian distribution at large distance, one finds[10] that this increase is much more gentle than the exponential Boltzmann's law. Near a charged wire the density of attracted particles does not change at all, whereas it decreases near a charged plane. These results are very different from the exponential increase of Boltzmann's law. (Note that this reasoning assumes that all trajectories connect to the unperturbed plasma at large distance, which is not a trivial restriction.[11])

These results hold for isotropic and sufficiently well-behaved distributions of incident particles, Maxwellian or not. Upon reflection, it comes as no surprise to learn that, even with a Maxwellian distribution at large distance, the attracted species nevertheless *do not* satisfy Boltzmann's law in the absence of collisions near the charge. The exponential density increase of Boltzmann's law for attracted particles is due to those of low velocity that are trapped in the potential well surrounding the charge and accumulate there. But without collisions, there are no such particles because at position r the attracted particles have a kinetic energy larger than $|e\Phi(r)|$ [from Eq. (19)], and are thus able to escape. Indeed Boltzmann's equilibrium requires collisional processes in which some particles can lose energy and get trapped in the potential.

Finally, therefore, in the nonlinear case the shielding is mainly produced by a density decrease of the repelled particles, somewhat aided in this task, albeit only in three dimensions, by a more gentle increase of the attracted ones.

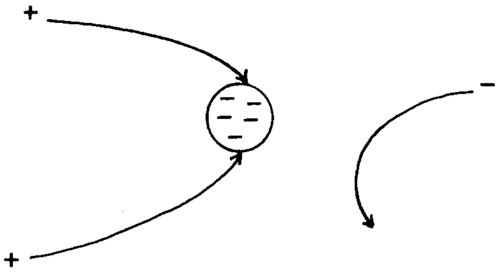

Fig. 2. Charging of an object in a plasma.

III. DRESSING OF REAL BODIES

We have so far studied ideal points, wires, or planes. But what happens with real objects? Is the problem linear, and how does the object's size affect the dressing?

This question is far from trivial,[12,13] but one can draw a rough picture to illustrate the physics. First of all, let us see whether the problem is linear in practice.

Consider an object isolated in the plasma. Its surface is being bombarded by ambient electrons and ions and in the simplest case,[14] it just collects the charges of the particles striking it. If these currents do not balance one another, the charge will change until an equilibrium is reached when the net current on the surface vanishes. Let us assume from now on that the plasma at large distances is not too far from equilibrium. In this case electrons and ions have kinetic energies of the same order of magnitude, and one can assume that they have a "typical" velocity denoted by v_e and v_i for electrons and ions, respectively (roughly equal to $\langle v^{-2}\rangle^{-1/2}$, $\langle v\rangle$ or $\langle v^2\rangle^{1/2}$ which have the same order of magnitude near equilibrium); we then have roughly [from (16)]

$$L_{De} \approx (\epsilon_0 m_e v_e^2/ne^2)^{1/2} \sim L_{Di} \sim L_D. \tag{20}$$

The electrons, being much lighter, move much quicker than ions, so that if the object is uncharged, their flux on its surface is much larger. The surface thus charges negatively, so that it repels the electrons and less of them can reach it (Fig. 2). Finally, it adjusts itself to a negative potential Φ (with respect to the plasma at large distance) which strongly repels the electrons in order that their flux become sufficiently small to just balance the ion flux. Thus Φ is of the order of a few times the typical electron kinetic energy:

$$-e\Phi/m_e v_e^2 = \eta_e > 1. \tag{21}$$

(In practice, η_e is roughly equal to 3.) Thus the problem is nonlinear.

We have seen that in this case, the density of the repelled particles, i.e., the electrons strongly decreases near the object, whereas the ion density is less strongly changed. Thus the object is surrounded by a sheath of perturbed plasma with a positive space charge density. In the spirit of this simple analysis, let us define a shielding length G so that the total positive charge in the sheath cancels out the negative charge q carried by the object—that is to say, the potential and the field are (nearly) zero farther out. We shall consider very small and very large objects and see how G is related to the Debye length.

A. Being small

Consider a small spherical object of radius $R \ll G$, charged at a negative potential given by (21). The (repelled) electron density is very small near the object, whereas the ion density is somewhat increased (as near a point charge), but not too strongly changed. Very crudely, we can view the electron density as being zero for $r < G$ and n farther out, whereas that of the ions remains roughly n everywhere. This gives a total charge volume density in the sheath of about ne. Since $G \gg R$, the sheath's volume is roughly $4\pi G^3/3$, so that its total charge is $q_{\text{sheath}} \sim ne \times 4\pi G^3/3$, which must cancel the charge q. Now, one has to be farther than an appreciable fraction of the sheath size G for the electrostatic field of the object to be appreciably, shielded. Thus the field remains nearly equal to the objects Coulomb field out to several radii R (since $R \ll G$), so that the capacitance has roughly the value *in vacuo*: $q/\Phi = 4\pi\epsilon_0 R$. Substituting $q \sim -q_{\text{sheath}}$, we find

$$G \sim L_{De}(3\eta_e R/L_{De})^{1/3}, \quad R \ll G. \tag{22}$$

Therefore, for very small objects, the shielding distance may be smaller than the Debye length, but not much more so since it varies only as $R^{1/3}$. This variation follows from the assumed charge density perturbation in the sheath to be about ne whatever the potential, instead of being proportional to it as in the linear case. Since, however, the density of attracted particles somewhat increases with Φ, the above result is just a rough estimate.

B. Being large

Consider now a very large object with $R \gg G$, so that the geometry is roughly plane. Let ions entering the sheath with initial velocity v_i arrive onto the object; conservation of flux (in plane geometry) and particle energy gives:

$$nv_i = n(R)v(R),$$

$$m_i v_i^2/2 = m_i v^2(R)/2 - e|\Phi|.$$

Eliminating $v(R)$, we find the ion density near the surface $n(R) \approx n/\sqrt{2\eta_i + 1} \sim n/\sqrt{2\eta_i}$, with

$$\eta_i = -e\Phi/m_i v_i^2 \sim \eta_e.$$

Approximating the ion density within the sheath by this value, since the variation is slow, the electron density still being neglected, one obtains the charge volume density in the sheath: $ne/\sqrt{2\eta_i}$. With $R \gg G$, the sheath's volume is now roughly $4\pi R^2 G$ so that its total charge is $q_{\text{sheath}} \sim 4\pi R^2 G \times ne/\sqrt{2\eta_i}$, which has to cancel out the charge q. The electric field at the surface $E = q/4\pi\epsilon_0 R^2$ is of order Φ/G since Φ decreases at the scale $G \ll R$. Substituting $q \sim -q_{\text{sheath}}$ and rearranging we deduce the order-of-magnitude estimate

$$G \sim L_{Di}(\eta_i)^{3/4}, \quad R \gg G. \tag{23}$$

This is an approximate version of the so-called Langmuir–Child's law, albeit in a different context.[15] Note that for the derivation to be consistent, the electron density, which we have assumed to be zero in the sheath, should fall faster than the ion density as the particles enter the sheath; since these density decreases are governed by the ratios $e|\Phi(r)|/m_e v_e^2$ or $e|\Phi(r)|/m_i v_i^2$, respectively, this requires, very roughly, that $m_e v_e^2 < m_i v_i^2$ (the lower the kinetic energy the larger the density perturbation). This very simplified version of the well-known Bohm's criterion shows that the picture of a well-defined sheath is too simplified: The ions must have been somewhat accelerated before entering the "sheath" to ensure that $m_i v_i^2 > m_e v_e^2$. This requires that the potential at the sheath edge be $\Phi(G) \neq 0$, so that the perturbed region is actually larger than G: the "sheath" size given by (23) is just an estimate of the scale over which the potential varies rapidly near the object.

Equation (23) shows that, for large potentials, the sheath of large bodies (with the above restriction) is somewhat larger than the Debye length, but not by many orders of magnitude for usual values of the potential.

Finally therefore, the Debye shielding length found in the linear approximation for a point charge remains in general roughly correct for actual finite objects. But it is important to remember that the particles responsible for the shielding may be distributed very differently from either the linear result (15) or the Boltzmann's law (3).

IV. THE DRESS OF A MOVING CHARGE

Consider now a point charge q moving with velocity $\mathbf{V}$ along the z axis. In order to shield the charge, the plasma particles must be capable of reacting to its motion. One might expect naïvely that if they move faster than the charge, they can adjust to the motion and still shield it, whereas if they move too slowly, they cannot react fast enough and the charge's potential remains Coulomb. But the reality is more subtle.

To illustrate the basic physics and keep the problem simple, we use the linear approximation and consider either very small or very large velocities.

A. Being slow

If the charge moves slowly compared to both electrons and ions, we expect its dressing to be roughly the normal Debye shielding. Now, suppose that it moves much *faster* than *ions*, but still much *slower* than *electrons*. From the charge's point of view, the ions have about the velocity V instead of v_i. We thus expect from Eq. (15) that the ion density perturbation is much smaller than with $V=0$, roughly by the factor $(v_i/V)^2$ so that they contribute negligibly to the shielding. On the other hand, from the electron point of view the charge barely moves, so that they respond nearly as if $V=0$. Hence, the shielding is due mostly to the electrons and the Debye length is just given by the electron contribution L_{De} in Eq. (7). In the intermediate case, when the charge moves at about the ion velocity, the ions also contribute to the shielding; we then expect the sheath to have a size somewhere in between L_D and L_{De} and to be somewhat deformed by the motion.

B. Being fast

Now assume that the charge moves much quicker than both electrons and ions, i.e.,

$$V \gg v_e \gg v_i.$$

Consider a potential of the form $\Phi(\mathbf{r}) \propto e^{i\mathbf{k}\cdot\mathbf{r}}$, $\mathbf{r}$ being the distance to the charge. This is a spatial Fourier component that does not vary with time *in the frame of the moving charge*. How do the electrons respond? Since the potential varies only along the direction of $\mathbf{k}$, it perturbs the electron velocity along that direction only, by a quantity $\delta v_\parallel$ given by: $m_e v_\parallel \delta v_\parallel = e\Phi$ (where the symbol $\parallel$ refers to the direction of $\mathbf{k}$). From flux conservation we have $\delta(n_e v_\parallel)=0$, so that the relative variation of electron density is just $\delta n_e/n = -\delta v_\parallel / v_\parallel$ (as in Sec. II B). Hence,

$$\delta n_e/n \approx -e\Phi/m_e V_\parallel^2, \tag{24}$$

where we have substituted $v_\parallel = V_\parallel$ since from the charge's point of view, the electrons have a velocity of about $\mathbf{V}$. The same result holds for ions (replacing m_e by m_i and $-e$ by $+e$), and so their density perturbation is smaller by a factor of m_e/m_i and of opposite sign.

Writing Poisson's equation, using $\nabla^2\Phi = -k^2\Phi$,

$$-k^2\Phi = e(\delta n_e - \delta n_i)/\epsilon_0, \tag{25}$$

and substituting the density perturbations, we find

$$k^2[1-\omega_p^2/(\mathbf{k}\cdot\mathbf{V})^2]\Phi = 0, \tag{26}$$

where

$$\omega_p = (\omega_{pe}^2 + \omega_{pi}^2)^{1/2} \text{ with } \omega_{pe,i} = (ne^2/\epsilon_0 m_{e,i})^{1/2},$$

is the so-called plasma frequency, and we have written $kV_\parallel = \mathbf{k}\cdot\mathbf{V}$. The term on the right-hand side of (26) is zero because we have omitted the charge q in Poisson's equation [in fact, (26) is just a dispersion equation in the charge's frame]. The zeros of the bracket indicate that the moving charge is emitting waves with wave vector $\mathbf{k}$, satisfying $\mathbf{k}\cdot\mathbf{V} = \omega_p$. Since the distance to a fixed origin is $\mathbf{r}' = \mathbf{r} + \mathbf{V}t$, the potential varies in the plasma frame as $e^{i\mathbf{k}\cdot(\mathbf{r}'-\mathbf{V}t)} = e^{i\mathbf{k}\cdot\mathbf{r}' - i\omega_p t}$ and thus at the frequency ω_p. So does the charge density perturbation. This means that the moving charge is exciting electrostatic plasma oscillations along its trajectory. This can also be viewed as a Cerenkov emission produced by a charge moving faster than the waves in the medium.[16] Therefore the field is not shielded; but it is not Coulomb, either: one can show that it decreases more slowly on the rear side, due to this wave emission.[17]

In summary, when the charge moves, the main change in its dress occurs if it moves faster than the electrons: The dress then trails a train of density oscillations whose wavelength along the trajectory is $\lambda = 2\pi/k_z = 2\pi V/\omega_p$; this is much larger than the Debye length which is of order $L_D \sim v_e/\omega_p$ [from (20)]. This shows that the notion of Debye length must be used with a pinch of salt: It becomes completely untrue for particles moving quicker than the average electrons.

V. THE IMPORTANCE OF BEING DRESSED

Since a moving (or variable) charge produces an electric potential which varies with time in the plasma frame, it can exchange energy with the plasma particles. The result depends on the range of influence of the charge, namely on its dressing.

A. Braking a dressed charge

Consider a pointlike charge q moving with velocity V smaller that the typical electron velocity v_e. We have seen that in this case the dressing is not too different from the normal Debye shielding, so that the potential may be crudely approximated by a Coulomb field for distances closer than L_D (or L_{De}), and zero farther out.

Let us estimate how the electrons brake the charge's motion. (The result is classic but the calculation will be

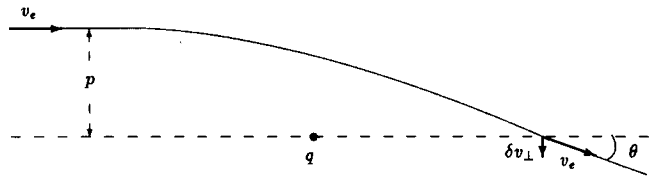

Fig. 3. Diagram illustrating the small deviation θ of the trajectory of an electron in the field of an electric charge q.

useful later.) Consider one electron arriving with impact parameter p (Fig. 3); since $V < v_e$, its velocity relative to the charge q is of the order of v_e. We only consider distances larger than the "close encounter" radius where the potential energy $qe/4\pi\epsilon_0 r$ is of order $m_e v_e^2$, i.e.,

$$p \gg p_0 \sim |q| e/4\pi\epsilon_0 m_e v_e^2. \tag{27}$$

Hence, the electron undergoes a small deflection, most of which takes place near closest approach. In this region, the force on the electron is roughly perpendicular to the direction of approach and of order $F_\perp \sim qe/4\pi\epsilon_0 p^2$. This force acts mostly during the time taken by the electron to travel a distance p along the approach direction on both sides of closest approach, i.e., $\delta t \sim 2p/v_e$. The electron thus acquires a perpendicular velocity $\delta v_\perp \sim \delta t F_\perp / m_e$, so that its angular deflection is

$$\theta \sim \delta v_\perp / v_e \sim 2pF_\perp / m_e v_e^2 \sim 2p_0/p.$$

(A more exact calculation turns out to give the same result as this estimate.)

The momentum change of the charge q (whose mass is assumed $\gg m_e$) along the direction of approach is just the opposite of that of the electron, i.e.,

$$\text{momentum change} \sim -m_e v_e (1-\cos\theta) \sim -2m_e v_e (p_0/p)^2.$$

The particles coming from the front side brake the motion, whereas those arriving from the rear side accelerate it; since the former are more numerous, the charge is slowed down. The difference between the rates of front side and rear side encounters with impact parameter between p and $p+dp$ is roughly $dN \sim nV \times 2\pi p\,dp$, which gives the braking force,

$$F \sim \int dN\; 2m_e v_e \left(\frac{p_0}{p}\right)^2 \sim 4\pi p_0^2 n m_e V v_e \int \frac{dp}{p} \sim 4\pi p_0^2 n m_e V v_e \ln(p_{\max}/p_{\min}), \tag{28}$$

Only impact parameters between p_0 and L_D do contribute (since the field of the charge is negligible farther than L_D), i.e.,

$$p_{\min} \sim p_0, \quad p_{\max} \sim L_D$$

(see Refs. 18 and 19).

This force is roughly equal to that produced by neutral particles impinging (with isotropic velocity v_e) onto a moving (and absorbing) object of geometrical cross section $\sim 4\pi p_0^2 \ln(L_D/p_0)$. This illustrates the fact that the long range interactions give an equivalent cross section larger than the figure πp_0^2 corresponding to close encounters only, by a factor $\sim 4\ln(L_D/p_0)$. This result also holds for ambient particles, taking the close encounter radius $p_0 \sim r_0$ (since their charge is $\pm e$ instead of q); thus their free path[21] is smaller by a factor of $\sim 4\ln(L_D/r_0)$ (generally of the order of 100) than the overestimate $1/n\pi r_0^2$ made in Sec. II.

B. Power loss of an oscillating dipole

The above results can be applied to an important case of variable charges. Consider a small electric dipole consisting of two point charges $+q$ and $-q$ distant by L and oscillating at the (angular) frequency ω. It is well known that it radiates electromagnetic waves at frequencies $\omega > \omega_p$. But the dipole also loses some energy *locally* in Coulomb encounters with ambient charged particles, just as moving charges do. Let us estimate this energy loss due to encounters with ambient electrons. This may be interesting, because this damping occurs also below the plasma frequency, and is often larger than the contribution of electromagnetic radiation.

By "small dipole," we mean that its length L (although larger than the close encounter radius p_0) is much smaller than the relevant scales which are here: the Debye length, the impact parameter p of incident electrons, and the distance they travel during one period of oscillation, i.e.,

$$p_0 < L \ll L_D,\ p, v_e/\omega. \tag{29}$$

Now consider electrons passing with impact parameter

$$p < v_e/\omega. \tag{30}$$

Since the encounter duration has the order of magnitude $\tau \sim p/v_e$, we have $\tau < 1/\omega$ so that any such electron sees less than one dipole oscillation. (For larger values of the impact parameter, there are several oscillations during an encounter, whose effect partially cancel out.) Thus, from the electron point of view, the dipole behaves as two charges $+q$ and $-q$ traveling a distance L during a half period π/ω with opposite velocities

$$V \sim L/(\pi/\omega) \sim \omega L/\pi. \tag{31}$$

Since $L \ll v_e/\omega$, we have $V \ll v_e$, so that the dipole power loss is roughly given by the work of the braking force (28) acting on both charges

$$P_{\text{dipole}} \sim 2 \times V \times F \sim 8\pi p_0^2 n m_e V^2 v_e \ln(p_{\max}/p_{\min}) \sim \frac{q^2\omega^2 L^2 \omega_p^2}{2\pi^3 \epsilon_0 v_e^3} \ln(p_{\max}/p_{\min}), \tag{32}$$

where we have substituted the expressions (27) of p_0 and (31) of V.

The minimum and maximum impact parameters are estimated as follows. From (29) we have $p_{\min} \sim L$. The upper limit $p_{\max}$ depends on the shielding of the dipole field, which in turn depends on the oscillation frequency. Since $\omega_p \sim v_e/L_D$, the electrons have enough time to travel a distance L_D during one period of oscillation if $\omega < \omega_p$, and thus to shield the field; hence $p_{\max} \sim L_D$. If, on the other hand, $\omega > \omega_p$, the field varies too quickly for the electrons to be able to shield it, so that we have just from (30): $p_{\max} \sim v_e/\omega$ (see Ref. 22).

Note that if the dipole is large, i.e., $L \gg L_D \sim v_e/\omega_p$, then at frequencies $\omega \sim \omega_p$ we have $L \gg v_e/\omega$, i.e., $V \gg v_e$. We thus expect that, just as fast moving charges do, it should excite plasma oscillations near the plasma frequency. This is indeed true, but since the dipole is not small, it no longer behaves as outlined above.[23]

C. Plasma fluctuations

We have so far studied the shielding of different kinds of charge perturbations. We have seen that the main response to a perturbation of large amplitude is a strong depletion of the repelled particles in a region of scale L_D. In three dimensions, this corresponds to a sphere containing a number of like charges of order $\mathcal{N} \sim 4\pi n L_D^3 \sim L_D/r_0 \gg 1$ (to an inessential numerical factor), hence a total charge of magnitude $\mathcal{N}e$ which produces in that region a mean field of order

$$E \sim \mathcal{N}e/4\pi\epsilon_0 L_D^2. \tag{33}$$

Substituting $L_D \sim v_e/\omega_p$, one may verify that the electrostatic energy density $\epsilon_0 E^2/2$ is of the order of the kinetic one $\sim n m_e v_e^2/2$. Under different disguises, this is the basis of standard elementary estimates of the Debye length.

But what happens in the absence of an imposed perturbation? In fact the plasma is permanently perturbing itself with its own agitation, so that it cannot be perfectly neutral locally. The motion of its particles produces electric field fluctuations; [20] the mean field is zero, but the mean square is not. If the number $\mathcal{N}$ of particles within a Debye sphere fluctuates by $\sqrt{\mathcal{N}}$, this produces a mean-square Coulomb field of order $E^2 \sim (\sqrt{\mathcal{N}}e/4\pi\epsilon_0 L_D^2)^2$ which is $\mathcal{N}$ times smaller than the square of the field (33) produced by a large imposed perturbation. Hence, the electrostatic energy density $\epsilon_0 E^2/2$ is $\mathcal{N}$ times smaller than the kinetic one. This reasoning, however, takes only into account the fluctuations at the scale L_D, and does not give the power spectrum. A more precise, albeit still simplified, estimate can be made as follows.

Whenever an electron passes at a distance $p < L_D$, the electric field increases to the value $|\delta E| \sim e/4\pi\epsilon_0 p^2$ during the time $\tau \sim p/v_e$. The time Fourier transform of the field is roughly that of a small impulse of area $\tau|\delta E|$:

$$|E(\omega)| \sim \tau|\delta E| \sim e/4\pi\epsilon_0 p v_e,$$

for frequencies $\omega < 1/\tau$, and decreases at larger frequencies. The power spectrum of these fluctuations is

$$E_\omega^2 \sim 2\sum |E(\omega)|^2,$$

where the sum is over the number N of (independent) events per unit time (and the factor 2 account for the fact that we consider as usual only positive frequencies). The rate at which electrons pass with impact parameters between p and $p+dp$ is $dN \sim n v_e \times 2\pi p dp$ and so

$$\begin{aligned} E_\omega^2 &\sim 2\int dN|E(\omega)|^2 \\ &\sim \frac{m_e\omega_p^2}{4\pi\epsilon_0 v_e}\int\frac{dp}{p} \\ &\sim \frac{m_e\omega_p^2}{4\pi\epsilon_0 v_e}\ln\left(\frac{p_{\max}}{p_{\min}}\right). \end{aligned} \tag{34}$$

For $\omega < 1/\tau \sim v_e/p$, $p_{\max}$ is the smaller of the two scales L_D and v_e/ω, just as in Sec. V B. The value of $p_{\min}$ is the smallest relevant scale. To determine it we have to precise how we measure the field. With, for example, an electric antenna of length L, one sees the voltage power spectrum $\Phi_\omega^2 = E_\omega^2 L^2$ if L is smaller than the fluctuation scales, i.e., if $p_{\min} = L(>r_0)$:

$$\Phi_\omega^2 \sim E_\omega^2 L^2 \sim \frac{m_e\omega_p^2 L^2}{4\pi\epsilon_0 v_e}\ln\left(\frac{p_{\max}}{p_{\min}}\right). \tag{35}$$

It is interesting to see how this value is related to the power loss of the same antenna working as an oscillating dipole. Since the instantaneous electric current is $dq/dt \sim q\omega$, the dipole power loss (32) corresponds to an electric resistance

$$R \sim \frac{2P_{\text{dipole}}}{\langle q^2\omega^2\rangle} \sim \frac{L^2\omega_p^2}{\pi^3\epsilon_0 v_e^3}\ln\left(\frac{p_{\max}}{p_{\min}}\right). \tag{36}$$

Comparing to the voltage power spectrum (35), we find $\Phi_\omega^2 \sim (\pi^2/4)m_e v_e^2 R$, i.e., $\Phi_\omega^2 \sim m_e v_e^2 R$ in order of magnitude. Therefore, the losses of the dipole antenna (represented by its electric resistance) are directly related to the power spectrum of the electric fluctuations that is measures in the plasma. This is a simplified generalization of Nyquist's fluctuation-dissipation theorem[24] $\Phi_\omega^2 \sim 4k_B TR$ which holds at thermal equilibrium. It is obtained here without any reference to thermodynamic concepts, but with the implicit assumption that the plasma is isotropic, and not too far from equilibrium in that we can define a "typical" electron velocity.

This very simple analysis does not take into account the electrons moving faster than average. We have seen that an electron moving at velocity $V \gg v_e$ produces electrostatic waves near the plasma frequency with a wavelength $\sim 2\pi V/\omega_p \gg L_D$. This gives another contribution to the fluctuations, corresponding to plasma waves whose power spectrum peaks at the plasma frequency. Since their wavelength is much larger than L_D, these waves must be observed with an antenna larger than the Debye length, just as an oscillating dipole can excite them only when it is larger than its Debye dress.

VI. CONCLUDING REMARKS

We have seen that the concept of Debye shielding does not require Boltzmann's equilibrium. With a non-Maxwellian (but isotropic) velocity distribution of plasma particles, the shielding length is obtained by replacing the thermal energy $k_B T$ by $m/\langle v^{-2}\rangle$ for each particle species. This quantity involves the mean inverse-squared velocity, which is more sensitive to the particles of low velocity than is the value $\langle v^2\rangle$ determining the pressure. Indeed, particles of low velocity have a larger ratio of potential to kinetic energy; they are thus more perturbed by the potential and contribute more to the shielding than the fast particles.

Even though it is a linear approximation, the concept still holds—with some restrictions, for large perturbations and in particular with finite charged objects. In that case the attracted and repelled species do not play symmetrical roles: The shielding is, in general, mainly produced by a strong density decrease of the repelled particles, and to a lesser extent—albeit only with a three-dimensional geometry, by a more gentle increase of the attracted ones.

The Debye shielding holds not only for charges at rest, but also for moving charges provided that they move slower than ambient electrons. Otherwise the plasma particles cannot react fast enough to shield the field. It is then neither shielded nor just Coulomb: The moving charge behaves as a Cerenkov emitter which excites plasma waves along its trajectory.

ACKNOWLEDGMENTS

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[1]P. Debye and E. Hückel, "Zur theorie der elektrolyte," Phys. Zeitschrift 9, 184–206 (1923).

[2]Lewi Tonks and Irving Langmuir, "Oscillations in ionized gases," Phys. Rev. 33, 195–210 (1929).

[3]We also neglect quantum effects. This requires that the particle encounter distances be also larger than their de Broglie wavelengths. The larger wavelength is in general that of electrons: $\lambda_{\rm Broglie}=\hbar/m_e v_e$, and from (1) we have $\lambda_{\rm Broglie}/r_0\sim(m_e v_e^2/2E_{\rm Bohr})^{1/2}$ where $E_{\rm Bohr}=m_e e^4/8\epsilon_0^2 h^2$ is the Bohr energy, so that this restriction is relevant when the electron kinetic energy is larger than the typical ionization energy (which is of the order of $E_{\rm Bohr}$).

[4]See for example, P. C. Clemmow and J. P. Dougherty, *Electrodynamics of Particles and Plasmas* (Addison-Wesley, Reading, MA 1969), pp. 233–237.

[5]Because, with $|e\Phi(r)/m_e v^2|\ll 1$, p always increases monotonically with r.

[6]This result can also be obtained from the linearized Vlasov equation.

[7]The particles strongly perturbed by the potential are those whose velocity, or its component perpendicular to a wire or a plane—depending on the geometry, is small enough. To get a problem strictly linear, one might choose a velocity distribution having no such particles. However, although it is possible to build an isotropic velocity distribution having no particles of velocity *modulus* smaller than a given value, any distribution isotropic in three dimensions does have particles whose velocity *component* perpendicular to an axis or a plane is arbitrarily small, and thus for which the problem is nonlinear in one or two dimensions.

[8]In this article, the plasma is three-dimensional, i.e., is made up of pointlike electrons and ions. In a plasma made up of wire or plane charges, the Debye shielding is very different; see Shang-Keng Ma, *Statistical Mechanics* (World Scientific, Singapore, 1985), pp. 337–341.

[9]A. J. M. Garrett, "Screening of point charges by an ideal plasma in two and three dimensions," Phys. Rev. A 37, 4354–4357 (1988).

[10]J. G. Laframboise and L. W. Parker, "Probe design for orbit-limited current collection," Phys. Fluids 16, 629–636 (1973).

[11]All the trajectories may not connect to the unperturbed plasma because, even if the shape of the potential profile does not allow bound orbits of positive energy (the so-called orbit-limited condition, see for example Ref. 10), there exists orbits of negative energy trapped around the charge; if the plasma is not strictly collisionless, they can be populated by collisions—however infrequent, if one waits long enough.

[12]See for example J. G. Laframboise, "Theory of spherical and cylindrical probes in a collisionless Maxwellian plasma at rest" (University of Toronto Institute for Aerospace studies Report 100, 1966).

[13]E. C. Whipple, "Potentials of surfaces in space," Rep. Prog. Phys. 44, 1197–1250 (1981).

[14]We neglect in particular the photoelectron and secondary emission (Ref. 13).

[15]See for example, Francis F. Chen, *Introduction to Plasma Physics* (Plenum, New York, 1974), pp. 248–249.

[16]In the present simple derivation we have neglected the electron agitation; otherwise we would have found propagating Langmuir waves instead of plasma oscillations at ω_p; see M. H. Cohen, "Radiation in a plasma. I. Cerenkov effect," Phys. Rev. 123, 712–721 (1961).

[17]N. Meyer-Vernet, "Surprises in classical physics: radiation problems in stable and linear plasmas," Eur. J. Phys. 5, 150–156 (1984).

[18]We have estimated the force by summing up the effects of individual long range encounters. But we could just as well have deduced it from the mean electric field acting on the charge;[20] we would have found that the field's Fourier component has a small imaginary part, indicating a damping which is a special case of Landau damping (L. D. Landau, "On the vibrations of the electronic plasma," J. Phys. USSR 10, 25 (1946), reprinted in D. ter Haar, *Men of Physics: L. D. Landau* (Pergamon, New York, 1965), pp. 96–116.) As the charge is slowed down, the ambient particles are accelerated in the direction of its motion; this process could be reversed if there were no collisions making particles lose their memory. Note, too, that if the ambient velocity distribution were so anisotropic that more particles were coming from the rear side than from the front side, the moving charge would be accelerated instead of being slowed down, just as, under similar conditions, a plasma wave is amplified instead of being Landau damped.

[19]This expression of the braking holds also in practice [and was in fact first derived: S. Chandrasekhar, Dynamical friction I. General considerations: the coefficient of dynamical friction, Astrophys. J. 97, 255–273 (1943)] for particles interacting via the gravitational force, at equilibrium—replacing $|q|e/4\pi\epsilon_0$ by its gravitational equivalent MmG (where M and m are the particle masses and G the gravitational constant), although there is no gravitational shielding since all masses attract each other. This is because in this case $p_{\max}$ is simply equal to the size of the system, which is itself of the order of the "gravitational" Debye length [obtained by replacing $e^2/4\pi\epsilon_0$ by m^2G in (8)]. Indeed, L_D represents the size of a nonneutral region, and in the gravitational context any system is nonneutral, so that L_D—called "Jeans length," turns out to be the typical size of a gravitational system near equilibrium. (This is a general consequence of virial theorem.)

[20]A. G. Sitenko, *Electromagnetic Fluctuations in Plasmas* (Academic, New York, 1967), pp. 125–138.

[21]Lyman Spitzer, Jr., *Physics of Fully Ionized Gases* (Wiley, New York, 1962), pp. 128–129.

[22]Just as for a moving charge, this power loss can also be viewed as due to Landau damping of electrostatic waves or fluctuations; this is indeed the basis of a more conventional derivation of this result (Ref. 17).

[23]See N. Meyer-Vernet and C. Perche, "Toolkit for antennae and thermal noise near the plasma frequency," J. Geophys. Res. 94, 2405–2415 (1989), where a more conventional derivation of the results of Sec. V.C can also be found.

[24]Aldert van der Ziel, *Noise* (Prentice-Hall, New York, 1954), pp. 8–9.

THE DIFFICULTY OF FINDING NEW IDEAS

If you can find any other view of the world which agrees over the entire range where things have already been observed, but disagrees somewhere else, you have made a great discovery. It is very nearly impossible, but not quite, to find any theory which agrees with experiments over the entire range in which all theories have been checked, and yet gives different consequences in some other range, even a theory whose different consequences do not turn out to agree with nature. A new idea is extremely difficult to think of. It takes a fantastic imagination.

Richard Feynman, *The Character of Physical Law* (M.I.T., Cambridge, 1967), pp. 171–172.