On Natural Noises Detected by Antennas in Plasmas

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A formal generalization of the Nyquist formula for an antenna in a possibly anisotropic equilibrium plasma is presented along with practically useful expressions derived from it. It is shown that this can explain some preliminary results of the recent three-dimensional radio mapping experiment (SBH) on the ISEE 3 spacecraft.

1. Introduction

Several types of natural noises (i.e., detected by passive experiments) have been observed in situ in the ionosphere and magnetosphere. Most of the interpretations [for example, *Shaw and Gurnett,* 1975; *Christiansen et al.,* 1978] invoke non-equilibrium processes, since the particle distribution functions are generally non-Maxwellian.

However, in some cases of practical interest, described in the present paper, the thermal noise itself can actually be measured.

This thermal noise has been previously calculated, by assuming [*Andronov,* 1966] or deriving [*De Pazzis,* 1969; *Fejer and Kan,* 1969] Nyquist formula for an antenna in an isotropic plasma. The present paper contains a generalization to an antenna in a possibly anistropic plasma. This derivation is basically analogous to that by *Fejer and Kan* [1969], but it directly uses the fluctuation-dissipation theorem, so that it avoids formulating explicitly the plasma dielectric tensor, and consequently the antenna resistance.

Some practical applications to geophysical plasmas are discussed, and it is shown that it can, in particular, successfully explain some results of the recent three-dimensional radio mapping experiment on ISEE 3.

2. Thermal Noise Measured by an Antenna

The voltage measured by a receiving passive antenna, in open circuit, is obtained in the usual way as

$$V(t) = \int \mathbf{E}(\mathbf{r}, t)\cdot \frac{\mathbf{J}(\mathbf{r})}{I_0}\, d\mathbf{r} \tag{1}$$

where $\mathbf{E}(\mathbf{r}, t)$ is the field to be measured, and $\mathbf{J}(\mathbf{r})/I_0$ the normalized current distribution in emission conditions. (In fact, in a magnetoactive plasma, as the dielectric permitivity tensor satisfies $\epsilon_{ij}(\mathbf{k}, \omega, \mathbf{B}_0) = \epsilon_{ji}(-\mathbf{k}, \omega, -\mathbf{B}_0)$, the application of the reciprocity theorem [*Ginzburg,* 1964], giving (1), shows that the current must be taken in a medium where the static magnetic field $\mathbf{B}_0$ has been reversed; this has no practical consequences in most applications.)

The autocorrelation of $V(t)$ is

$$\langle V(t_1)V(t_2)\rangle = \frac{1}{I_0^2}\iint d\mathbf{r}_1\, d\mathbf{r}_2\, \mathbf{J}(\mathbf{r}_1) < \mathbf{E}(\mathbf{r}_1, t_1)\mathbf{E}(\mathbf{r}_2, t_2) > \mathbf{J}(\mathbf{r}_2)$$

If the medium is homogeneous and stationary, the quadratic space-time correlation function depends only on $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and $t = t_1 - t_2$. Hence, Parseval theorem gives the following spectral density:

$$V_\omega^2 = \frac{1}{I_0^2(2\pi)^3}\int d\mathbf{k} J_i(\mathbf{k})E_{ij}(\mathbf{k}, \omega)J_j^*(\mathbf{k}) \tag{2}$$

where the usual convention of sommation on indices is implied, and

$$V_\omega^2 = \int \langle V(t_1)V(t_1 + t)\rangle\, e^{i\omega t}\, dt$$

$$\mathbf{J}(\mathbf{k}) = \int \mathbf{J}(\mathbf{r}) \exp(-i\mathbf{k}\cdot\mathbf{r})\, d\mathbf{r} \tag{2'}$$

$$E_{ij}(\mathbf{k}, \omega) = \int dt\, d\mathbf{r} < E_i(\mathbf{r}_1, t_1)E_j(\mathbf{r}_1 + \mathbf{r}, t_1 + t) > \exp[i(\omega t - \mathbf{k}\cdot\mathbf{r})]$$

($\mathbf{J}(\mathbf{r})$ and $\mathbf{E}(\mathbf{r}, t)$ are taken as real).

In an equilibrium plasma, the tensor of the spectral distribution of the electric field fluctuations in the medium is given, in the classical limit (high temperature), from the fluctuation-dissipation theorem, as [*Sitenko,* 1967]

$$E_{ij}(\mathbf{k}, \omega) = \frac{i\chi T}{\epsilon_0\omega}[\Lambda_{ji}^{-1} - \Lambda_{ij}^{-1*}] \tag{3}$$

where

$$\Lambda_{ij}(\mathbf{k}, \omega) = \frac{k^2c^2}{\omega^2}\left(\frac{k_ik_j}{k^2} - \delta_{ij}\right) + \epsilon_{ij}(\mathbf{k}, \omega) \tag{4}$$

(χ, c, ϵ_0 are the Boltzmann constant, velocity of light, and vacuum permittivity; and T is the plasma temperature; rational unit system).

Substituting (3) in (2) and interchanging the dummy indices give

$$V\omega^2 = \frac{-2\chi T}{(2\pi)^3\epsilon_0\omega I_0^2}\int d\mathbf{k}\, \mathrm{Im}\,\{J_i^*(\mathbf{k})\,\Lambda_{ij}^{-1}(\mathbf{k}, \omega)J_j(\mathbf{k})\} \tag{5}$$

(where Im denotes the imaginary part).

On the other hand, the antenna resistance is given, by the usual emf method, as

$$R = -\frac{1}{I_0^2}\mathrm{Re}\left(\int d\mathbf{r}\mathbf{E}(\mathbf{r})\cdot\mathbf{J}(\mathbf{r})\right) = \frac{-1}{(2\pi)^3I_0^2}\mathrm{Re}\left(\int d\mathbf{k}\mathbf{E}(\mathbf{k})\cdot\mathbf{J}(\mathbf{k})^*\right)$$

where $\mathbf{E}(\mathbf{r})$ is the field of the harmonic source $\mathbf{J}(\mathbf{r})$ (antenna current, $e^{-i\omega t}$), thus satisfying

$$\Lambda(\mathbf{k}, \omega)\cdot\mathbf{E}(\mathbf{k}) = -i\frac{\mathbf{J}(\mathbf{k})}{\omega\epsilon_0}$$

where $\Lambda(\mathbf{k}, \omega)$ is the tensor written in (4). So, we obtain

$$R = \frac{-1}{(2\pi)^3\epsilon_0\omega I_0^2}\int d\mathbf{k}\, \mathrm{Im}\,\{J_i^*(\mathbf{k})\Lambda_{ij}^{-1}(\mathbf{k}, \omega)J_j(\mathbf{k})\} \tag{6}$$

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Substituting (6) in (5) gives

$$V_\omega^2 = 2\chi TR \qquad (6')$$

which is the usual Nyquist formula. The factor 2, instead of 4, in the well-known formula stems from the definition (2′) of V_ω^2 [see, for example, *Papoulis,* 1965].

We stress that, owing to the hypotheses involved, this derivation is strictly valid for any antenna immersed in an homogeneous stationary, and possibly anisotropic, equilibrium plasma and in the absence of external fields. It permits the calculation of the thermal noise by using (6′), together with previous theoretical derivations of the antenna resistance R.

If the plasma is not in equilibrium, (3) and thus (6′) are not valid, and, in the general case, the calculation is not straightforward; if the plasma is stable, one can use (2) and insert the proper expression for the tensor $E_{ij}(\mathbf{k}, \omega)$, taking account of the actual particle distribution function [*Sitenko,* 1967], like in *Grabowski and Slavik* [1976], for example (whose derivation is restricted to a two-element-point-dipole antenna). In some special cases, (7) can be generalized; we will return to this point in section 3.

The other restrictive hypothesis is the plasma homogeneity; the calculation is thus strictly valid for the so-called grid antenna. In practice, the antenna is surrounded by a sheath, which may modify the result: for example, it is known [*Meyer-Vernet et al.,* 1978] that an electron-depleted sheath (typical for ionospheric applications) may increase the high-frequency resistance by several orders of magnitude, below the electron plasma frequency; on the other hand, in some magnetospheric or solar wind applications, the photoelectron noise may play a part.

Finally, we note that a relative velocity **V** between the antenna and the plasma, does not change (6′); of course, in this case, R must be calculated by performing the transformation $\omega \rightarrow \omega - \mathbf{k}\cdot\mathbf{V}$ (nonrelativistic case) in (6).

3. Applications to an Equilibrium Electron Plasma

Isotropic plasma. In an isotropic plasma (dielectric permittivities longitudinal ϵ_L and transverse ϵ_T), the tensor E_{ij} decouples in the well-known two parts

$$E_{ij}(\mathbf{k}, \omega) = \frac{2\chi T}{\epsilon_0 \omega}\left[\frac{k_i k_j \operatorname{Im}(\epsilon_L)}{k^2|\epsilon_L|^2} + \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\frac{\operatorname{Im}(\epsilon_T)}{|\epsilon_T - k^2c^2/\omega^2|^2}\right] \qquad (7)$$

For a small magnetic antenna, $\mathbf{k}\cdot\mathbf{J}(\mathbf{k}) = 0$; and putting (7) in (2) shows explicitly that the longitudinal term does not contribute and the measured noise will be negligible.

On the other hand, for a small electric antenna, the first term is dominant, and one expects a noise band, peaking in the vicinity (and above) the plasma frequency, due to the contribution of the first Landau pole of ϵ_L, and of the others. The noise bandwidth and amplitude depend on the antenna length, owing to the factor $J(\mathbf{k})$ in (2) (increasing the antenna length decreases the width); a small residual noise is also expected below the plasma frequency, due to the contribution of the other Landau poles.

To be more explicit, consider a short filamental antenna with triangular current distribution, operating at frequencies near the electron plasma frequency. The calculation of R has been performed by *Kuehl* [1967] (and later generalized to finite antenna's radius [*Schiff,* 1970]); this involves a numerical integration in **k**, like (6), using the usual expression of the permittivity, with the Fried and Conte function (the so-called kinetic description).

In the special case when small values of k give the dominant contribution in the integral giving R, the so-called hydrodynamic approximation can be used. This leads to the following explicit expression [*Balmain,* 1965]:

$$R = 2\{k_pLF(k_pL)\}/[\pi\epsilon_0\omega L(\omega^2/\omega_p^2 - 1)] \qquad (8)$$

where $k_p = (\omega^2/\omega_p^2 - 1)^{1/2}/(3)^{1/2}L_D$, L_D, L and $\omega_p/2\pi$ are, respectively, the Debye length, antenna half-length, and plasma frequency; F is defined in Appendix 2 (12′), and the term $\{\ \} \rightarrow \pi/4$ in the limit $k_pL \rightarrow \infty$.

This concerns a filamental antenna. For an antenna consisting of two small spheres, approximate calculations are performed in Appendix 1.

An interesting feature of these results is that if $L/L_D \gg 1$ and $\omega/\omega_p \gg 1$, the expected noise power-spectrum varies as ω^{-3} for the filament antenna and as ω^{-2} for the two-spheres antenna. This could be easily verified by experiments.

Anisotropic plasma. For an anisotropic plasma the tensor $E_{ij}(\mathbf{k}, \omega)$ takes a much more complicated form, which makes the resistance calculations more difficult. For the short filamental antenna, R has been calculated by *Nakatani and Kuehl* [1976] (kinetic description); they give some numerical results for parameters typical of laboratory plasmas, in the case of a dipole parallel to the static magnetic field.

When the hydrohynamic with tensor pressure (or full adiabatic) approximation of ϵ_{ij} is valid for calculating the resistance, one can use results by *Meyer and Vernet* [1974], which give also the variation with antenna orientation. For conditions typical of ionospheric experiments, this approximation gives correct results [*Meyer-Vernet,* 1978] for frequencies near the plasma and upper hybrid frequencies.

In the general case, this approximation must be taken with caution: it involves both an asymptotic expansion excluding the vicinity of the gyrofrequency harmonics and a truncated series expansion for low values of k; it should not be valid in many magnetospheric applications when, in particular, the antenna length is of the order of the Debye length.

4. Applications to High-Frequency Noises in Geophysical Plasmas

Let us explain the previous results to interpret some data of the recent three-dimensional radio mapping experiment on ISEE 3 [*Knoll et al.,* 1978] in the solar wind. Neglecting the anisotropy, the relevant parameters are $f_p = \omega_p/2\pi \sim 2.4\ 10^4$ Hz; $T \sim 10^5$ °K; $L = 45$ m (cylindrical dipole); thus $L/L_D \sim 10$ (and the antenna radius $r = 2.10^{-4}$ m satisfies $r/L_D \ll 1$).

Of course, an obvious objection to thermal noise calculation is that the plasma is not in equilibrium. Broadly speaking, the actual electron distribution function can be described as a bi-Maxwellian one (with the colder component containing the main part of the total kinetic energy). In the close vicinity of the plasma frequency, the presence of the hot component is expected to change considerably the noise level [see, for example, *Fejer and Kan,* 1969]. However, as shown in Appendix 2, the presence of the hot component (described in *Feldman et al.* [1975], for instance) does not change the noise levels very much for $\omega/\omega_p \gg 1$, $L/L_D \gg 1$.

Figure 1 shows an example of the measured noise spectrum $2V_\omega^2$: the curve drawn is the actually measured noise, multiplied by the factor $(1 + Z/Z_v)^2$ which arises from the finite impedance Z_v of the measuring device (input capacity: 40 pF), where Z is the antenna impedance. (In the main part of the

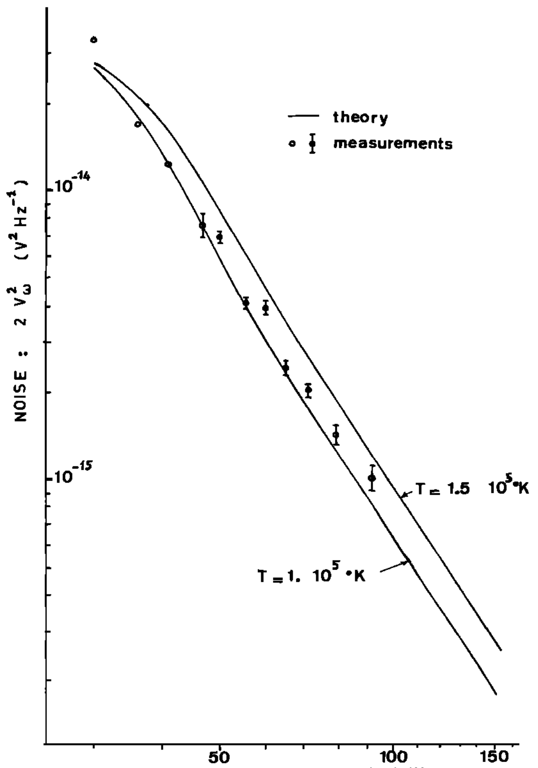

Fig. 1. Comparison between the measured noise level brought to the open circuit antenna terminals ($2V_\omega^2$), and the theoretical thermal noise level (for f_p = 24 kHz and two electron temperatures 10^5 and 1.5 10^5 °K, respectively).

curve, $\omega/\omega_p >> 1$ and Z is calculated as in a vacuum; nearer to ω_p, the hydrodynamic approximation has been used: since its validity is very questionable with our parameters for calculating Im (Z), the corresponding data points are shown without explicit error bars.)

On the same figure is drawn the theoretical noise $2V_\omega^2$, calculated from (8) and (6′) with $T = 10^5$ °K and 1.5 10^5 °K, and f_p deduced from the $2f_p$ line shown in Figure 2. One sees that the agreement is good. At high frequencies, the law ω^{-3} is a good fit to the data and the amplitude could even serve as a temperature measuring device. This is of interest, since such a passive noise measurement is generally not recommended to measure the thermal population's parameters (*Pottelette et al.* [1977], for example).

Figure 2 shows an example of the data variation. In some cases (which are probably related to a variation in the nonthermal electrons), a line at $2f_p$ appears, but the main part of the spectrum is only slightly modified. The data in Figure 1 are those of curve 1 of Figure 2, where no such nonlinear feature appears. It is important to note that in these examples the immediate vicinity of f_p does not fall in the frequency range of the receiver. Of course, these are preliminary results, and a more detailed interpretation should be made taking in account such features as, for example, the relative bulk velocities (between the satellite and the plasma, and/or the different components of the distribution function), and including the immediate vicinity of the plasma frequency.

Incidentally, we remark that some low-level noises have been previously reported in the magnetosphere [*Christiansen et al.*, 1978; *Shaw and Gurnett*, 1975], for example, and tentatively ascribed by these authors to nonthermal processes. Let us calculate an order of magnitude of the expected thermal noise in these cases: take parameters $f_p = 3.10^4$ Hz, $T = 10^4$ °K, L = 20 m and neglect (unrealistically) the anisotropy and the nonthermal electrons. We obtain, for a filament dipole ((8) and (6′)), a noise peaking (slightly above the plasma frequency) at about $2V_\omega^2 \sim 3 \times 10^{-14} V^2$ Hz^{-1}. For the two-spheres antenna, equations (10) and (6′) give a noise peaking at about $2\ V_\omega^2 \sim 10^{-13} V^2$ Hz^{-1} (slightly above the plasma frequency). In both cases these figures are of the same order of magnitude as the data or, at least, not negligible as compared to them.

This suggests that it should be interesting to perform a more correct calculation (taking account, in particular, of the anisotropy) to be sure that the nonthermal contribution is always as important as implicitly assumed by the above authors, for these low-level noises.

Finally, it is worth noting that the quantity $2V_\omega^2/L^2$, which is the result generally quoted by experimenters, has no intrinsic physical meaning except in the limit $L \rightarrow 0$, which is not at all achieved for the thermal noise in the above examples. For instance, in the case drawn in Figure 1, $2V_\omega^2$ is expected to behave approximately in $1/L$.

CONCLUSION

Contrary to what is generally assumed by space experimenters, the thermal contribution to the noise measured in passive experiments should not be overlooked. We have shown that approximate calculations of the thermal noise can explain some preliminary results of the recent three-dimensional radio mapping experiment on ISEE 3.

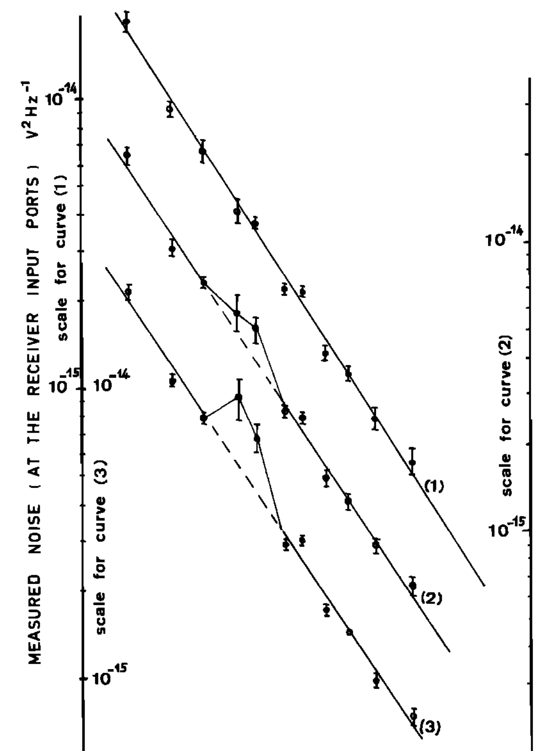

Fig. 2. Noise levels at the receiver input measured at three consecutive times, 2000 s apart. The $2f_p$ line ($f_p \sim$ 24 kHz) appears on curve 2 (~2000 s after curve 1) and curve 3 (~2000 s after curve 2). The line does not appear on curve 1, which corresponds to the same data as used in Figure 1.

APPENDIX 1: THE PAIR OF SPHERICAL PROBES IN A WARM ISOTROPIC PLASMA

In the case of an antenna consisting of a pair of small spherical probes, approximate values of R can be obtained as follows.

Let $2L$ be the distance between the probes (on the Z axis) and let us neglect the individual probe dimension, and assume an $\exp(-i\omega t)$ time dependence.

The charge distribution is written as

$$Q\,\delta(x)\,\delta(y)\,[\delta(z-L)-\delta(z+L)]$$

thus in Fourier space,

$$\tilde{\rho}(\mathbf{K}) = -Q2i\sin(k_z L)$$

In the so-called quasi-static approximation, expected to be valid here for most applications (it retains the longitudinal part in (6)), the potential is given, in Fourier space:

$$\tilde{V}(\mathbf{K}) = \frac{\tilde{\rho}(\mathbf{K})}{\epsilon_0\epsilon_L(\mathbf{K},\omega)K^2}$$

The resistance is obtained as

$$R = \mathrm{Re}\,\{[V(0,0,L)-V(0,0,-L)]/-i\omega Q\}$$

The $\mathbf{K}$ integration is easily performed in spherical coordinates and reduces to

$$R = \frac{-1}{\pi^2\omega\epsilon_0}\,\mathrm{Im}\int_0^\infty \frac{dk}{\epsilon_L(k,\omega)}\left[1-\frac{\sin(2kL)}{2kL}\right] \qquad (9)$$

The calculation of R requires a numerical integration; however, inspection of (9) leads to some approximate results.

First, using approximations of the *Fried and Conte* [1961] function as in *Kuehl* (1966), gives the high and low frequencies limiting values

$$R \sim \omega_p/\pi^{3/2}2^{1/2}\epsilon_0\omega^2 L_D \qquad \omega/\omega_p \gg 1 \qquad L/L_D \gg 1$$

$$R \sim 1/(2\pi)^{3/2}\epsilon_0\omega_p L_D \qquad \omega/\omega_p \ll 1 \qquad L/L_D \gg 1$$

These are very different from those relevant for a filamental antenna [*Kuehl*, 1966]; of course, both formulations are equivalent only in the limit $L \to 0$ (except for a factor 4 which stems from the difference in effective lengths).

Second, in the hydrodynamic approximation, (9) gives

$$R = \left[1-\frac{\sin(2k_pL)}{2k_pL}\right]\Big/6\pi\epsilon_0\omega L_D^2 k_p \qquad (10)$$

for $\omega/\omega_p > 1$.

We note that, as expected, this reduces in the limit $L \to \infty$, to 2 times the corresponding result for one single sphere [*Fejer*, 1964]; and, in the limit $L \to 0$, to 4 times the corresponding limit for the filament. We note also that the derivation above is valid only for small spheres radii: in particular, it assumes implicitly the condition $k_pR \ll 1$ (where R is the radius of one individual sphere).

APPENDIX 2: THE HIGH-FREQUENCY NOISE LIMIT FOR A BI-MAXWELLIAN ELECTRON DISTRIBUTION FUNCTION

Let us consider a bi-Maxwellian electron distribution function (with parameters ω_{pk}, T_k, $k = 1, 2$, $\omega_{p1} \gg \omega_{p2}$, $T_2 > T_1$) in an isotropic plasma. Equation (7) can be generalized as [*Sitenko*, 1967] (neglecting the *EM* part)

$$E_{ij}(\mathbf{k},\omega) = \frac{2\chi}{\epsilon_0\omega k^2}\frac{k_ik_j}{|\epsilon_L|^2}\sum_k T_k\,\mathrm{Im}(\epsilon_{Lk}) \qquad (11)$$

with

$\epsilon_L = 1 - \sum_k Z'(\omega/(2^{1/2}kL_{Dk}\omega_{pk}))/(2k^2L_{Dk}^2)$ (L_{Dk} is the Debye length for the population k, Z is the plasma dispersion function [*Fried and Conte*, 1961].)

For the filamental antenna (0**Z** direction, half-length L) with triangular current distribution, i.e., in Fourier space,

$$\mathbf{J}(k) = \mathbf{z}4I_0\sin^2(k_zL/2)/(k_z^2L)$$

(2) and (11) reduce to

$$V_\omega^2 = \frac{8\chi}{\pi^2\epsilon_0\omega}\int_0^\infty dk\,\frac{F(kL)}{|\epsilon_L|^2}\sum_k T_k\,\mathrm{Im}(\epsilon_{Lk}) \qquad (12)$$

where

$$F(x) = [Si(x) - 0.5\,Si(2x) - 2\sin^4(x/2)/x]/x \qquad (12')$$

(Si is the sine integral). The calculation of (12) for arbitrary parameters requires numerical integration. A similar expression has been evaluated by *Fejer and Kan* [1969] in the close vicinity of the plasma frequency. We shall evaluate (12) in another limiting case, namely, for high frequencies.

It is easily seen that if the condition $\omega/\omega_{p1}(T_1/T_2)^{1/2} \gg 1$ is satisfied, then $|\epsilon_L|^2$ is of order 1 when the integrand in (12) is not negligible, and thus (12) can be approximated by

$$V_\omega^2 = \frac{8\chi(2)^{1/2}}{\pi^{3/2}\epsilon_0\omega^2}\sum_k\frac{\omega_{pk}}{L_{Dk}}T_k\int_0^\infty d\zeta_k\zeta_k \cdot \exp(-\zeta_k^2)F[\omega L/((2)^{1/2}\omega_{pk}L_{Dk}\zeta_k)]$$

Then, if $L/L_{D1} \gg 1$ (and thus $(\omega/\omega_{pk})(L/L_{Dk}) \gg 1$ for $k = 1, 2$), the function F can be replaced by its approximation for large arguments, giving finally

$$V_\omega^2 = \frac{\chi}{\epsilon_0 L\omega^3}\sum_k T_k\omega_{pk}^2 \qquad (13)$$

Thus with typical parameters such as $\omega_{p1}^2/\omega_{p2}^2 > 20$, $T_1/T_2 \sim 1/6$ [see, for example, *Feldman et al.*, 1975], the noise is approximately given by the 'thermal' contribution (ω_{p1}, T_1), for frequencies and antenna lengths satisfying

$$\frac{\omega}{\omega_{p1}}\left(\frac{T_1}{T_2}\right)^{1/2} \gg 1 \qquad \frac{L}{L_{D1}} \gg 1$$

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REFERENCES

Andronov, A. A., Antenna impedance and noise in a space plasma, *Komich. Issled.*, *4*, 558, 1966.
Balmain, K. G., Impedance of a short dipole in a compressible plasma, *J. Res. Nat. Bur. Stand., Sect. D*, *69*, 559, 1965.
Christiansen, P., et al., Geos I: Identification of natural magnetospheric emissions, *Nature*, *272*, 682, 1978.
De Pazzis, O., Shot noise in antennas, *Radio Sci.*, *4*, 91, 1969.
Fejer, J. A., Interaction of an antenna with a hot plasma and the theory of resonance probes, *J. Res. Nat. Bur. Stand., Sect. D*, *68*, 1171, 1964.
Fejer, J. A., and J. R. Kan, Noise spectrum received by an antenna in a plasma, *Radio Sci.*, *4*, 721, 1969.
Feldman, W. C., J. R. Ashbridge, S. J. Bame, H. D. Montgomery, and S. P. Gary, Solar wind electrons, *J. Geophys. Res.*, *80*, 4181, 1975.
Fried, B. D., and S. D. Conte, *The Plasma Dispersion Function*, Academic, New York, 1961.
Ginzburg, V. L., *The Propagation of Electromagnetic Waves in Plasmas*, p. 342, Pergamon, New York, 1964.
Grabowski, R., and J. Slavik, Theoretical power spectrum of measur-

able potential difference fluctuations in a stable magnetized plasma, *Plasma Phys.*, *18*, 641, 1976.
Knoll, R., et al., The three-dimensional radio mapping experiment on ISEE-C, *IEEE Trans.*, *16*(GE), 199, 1978.
Kuehl, H. H., Resistance of a short antenna in a warm plasma, *Radio Sci.*, *1*, 971, 1966.
Kuehl, H. H., Computations of the resistance of a short antenna in a warm plasma, *Radio Sci.*, *2*, 73, 1967.
Meyer, P., and N. Vernet, Impedance of a short antenna in a warm magnetoplasma, *Radio Sci.*, *9*, 409, 1974.
Meyer-Vernet, N., Impedance of a short antenna in a warm magnetoplasma: Experiment and comparison with theory, *Radio Sci.*, *13*(6), 1059, 1978.
Meyer-Vernet, N., P. Meyer, and C. Perche, Losses due to the inhomogeneous sheath surrounding an antenna in a plasma, *Radio Sci.*, *13*, 69, 1978.
Nakatani, D. T., and H. H. Kuehl, Input impedance of a short dipole antenna in a warm isotropic plasma, 1, Kinetic theory, *Radio Sci.*, *11*, 433, 1976.
Papoulis, A., *Probability, Random Variables and Stochastic Processes*, p. 362, McGraw-Hill, New York, 1965.
Pottelette, R., C. Chauliaguet, and L. R. O. Storey, Theoretical cross-spectrum of the microfield measured by two small dipoles in a warm isotropic plasma, *J. Plasma Phys.*, *17*, 201, 1977.
Schiff, M. L., Impedance of a short dipole antenna in a warm isotropic plasma, *Radio Sci.*, *5*, 1489, 1970.
Shaw, R. R., and D. A. Gurnett, Electrostatic noise bands associated with the gyrofrequency and plasma frequency in the outer magnetosphere, *J. Geophys. Res.*, *80*, 4259, 1975.
Sitenko, A. G., *Electromagnetic Fluctuations in Plasma*, p. 28, Academic, New York, 1967.

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