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Volcanoes on Io

L M Celnikier

Laboratoire associé au CNRS No 326 et Laboratoire d'Astronomie de l'Université Pierre et Marie Curie, Observatoire de Paris-Meudon, Place J Janssen, 92195 Meudon Principal Cedex, France

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Abstract The tidal forcing phenomenon, used to predict the existence of volcanic activity on Io, is treated from a very simple point of view which nevertheless reproduces to a sufficient accuracy the final conclusions of the prediction, while highlighting the essential physical processes involved.

Résumé L'existence d'une activité volcanique sur Io a été dévinée comme étant une conséquence des effets de marée. Le phénomène est traité ici par une méthode très simple, qui permet de retrouver les resultats quantitatifs de façon élémentaire, tout en dégageant les principes physiques fondamentaux.

One of the more striking results of the Voyager missions was the discovery of intense volcanic-type activity on the surface of the jovian satellite Io. No less remarkable was the prediction by Peale *et al* (1979), published just three days before the flyby, that Io could well be the seat of such phenomena.

Since then the theoretical conclusions have been quoted, requoted and even criticised (Gold 1979). However, apart from purely verbal accounts, the basic physics of the analysis has remained outside the scope of most discussions, since the theoretical formula from which the prediction follows is based on a very technical computation (Peale and Cassen 1978) which itself relies on previous no less involved work (for example, Kaula 1966). This is a pity, since the central idea is simple and represents an elegant application of elementary mechanics and heat transfer theory to planetary physics.

In this paper it is shown how the essential result may be obtained directly from an elementary analysis using a somewhat simplified model for Io: one may think of the calculation as an example of how planetary physics may be brought to a wider audience than before. Most modern astrophysics textbooks contain order of magnitude estimates of stellar structure based simply on the gas laws and quantum mechanics as exemplified by the uncertainty principle: these calculations, divorced as they are of technical (but in the final analysis, irrelevant) computational difficulties, give one considerable insight into both the fundamental physics involved and the particular astrophysical application (see, for example, Weisskopf 1975, or Celnikier 1979). Planetary physics has long remained outside this type of approach, presumably because of the apparent complexity of solids and liquids. One notable exception (Weisskopf 1975) has shown, however, that qualitative physics can be applied to planetary problems with considerable success; the present paper is an extension of the same type of approach.

The best verbal description I have seen of the basic heating mechanism invoked for Io may be found in Morrison and Samz's popular book (1980) from which I quote verbatim:

'Io is about the same distance from Jupiter as the Moon from the Earth, but the much greater mass of Jupiter raises enormous tides in its satellite. These tides distort its shape, but no other effect would be present if Io remained at a constant distance from Jupiter. What Peale, Cassen and Reynolds realized was that the distance of Io from Jupiter varies as the result of small gravitational perturbations from the other Galilean satellites. Therefore, the tidal distortions also vary, in effect squeezing and unsqueezing Io each orbit. Such flexing pumps energy into the interior of Io in the form of heat.'

Translating this simple scheme into physics is far from trivial if the result is to be completely general—for example, for a satellite of arbitrary shape, internal structure, and elastic moduli. However, the essential phenomenon is the longitudinal 'squeezing and unsqueezing' effect: this allows one to replace the real geometrical configuration by a much simpler, albeit rather artificial one, without any significant alteration of the physics involved.

Consider a homogeneous cubic satellite of side L, density ρ and mass $m = L^3\rho$—this will be taken as a first approximation to a real body of the same mass but different shape.

Suppose that one of the sides is permanently aligned along the orbital radius vector: this corresponds to the fact that Io's axial rotation is synchronised with its orbital motion. The satellite can be thought of as two halves, each of mass $m/2$, whose barycentres are separated by $L/2$. If the satellite is instantaneously at a distance R from a central body of mass M, the two 'halves' of the satellite will experience a different gravitational force, the difference ΔF being given by:

$$\begin{aligned}\Delta F &= \Delta(GMm/2R^2)\\ &\cong GMm\ \Delta R/R^3\\ &= GMmL/2R^3.\end{aligned}$$

This differential force acts on two points separated by $L/2$ and is responsible for deforming the satellite along the radius vector.

A simple longitudinal elastic body of length l under the influence of a stretching force f satisfies Hooke's law:

$$\Delta l/l = f/SY$$

where: l = induced extension; S = cross section; Y = Young's modulus of elasticity. It follows that the work W done to extend the (cubic) satellite by a length ΔL under the influence of the force ΔF is given by:

$$\begin{aligned}W &= \Delta L\ \Delta F/2\\ &= L(\Delta F)^2/4SY \qquad (1)\end{aligned}$$

which reduces in our case to:

$$\begin{aligned}W &= \frac{L}{4SY}\left(\frac{GMmL}{2R^3}\right)^2\\ &= \frac{G^2M^2m^2L}{16YR^6}.\end{aligned}$$

Now, the satellite is in an elliptical orbit about the planet, so that R satisfies the polar equation:

$$R = a(1-e^2)/(1+e\cos\theta)$$

where:

a = semi-major axis of the orbit

e = orbital eccentricity

θ = polar angle.

We see that the deformation of the satellite varies periodically, as does therefore the work done on the elastic material:

$$\begin{aligned}W &= \frac{G^2M^2m^2L}{16Ya^6}\frac{(1+e\cos\theta)^6}{(1-e^2)^6}\\ &= \frac{G^2M^2m^2L}{16Ya^6}(1-6e^2+\ldots)\\ &\qquad\times(1+6e\cos\theta+15e^2\cos^2\theta+\ldots).\end{aligned}$$

Now, the eccentricity is much smaller than one, so that in the expansion for W we may neglect all terms higher than e^2, which leads to:

$$W \cong \frac{G^2M^2m^2L}{16Ya^6}[(1+6e^2)+6e\cos\theta+15e^2\cos^2\theta].$$

We are interested in the work done on the satellite during its orbital motion, so that the θ-independent term—which represents the average fixed distortion—may be left out of further consideration, giving us the secular contribution to the work, $W_s(\theta)$:

$$W_s(\theta) = \frac{G^2M^2m^2L}{16Ya^6}[6e\cos\theta+15e^2\cos^2\theta].$$

The total secular contribution $W_s(\text{tot})$ during one orbit is obtained by integrating this expression over 2π:

$$\begin{aligned}W_s(\text{tot}) &= \int_0^{2\pi} W_s(\theta)\,d\theta\\ &= \frac{15\pi G^2M^2m^2Le^2}{16Ya^6}.\end{aligned}$$

In short, the tide raising potential is quadrupolar and so proportional to $1/R^3$, as is therefore the distortion. The work done must thus be proportional to the square of the distortion, integrated over the volume of the body and divided by an elastic modulus related to Young's modulus; finally, the net work done in one orbital period when the eccentricity is small is proportional to the integral of $e^2\cos^2\theta$.

Consequently, the result is proportional to $e^2/(Ya^6)$, and the value of the constant depends on the geometrical configuration assumed.

We may think of W_s(tot) as being the energy stored in the satellite in the form of the forced oscillations induced by the periodically varying tidal force. Various dissipative effects will cause a fraction of this energy to be lost to the oscillations and ultimately to appear as heat within the satellite: in the theory of forced oscillators, the inverse of this fraction is commonly called the 'Q-factor' or the 'dissipation function'. Therefore, the thermal energy E appearing per orbit in the satellite as a consequence of dissipating the stored elastic energy of the tidal forcing is given by:

$$E = \frac{15\pi G^2M^2m^2Le^2}{16Ya^6Q}.$$

The orbital period P is given by:

$$P = (4\pi^2a^3/GM)^{1/2}$$

but it is in fact more convenient to work in terms of the angular velocity n:

$$\begin{aligned}n^2 &= (2\pi/P)^2\\ &= GM/a^3\end{aligned}$$

whence:

$$E = \frac{15\pi n^4 m^2 L e^2}{16YQ}.$$

Dividing this expression by the orbital period gives the mean rate dE/dt at which thermal energy is dissipated within the body of the satellite:

$$dE/dt \cong E/P = \frac{15n^5 m^2 L e^2}{32YQ}$$
$$= \frac{15n^5 \rho^2 L^7 e^2}{32YQ}.$$

This is the basic result used by Peale *et al* (1979); however, to simplify the comparison, it is better to present the result in terms of:

(i) The shear modulus of elasticity μ, since this is the quantity which Peale *et al* actually used; in the case of typical terrestrial materials, $Y \cong 2.5\ \mu$. One might note in passing that an intrinsically more meaningful parameter to use would have been the bulk modulus, since it is better determined for planetary material than is μ; since the bulk modulus is greater than the shear modulus, retaining the latter gives one an upper limit to the energy dissipation.

(ii) The satellite radius R_s: if one considers a radius to be essentially half of a characteristic dimension, we have:

$$R_s = L/2.$$

Finally, therefore, the average rate of production of thermal energy in our homogeneous cubic satellite under the action of the tidal forcing phenomenon is given by:

$$dE/dt = \frac{15 \times 2^7}{96} \cdot \frac{n^5 \rho^2 R_s^7 e^2}{\mu Q}$$
$$\cong 20 \frac{n^5 \rho^2 R_s^7 e^2}{\mu Q}.$$

The corresponding result for a homogeneous spherical satellite found by Peale *et al* (1979) is:

$$dE/dt = \frac{36\pi}{19} \frac{n^5 \rho^2 R_s^7 e^2}{\mu Q}$$
$$\cong 6 \frac{n^5 \rho^2 R_s^7 e^2}{\mu Q}.$$

The difference between the two results is really not very large, especially when one considers the economy of effort involved in the method presented here. The difference can be reduced even farther if one wishes by noting that the total energy refers to the volume occupied by the satellite: since a cubic satellite of characteristic dimension $2R_s$ occupies about twice the volume of a spherical satellite having the same characteristic scale, our result should be divided by about two in order to reduce it to the geometry one normally associates with natural satellites.

The small numerical difference between the simple derivation and the complete one is of even less importance than one might think, since the entire result depends on the value of Q. This number can only be calculated theoretically for the most elementary laroratory oscillators; in the case of bulk planetary material its value must be determined empirically, for example by timing free planetary oscillations or by measuring the decay of seismic waves with distance from the source (Cook 1973). Reliable measurements are available only for the Earth: a representative sample of results given by Cook (1973) shows values spread over a range from about 10 to nearly 1000. The 'typical' value used to calculate the internal structure of Io was 100, which raises a certain number of questions:

The value could be wrong either way by an order of magnitude.

It might actually be on the low side, since 100 is the lower limit of results obtained from free terrestrial oscillations which presumably are more indicative of global planetary conditions than are other types of determination. Measurements on the moon made by the Apollo seismometers are rather hard to interpret: in the crust Q is about 5000 and drops to about 1500 in the lower mantle—the high values relative to the Earth are attributed to the absence of water. There is no evidence which requires the solid body Q to be low. The internal structure of the Moon is not known: studies of free librations (Yoder 1981) suggest (but do not impose) the presence of a small (radius <400 km) fluid core; seismic results are ambiguous, relate essentially to one backside impact event, and their interpretation has been contested.

The density of Io is slightly less than the Moon's, and one might be inclined to assimilate its constitution to that of the Moon. However, Io is an outer solar system body, whose initial composition may therefore have been enriched in volatiles. How relevant are terrestrial and lunar measurements to Io?

The small error in the simple derivation pales in significance when compared to the known uncertainty in Q!

It is also important to note that the shear modulus of arbitrary planetary material is actually a rather poorly known quantity. $6.5 \times 10^{10}\ \mathrm{Nm^{-2}}$ is often used in calculations of lunar sized bodies, the use of two significant figures giving an impression of great accuracy. Actually, reliable measurements are available only for the Earth's upper crust (Cook 1980); these range from 7.2×10^{10} to 6.4×10^{10} over the depth range 40–170 km, while for the moon lower limits for the depth range 60–800 km are 7.6×10^{10} to 5.3×10^{10} respectively; no values are available for a 'global' shear modulus. Two significant figures may be appropriate for the individual localised measurements, but one has no right to take an average value when considering global properties: two significant figures and possibly even one are illusory when applied to a satellite of unknown composition under clearly exotic conditions.

The heat generated by the tidal forcing must ultimately appear at the surface, where it will be radiated; calculations suggest that the orbital eccentricity of Io is about 0.004, so that substituting the 'canonical values' of $\mu \cong 10^{11}$ and $Q \cong 100$ we obtain a total heat flow at the satellite surface of the order of 10^{12} W.

It is interesting to compare this value with the energy E absorbed by Io from the Sun. This is given by:

$$E \cong R_s^2 \left(\frac{R_\odot}{d}\right)^2 T_\odot^4 \pi\sigma(1-\gamma)$$

where: σ = Stefan's constant; γ = fraction of the incident energy which is reflected. The value γ for Io in the visible part of the spectrum is about 0.8.

$R_\odot$ and $T_\odot$ are the solar surface radius and temperature respectively, d = distance of Jupiter from the Sun, which leads to:

$$E \cong 10^{14}\ \mathrm{W}$$

This value is about 100 times the power generated by the tidal forcing for the simplest model; consequently, if no other phenomena intervene, the surface temperature of Io would be very close to its simple 'solar illuminated' value of about 120 K.

The heat generated within Io will necessarily set up a temperature gradient from the inside to the outside which will drive the heat flow. Within the limits of the cubic satellite model, the heat will flow in one direction; if we consider that the heat is generated uniformly within the satellite at a rate H per unit volume ($\cong (1/L^3)(\mathrm{d}E/\mathrm{d}t)$ in our model), elementary unidimensional heat conduction theory shows that when thermal equilibrium has been reached:

$$H = 2K\,\Delta T/(L/2)^2$$

where: $L/2$ is the distance from the centre to the edge, K is the thermal conductivity.

The thermal conductivity of planetary material is not well known; the values depend on at least depth and temperature. In the case of the Earth, the quoted values increase from 1 to about $4\ \mathrm{W\,m^{-1}\,K^{-1}}$, up to a depth of some 2000 km. There is really no reason why any of these values should apply to Io; however, taking (with Peale *et al*) $K = 4 \times 10^5$ one finds:

$$\Delta T \cong 10^4.$$

Since the surface temperature is very low, this is effectively the central temperature, and we note that it exceeds the melting point of terrestrial surface rocks. This is the basis for believing that the internal state of Io could be partly or wholly molten.

One should emphasise that this result is subject to considerable uncertainty. 10^4 K is surely an upper limit to the internal temperature of Io since:

$Q = 100$ may well be a lower limit; higher values of Q reduce the dissipation and in particular if Q were as high as 1000 the final temperature would be below the melting point of terrestrial surface rocks.

The rate of transfer of heat towards the surface may be somewhat more efficient than is suggested by the coefficient of thermal conductivity. 'Solid state' convection has been proposed as a possible mechanism, but it is poorly understood theoretically and not at all studied empirically; using order of magnitude estimates of the relevant parameters for the case of the moon (Schubert *et al* 1977), one finds that the effective thermal conductivity could be raised by as much as about 10 or as little as about two—the applicability of these numbers to the case of Io is in any case questionable. If solid state convection is as efficient as it could be no melting will occur unless $1/Q$ is remarkably high.

Even if one accepts an order of magnitude of a few thousand degrees as a 'best buy' estimate of the central temperature, it is by no means clear that melting must occur: the phase changes occurring inside planetary bodies are not well understood and do not depend only on temperature. It is instructive to recall that up to a depth of 3000 km the Earth is not globally molten although the temperature probably reaches a value close to 4000 K. Of course, this may or may not be relevant, since for an Io sized body, the central pressure is that which occurs at a depth of only about 200 km in the Earth; moreover, melting points vary widely from mineral to mineral, so that for example an iron–sulphur mixture melts at several hundred degrees less than pure iron.

Let us assume, with Peale *et al* (1979), that melting is initiated in a core region of Io. One can see immediately that this will weaken the global rigidity of the satellite, whose forced oscillations will therefore increase in amplitude so that more energy will be pumped in. We can gauge the importance of the effect using our cubic satellite model once again. Suppose that a central cubic volume of side $l < L/2$ is in a fluid state. The 'stretched' solid material then consists of a segment, of length l, whose cross section is $L^2 - l^2$, and an effective segment of length $L/2 - l$ whose cross section is still L^2. Consequently, the total extension under a stretching force ΔF is given by:

$$\text{extension} \cong \frac{l\,\Delta F}{Y(L^2 - l^2)} + \frac{L/2 - l}{YL^2}\Delta F$$

$$= \frac{\Delta F}{2LY}\left[\frac{2l/L}{1-l^2/L^2}+(1-2l/L)\right]$$

so that the ratio of the energy dissipated in this case to that dissipated in a completely solid satellite is given by:

$$\frac{2l/L}{1-l^2/L^2}+(1-2l/L).$$

Similarly, when the core dimension exceeds $L/2$, this ratio simplifies to:

$$\frac{2l/L}{1-l^2/L^2}.$$

For small core sizes, the gain in energy is rather small; for example, for $l/L=0.5$ the above ratio is only 1.3 (this compares with the value of about 3 quoted by Schubert *et al* (1977) for a proper two-layer model). The ratio can of course become important when the core is large: for $l/L=0.95$, our simple estimate gives an enhancement of about 20 (the full calculation gives 15).

One might plausibly suppose that this energy is dissipated entirely in the remaining solid mantle of the satellite. If this is true, the energy per unit volume in the mantle increases as:

$$\frac{1}{1-l^3/L^3}\left[\frac{2l/L}{1-l^2/L^2}+(1-2l/L)\right]$$

for $l<L/2$, and

$$\frac{2l/L}{(1-l^3/L^3)(1-l^2/L^2)}$$

for $l \geqslant L/2$.

As the mantle thickness decreases, energy is transferred through it more efficiently: the two extreme cases are conduction and solid state convection.

In the case of conduction, the heat transfer rate varies as the inverse square of the mantle thickness since the process is essentially a random scattering. Therefore, the ratio of the excess heat generated to the heat transfer rate is given by:

$$\frac{(1-l/L)^2}{(1-l^3/L^3)}\frac{2l/L}{(1-l^2/L^2)}$$

for $l \geqslant L/2$. This ratio is less than one for all values of l/L, whence it follows that heat conduction can successfully cope with any enhancement in dissipated energy which is produced by a molten core. Therefore, even if for some reason melting did occur in the satellite centre, conduction would prevent any spreading of the effect, in spite of a thereby weakened structure.

Convection, however, introduces a new aspect. Convective heat transfer is essentially a nonrandom, directed motion of matter so that the rate at which energy is transported must vary as something like l/(time to cross the mantle), which reduces to l/(mantle thickness). The velocity of the transfer, as well as the various constants, depends on the detailed (badly known) theory of solid state convection, but we do not need these numbers here: the ratio of the excess heat generated to the heat transfer rate by convection is given by:

$$\frac{(1-l/L)}{(1-l^3/L^3)}\frac{2l/L}{(1-l^2/L^2)}$$

for $l \geqslant L > 2$, assuming (with Peale *et al* 1979) that the convection constants do not change throughout the satellite.

This relation is roughly constant, rising very slowly up to values of $l/L=0.9$, where the rise becomes more important. This contrasts sharply with the behaviour for the case of conduction: convection is not really able to cope with the excess energy generated in a weakened satellite. The exact calculation of Peale *et al* (1979), based on a particular model of solid state convection, shows that in fact it copes even less well than our simple analysis would suggest.

Therefore, if a central region of the satellite does reach melting point, and if energy is transferred through the satellite by convection, melting will continue outwards from the core—this is what Peale *et al* have called the 'runaway melting process'.

Can the central parts of Io have reached melting point through the tidal forcing process in a reasonable interval of time?

A lower limit on the rate of increase of temperature, dT/dt, can be obtained by ignoring all energy transfer mechanisms; in this case:

$$C_p\frac{dT}{dt}=\frac{1}{m}\frac{dE}{dt}$$

where C_p is the specific heat of planetary material. Using a somewhat *ad hoc* lunar value for $C_p -$ $10^3\,J\,kg\,K^{-1}$ (Schubert *et al* 1977)—one obtains $dT/dt \simeq 10^{-14}\,K\,s^{-1}$. Consequently, to raise the temperature of a cold satellite to 1000 K by this mechanism alone will take rather more than 3×10^9 years. We note that the time estimate will not be much reduced by the run away process, which becomes significant only when the molten core is an important fraction of the satellite. On the other hand, the time estimate could be increased considerably if solid state convection does indeed act within Io, since then the heat deposition rate in the centre will be much lower. The run away melting only works if there is solid state convection, in which case however the time at which it might start is pushed back: in fact, the 'run away' is more a kind of 'crawl away'.

3×10^9 years is uncomfortably close to the age of the solar system, since it leaves no lee-way for the probable errors in some of the constants such as Q. It would seem necessary to invoke an already quite warm Io (heated by mechanisms not specified and over an unknown time interval) for tidal forcing to be able to create the observed volcanic activity.

But in that case, do we need the tidal forcing? This is the important point made by Gold (1979), irrespective of whether his alternative proposal is right or not. The extraordinary success of the prediction of volcanic activity on Io has unfortunately led to a loss of perspective: one has tended to overlook the fact that only a rather particular choice of thermal and mechanical planetary constants allow tidal forcing to play a direct role.

In conclusion, I have shown how qualitative physics can be used to attack a seemingly complex planetary problem. The mathematical and numerical results are substantially the same as those found through detailed analysis; this of course no more nullifies the latter's importance than Weisskopf's simple analyses (1979) eliminate the need for full computations of stellar structure, but I submit that one's understanding is enhanced.

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