Improved GonioPolarimetry with JUNO

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Goniopolarimetry (GP – a more adapted name for Direction-Finding) allows to retrieve the wave parameters from a set of measurements. The wave parameters consist of two angles (θ,ϕ) defining the **k**-vector, and the 4 Stokes parameters defining the intensity (S) linear polarization (U,Q) and circular polarization (V), thus a total of 6 parameters to be determined. Each measurement performed by a receiver connected to an antenna (monopole or dipole at low frequencies) can be expressed as a function of the waves parameters through an equation.

A given receiver can perform instantaneously one or several measurements :

- the simplest case is a total flux receiver connected to one antenna, which performs one measurement (equivalent to a simple autocorrelation of the antenna signal), and provides thus a single equation allowing only to derive a rough approximation of the wave intensity S ; this is the case for the present JUNO-WAVES experiment.
- a more elaborate receiver with 2 inputs connected each to one antenna can provide instantaneously 4 measurements - the autocorrelation A_{ii} of each antenna signal, and the real and imaginary part of their complex cross-correlation C_{ij} – providing thus a system of 4 equations combining wave parameters ; such a system is given below ; this is the case for the proposed JUNO-WAVES-RAR experiment [see Ladreiter et al., 1995 ; Cecconi and Zarka, 2005 ; and references therein].

$$A_{11} = \frac{Sh_1^2}{2}\Big[(1+Q)\Omega_1^2 + 2U\Omega_1\Psi_1 + (1-Q)\Psi_1^2\Big] \quad (1)$$

$$A_{22} = \frac{Sh_2^2}{2}\Big[(1+Q)\Omega_2^2 + 2U\Omega_2\Psi_2 + (1-Q)\Psi_2^2\Big] \quad (2)$$

$$C_{12}^r = \frac{Sh_1h_2}{2}\Big[(1+Q)\Omega_1\Omega_2 + U(\Omega_1\Psi_2 + \Omega_2\Psi_1) + (1-Q)\Psi_1\Psi_2\Big] \quad (3)$$

$$C_{12}^i = \frac{Sh_1h_2}{2}V(-\Omega_1\Psi_2 + \Omega_2\Psi_1) \quad (4)$$

with

$$\Omega_i = \cos\theta_i\sin\theta - \sin\theta_i\cos\theta\cos(\phi-\phi_i) \quad (5)$$

$$\Psi_i = -\sin\theta_i\sin(\phi-\phi_i) \quad (6)$$

where h_i, θ_i and ϕ_i are the effective length, colatitude and azimuth or antenna i (which must be first calibrated by rheometry [Rucker et al., 1996], wire-grid modelling [Fischer et al., 2001], and/or observations of a reference radio source [Vogl et al., 2004]).

Let us call N_e the number of instantaneous measurements (or equations) provided by the receiver and N_p the number of wave parameter (or unknowns) to be determined from the measurements. Obviously, a unique solution may exist only if $N_e > N_p$. This is not true in the general case (e.g. $N_p=6$ versus $N_e=4$ with a 2 channel correlation receiver).

Various methods have been devised to overcome this limitation :

(1) assume that some wave parameters are known a priori (e.g. the source direction θ,ϕ, and/or the absence of linear polarization with U=Q=0) in order to reduce N_p, and solve the system (1-4) for the remaining parameters ;

(2) perform (quasi-)simultaneous measurements with several pairs of antennas : for example, Cassini/RPWS has 3 monopoles antennas (u,v,w) which can be used as 2 pairs of antennas

((u,w) and (v,w)) connected alternatively to a 2-input receiver [Gurnett et al., 2004]. This system provides instantaneously a set of 4 independent measurements (e.g. A_{uu}, A_{ww}, C_{uw}^{r}, C_{uw}^{i}), and quasi-instantaneously – within the time of an electronic switch and another integration, i.e. a few 10's of msec – a set of N_e=7 independent measurements (A_{uu}, A_{vv}, A_{ww}, C_{uw}^{r}, C_{uw}^{i}, C_{vw}^{r}, C_{vw}^{i}). A 3-input receiver would provide truly instantaneous measurements, but at a too high resource cost.

(3) perform a series of measurements with one pair of antennas on a spinning spacecraft, at several (N) measurements per spin, and consider this series as simultaneous measurements of the same emission. One thus gets N_e=N×4 independent equations. This technique was applied by the Ulysses/URAP experiment, with an analog synthesis of C_{12}^{r} and C_{12}^{i} as the sum in phase (S_0) and in quadrature (S_{90}) of the signals from the 2 antennas [e.g. Ladreiter et al., 1994]. It will be used with direct digital computation of measurements (1-4) for the proposed JUNO-WAVES-RAR experiment.

Method (1) imposes severe assumptions on the wave parameters. Method (2) assumes that the wave parameters remain constant over the duration of a pair of consecutive measurements at the same frequency (e.g. 25 to 325 msec with Cassini/RPWS). Method (3) assumes that the wave parameters remain constant over at least a fraction of the spacecraft spin period (12 sec for Ulysses). The latter assumption is far from granted for planetary emissions which are known to be very sporadic [Ladreiter et al., 1994 ; Zarka, 1998 ; and references therein]. It will be even less justified for JUNO, whose spin period is expected to be 20-30 sec. If wave parameters do vary significantly during the spin, then GP results are wrong.

However, it is now known that for Jovian low-frequency radio emissions, all wave parameters do not fluctuate with the same timescale, the most variable parameter being the intensity (S), and the least variable the polarization (U,Q,V) [Dulk et al., 1992, 1994]. We propose thus a new inversion of GP measurements on a slowly spinning spacecraft as JUNO, assuming that a subset of wave parameters only remain constant during the spacecraft spin.

If we assume that U, Q, and V remain constant over several seconds during which N sets of 2-antenna measurements are recorded, but let S, θ, and ϕ vary from one measurement to the next, then we get N_e=N×4 independent equations to derive N_p=N×3 + 3 parameters. $N_e > N_p$ implies N>3. Thus it will be possible to invert each group of 3 consecutive sets of measurements with different antenna-source geometries to derive reliable wave parameters. If JUNO-WAVES-RAR performs 8 sets of measurements at each frequency per spacecraft spin, then 3 consecutive sets correspond to relative antenna configurations of 0, 45 and 90°, well adapted to GP inversion. The assumption on wave parameters is simply a polarization stable over ¼ spin, i.e. 5 to 7.5 sec, which is very reasonable.

One may of course reanalyze the same series of GP measurements by varying the triplets considered together (i.e. (1-2-3), (4-5-6), (7-8-9), … and then (2-3-4), (5-6-7)… etc.), by varying the assumptions on wave parameters (U and Q only constant ; or U, Q, V, θ, and ϕ constant) and thus the number of consecutive measurement sets inverted together (from 1 to 8 or more). The resolution of a given system of N_e equations with N_p unknown can be done by any method (analytical inversion, least squares fit, singular value decomposition…).

The underlying assumptions to the above discussion is that the parameters h_i, θ_i and ϕ_i characterizing the electrical antennas are well-known (through rheometry [Rucker et al., 1996], wire-grid modelling [Fischer et al., 2001], and/or observations of a reference radio source [Vogl et al., 2004]), and that the antennas are more-or-less assimilable to short dipoles, i.e. presenting a marked minimum along a given direction in space, which is thus the direction of the electric antenna equivalent to the antenna+spacecraft system.

Note also that one set of instantaneous measurements (1-4) will actually correspond to an integration time of 20-80 msec with JUNO-WAVES-RAR. This represent 0.24° to 1.44° of spacecraft spin during the measurements, i.e. it is negligible for short integrations (20 msec), and may be corrected in real-time inside the instrument for longer integrations (80 msec).

Source direction (θ,ϕ) should be retrieved with an accuracy $\leq$2°, and Stokes parameters with a relative accuracy $\leq$10%.

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