# MAGNETIC ENERGY AND HELICITY FLUXES AT THE PHOTOSPHERIC LEVEL

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**Abstract.** The source of coronal magnetic energy and helicity lies below the surface of the sun, probably in the convective zone dynamo. Measurements of magnetic and velocity fields can capture the fluxes of both magnetic energy and helicity crossing the photosphere. We point out the ambiguities which can occur when observations are used to compute these fluxes. In particular, we show that these fluxes should be computed only from the horizontal motions deduced by tracking the photospheric cut of magnetic flux tubes. These horizontal motions include the effect of both the emergence and the shearing motions what ever the magnetic configuration complexity is. We finally analyze the observational difficulties involved in deriving such fluxes, in particular the limitations of the correlation tracking methods.

**Keywords:** magnetic fields, photosphere

## 1. Introduction

It is generally thought that coronal phenomena like flares and Coronal Mass Ejections (CMEs) are a consequence of magnetic energy release. However, so far, a quantitative proof (or disproof!) of this statement has not yet been put on firm ground mainly because the magnetic field vector is observed primarily at the photospheric level. Computing the coronal magnetic energy relies either on magnetic extrapolation techniques or more directly on the use of the virial theorem, which transforms the volume integral of the coronal energy density to a surface integral at the photospheric level. Presently, the uncertainties in the measurements of the photospheric magnetic fields preclude a precise determination of the coronal magnetic energy (see e.g. Klimchuk, Canfield & Rhoads, 1992). Deviations from the force-free field approximation increase the uncertainties. When photospheric velocities are observed an alternative method becomes available. This method computes the Poynting flux, thus obtaining the time-derivative of energy rather than the energy itself (Kusano et al., 2002).

Another important MHD quantity to derive from observations is the magnetic helicity, which plays a key role in magnetohydrodynamics (MHD) because it is one of the few global quantities that are preserved in the absence or near absence of resistivity. Observations of helical magnetic structures in the photosphere, corona and solar wind have attracted considerable attention, with the consequent interest in magnetic helicity studies (see reviews in Brown, Canfield & Pevtsov, 1999). Only relatively recently, however, it has been fully realized that magnetic helicity can be derived directly from observations (e.g. Berger & Ruzmaikin, 2000). Indeed, presently several groups of researchers have computed the photospheric flux of helicity: by differential rotation (DeVore, 2000; Démoulin et al., 2002; Green et al., 2002), by local horizontal motions (Chae, 2001; Chae et al., 2001; Kusano et al., 2002; Moon et al., 2002a, 2002b; Nindos & Zhang, 2002) and by vertical motions (Kusano et al., 2002).

Both magnetic energy and helicity are brought into the corona with the magnetic flux as the field emerges. Horizontal (e.g. shearing or twisting) motions can also provide both energy and helicity to the corona. These two kinds of contributions are present in the theoretical flux equations. The direct application of these equations to observations requires the availability of the three components of the plasma velocity. However, at most, only one component is deduced from the Doppler shift of spectral lines.

On the other hand, other observations allow us to follow magnetic entities which represent the photospheric cuts of magnetic flux tubes. Combining these velocity measurements and magnetograms should help us compute the magnetic energy and helicity fluxes. What are the relevant observations to use in flux estimations? Do we have enough information to fully compute the fluxes?

In order to answer the above questions we proceed by steps. First, we recall the definitions of the magnetic energy and helicity (Section 2.1) and their evolutions, in particular, via boundary motions (Section 2.2). Then, we summarize the presently available observations (Section 3.1), link them to the theoretical equations (Section 3.2) before clarifying which kind of observations permit us to estimate the energy and helicity fluxes (Section 3.3). We comment on the limitations encountered in deriving the fluxes from the observations (Section 3.4) and we conclude (Section 4).

## 2. Magnetic energy and helicity

### 2.1. Definitions

We recall below the definitions of the magnetic energy and helicity in a volume $V$ which is simply connected and bounded by a closed surface, $S$, with a normal vector $\hat{n}$ pointing inside $V$. In the application to the solar case, $V$ is the coronal volume or a part of it. The lower part of the surface $S$, called $Sp$, is located at the photospheric level where velocity and magnetic field observations are frequently available. On the remaining part of $S$, or outer part $So$ ($S = Sp + So$), no observations are available.

The magnetic energy of any field $\vec{B}$ is:

$$E = \frac{1}{2\mu_0} \int_V B^2 dV \,, \tag{1}$$

We can also define the potential field $\vec{B}_P$ (without current: $\vec{\nabla} \times \vec{B}_P = 0$) which has the same normal field component at the boundary $Sp$:

$$\vec{B}_P.\hat{n}|_{Sp} = \vec{B}.\hat{n}|_{Sp} \,. \tag{2}$$

Usually $So$ is put at enough large distance so that $\vec{B}_P$ is not significantly influenced by the boundary conditions selected on $So$.

The magnetic helicity of a divergence-free field $\vec{B}$ within a volume $V$ was first defined by:

$$H_{fc} = \int_V \vec{A}.\vec{B} dV \,, \tag{3}$$

where the vector potential $\vec{A}$ satisfies

$$\vec{B} = \vec{\nabla} \times \vec{A} \,. \tag{4}$$

However Eq. (3) is physically meaningful only when the magnetic field is fully contained inside the volume $V$ (i.e. at any point of the surface $S$ surrounding $V$, the normal component $B_n = \vec{B}.\hat{n}$ vanishes); this is because the vector potential $\vec{A}$ is defined only up to a gauge transformation ($\vec{A'} = \vec{A} + \vec{\nabla}\Phi$) and $H_{fc}$ is gauge-invariant only when $B_n = 0$.

In the solar case, we clearly have magnetic fluxes crossing S, in particular $Sp$ (as shown by the photospheric magnetograms). Berger & Field (1984) have shown that for cases where $B_n \neq 0$ on S one can define a relative magnetic helicity, $H$, subtracting the helicity of a reference field, which is classically taken as the potential field $\vec{B}_P$ as defined above. An expression for $H$, valid for any gauge (Finn & Antonsen, 1985) is

$$H = \int_V (\vec{A} + \vec{A}_P).(\vec{B} - \vec{B}_P) dV \,. \tag{5}$$

While this expression is gauge-invariant, some gauges are easier to use than others! In many situations, especially when computing fluxes, the most convenient gauge conditions are the following (Barnes, 1988; Berger, 1988):
on the boundary $S$,

$$(\vec{\nabla} \times \vec{A}_P).\hat{n} \;=\; \vec{B}.\hat{n}\,, \tag{6}$$

$$\vec{A}_P.\hat{n} \;=\; 0\,, \tag{7}$$

and in $V$,

$$\vec{\nabla}.\vec{A}_P \;=\; 0\,, \tag{8}$$

$$\vec{\nabla} \times \vec{A}_P \;=\; \vec{B}_P\,. \tag{9}$$

These conditions define $\vec{A}_P$ uniquely. If, in addition, the vector potential $\vec{A}$ satisfies on $S$

$$\vec{A} \times \hat{n} = \vec{A}_P \times \hat{n}\,, \tag{10}$$

then Eq. (5) reduces to $H = \int_V \vec{A}.\vec{B} dV$.

## 2.2. Fluxes

The changes of the magnetic energy and helicity content within the volume $V$ are due to both fluxes across S and dissipation within $V$. In the following equations we keep all the terms, but we write explicitly only the fluxes through the photospheric boundary, $Sp$, since they are the only ones that can be computed directly from present observations.

Let $\vec{v}$ be the plasma velocity. At the photosphere $\vec{v}$ separates into tangential and normal components $\vec{v} = \vec{v}_t + \vec{v}_n$. Here the unit normal $\hat{n}$ points upwards into the corona. The magnetic energy evolution inside $V$ satisfies

$$\frac{dE}{dt} \;=\; \frac{1}{\mu_0}\int_S \vec{B} \times (\vec{v} \times \vec{B}).\hat{n}\, dS + \left.\frac{dE}{dt}\right|_{\rm dis.}\,, \tag{11}$$

$$\;=\; \left.\frac{dE}{dt}\right|_t + \left.\frac{dE}{dt}\right|_n + \left.\frac{dE}{dt}\right|_{So} + \left.\frac{dE}{dt}\right|_{\rm dis.}\,, \tag{12}$$

where we explicitly write, at the photospheric level, the tangential and normal fluxes:

$$\left.\frac{dE}{dt}\right|_t \;=\; -\frac{1}{\mu_0}\int_{Sp} (\vec{B}_t.\vec{v}_t) B_n\, dS\,, \tag{13}$$

$$\left.\frac{dE}{dt}\right|_n \;=\; +\frac{1}{\mu_0}\int_{Sp} B_t^2 v_n\, dS\,. \tag{14}$$

They corresponds respectively to energy input in $V$ from horizontal (e.g. shearing or twisting) motions and from vertical motions (e.g. emergence of new flux).

The evolution of the magnetic helicity inside $V$ is (Berger and Field, 1984):

$$\frac{dH}{dt} = 2\int_S \vec{A}_P \times (\vec{v} \times \vec{B}).\hat{n}\, dS + \left.\frac{dH}{dt}\right|_{\rm dis.}, \tag{15}$$

$$= \left.\frac{dH}{dt}\right|_t + \left.\frac{dH}{dt}\right|_n + \left.\frac{dH}{dt}\right|_{So} + \left.\frac{dH}{dt}\right|_{\rm dis.}. \tag{16}$$

Here, as in the energy case, we have separated the tangential and normal flux across $Sp$:

$$\left.\frac{dH}{dt}\right|_t = -2\int_{Sp} (\vec{A}_P.\vec{v}_t) B_n\, dS\,, \tag{17}$$

$$\left.\frac{dH}{dt}\right|_n = +2\int_{Sp} (\vec{A}_P.\vec{B}_t) v_n\, dS\,. \tag{18}$$

The expressions of the magnetic helicity fluxes on $S$ are closely linked to the energy fluxes since one can get the helicity fluxes of Eq. (15) from Eq. (11) by replacing $\vec{B}/\mu_0$ by $2\vec{A}_P$.

The fluxes on the outer boundary $So$ cannot be deduced from current observations, simply because we do not have the magnetic and velocity fields measurements in the corona. The flux of magnetic helicity has been only computed indirectly from the estimated number of CMEs leaving the volume $V$ (defined by the coronal field extension of an AR magnetic field), assuming that the average magnetic helicity of a CME equates the average magnetic helicity found in magnetic clouds (see Rust & Kumar, 1996, DeVore, 2000, Démoulin et al., 2002).

The dissipation terms in Eq. (11) and Eq. (15) are only formally equivalent. The energy dissipation, $dE/dt|_{\rm dis.}$, is thought to be fundamental in the origin of the corona itself (as the heating of the corona depends on the dissipation of magnetic energy) but also in dynamic events such as flares. At the opposite, helicity dissipation, $dH/dt|_{\rm dis.}$, is negligible in all the processes occurring in the corona, including the non-ideal ones (Berger, 1984). The dissipation time scale of magnetic helicity is the global diffusion time scale. For example, with a classical resistivity, Berger (1984) found a minimum helicity dissipation time of the order of $10^5$ years in a typical coronal loop! An upper bound on the helicity dissipation scales as $R_M^{-1/2}$, where $R_M$ is the magnetic Reynolds number, but the dissipation can be even smaller at high $R_M$ since it scales as $R_M^{-2}$ in magnetic reconnection events (Freedman & Berger, 1993).

## 3. Photospheric fluxes

### 3.1. Available observations

The most frequently available photospheric data are the longitudinal magnetograms which are maps of the magnetic field component, $B_{\parallel}$, along the line of sight. A frequently used approximation assumes that the photospheric field is vertical (due to the buoyancy of magnetic flux tubes). This permits us to compute the normal component $B_n$ from $B_{\parallel}$ (obviously the closer to disk centre observations are, the better the approximation).

Vector magnetographs have also been developed all around the world. They give the components along, $B_{\parallel}$, and orthogonal, $\vec{B}_{\perp}$, to the line of sight (at least when the magnetic flux tubes are spatially resolved). After calibration and removal of the 180° ambiguity, the data can be transformed to photospheric maps of the normal, $B_n$, and tangential, $\vec{B}_t$, components.

Longitudinal velocities, $v_{\parallel}$, are classically derived from the Doppler shift of spectral lines (e.g. Strous et al., 1996 and references therein). On the other hand, when the studied region is not very close to the centre disk, there is no safe (or clear) way to transform $v_{\parallel}$ to $v_n$ (this is different from the magnetic field case above where $B_{\parallel}$ can be transformed to $B_n$).

Consider a small magnetic element moving on the surface of the sun. In principle, a time-series of magnetograms will track this motion. Let us call the tracking velocity $\vec{u}$, to distinguish it from the plasma velocity $\vec{v}$. By definition $\vec{u}$ is parallel to the surface, i.e. $\vec{u} = \vec{u}_t$. However, only part of $\vec{u}$ corresponds to the transverse plasma velocity $\vec{v}_t$. The normal plasma velocity can also contribute (see Figure 1). If a flux tube rises through the photosphere, the point $\vec{x}$ where it crosses the photosphere moves (unless the tube is vertical).

One method for observing $\vec{u}$ identifies the local brightness (or the magnetic field) extremum and follows it with time (e.g. Strous et al., 1996), so this method can be called Track Individual Entities (TIE). A second method, more widely used, is called Local Correlation Tracking (or LCT, see Chae et al., 2001 and references therein). The velocity is locally determined by cross correlating a small fraction of two subsequent images shifted by a variable displacement. The relative displacement corresponds to the shift having the highest correlation among all of them; this provides an estimation of $\vec{u}$.

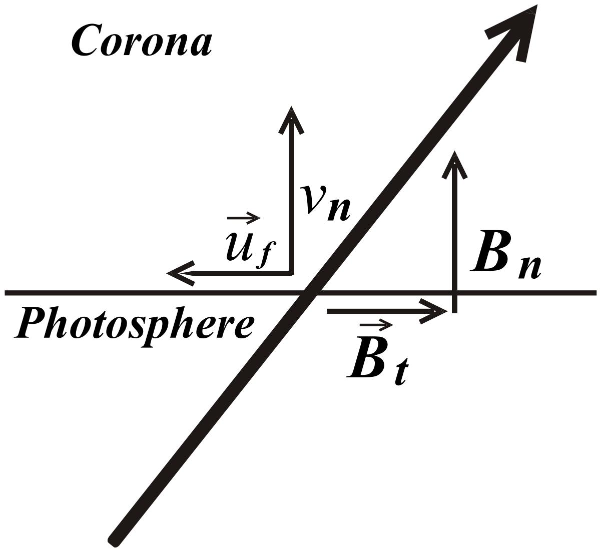


*Figure 1.* A flux tube rising through the photosphere. The point where the tube crosses the photosphere moves with velocity $\vec{u}_f$. From the figure, the ratio $|\vec{u}_f|/v_n = |B_t|/B_n$. Also, $\vec{u}_f$ points in the opposite direction to $\vec{B}_t$, thus $\vec{u}_f = -(v_n/B_n)\vec{B}_t$ .

### 3.2. Estimation of photospheric fluxes

The use of the observed velocities in the theoretical equations for the fluxes of Sect. 2.2 is the most delicate part, so we mostly concentrate on that below.

It is, at first, appealing to use the information from Doppler velocities since they are measuring plasma motions. However, only longitudinal velocities $v_\parallel$ are available, which gives no clear information about the velocity component orthogonal to $\vec{B}$, a quantity that enters explicitly in both energy and helicity fluxes (Eqs. (11) and (15)) as the $\vec{v} \times \vec{B}$ term. Only when the studied region is close to disk centre one can use $v_n \approx v_\parallel$. But this needs to be used with caution since, e.g., in emerging regions both up flows and down flows are observed, the latter being a factor [2-5] larger in magnitude (e.g. Lites, Skumanich & Martinez Pillet, 1998, see van Driel (2003) for a review of magnetic flux emergence). Indeed Doppler velocities have contributions both from the emergence of the flux tube and from the down flows in the feet of the flux tube (where $\vec{v}$ is mostly parallel to $\vec{B}$). We conclude that there is no safe way to deduce $v_n$, nor $\vec{v} \times \vec{B}$, from Doppler measurements.

The tracking velocities, $\vec{u}$, are supposed to track the motions of specific magnetic entities at a given height in the atmosphere (eventually, after correcting for the height change with disk position for a given optical depth). Then, the transformation from $\vec{u}_\perp$ (orthogonal to the line of sight) to $\vec{u} = \vec{u}_t$ (tangential to the photosphere) is only a geometrical one (indeed in practice, the photospheric observations are first transformed to the local frame by stretching the images taking into account the observed location on the Sun; $\vec{u}$ is derived directly from these stretched images). The tracking velocity $\vec{u}$ has been used as a proxy for the tangential plasma velocity $\vec{v}_t$ (Chae, 2001; Chae et al., 2001; Kusano et al., 2002; Moon et al., 2002a, 2002b; Nindos & Zhang, 2002). A priori, the observed transverse velocities allow us to compute only the two flux terms $dE/dt|_t$ and $dH/dt|_t$ (Eqs. (13) and (17)). However, it is important to note that $\vec{u}$ and $\vec{v}_t$ are not necessarily equal ! We investigate this in the next section.

### 3.3. Do we need to add the injection by vertical motions ?

In the previous section we have, a priori, deduced only part of the energy and helicity fluxes from the observations which track magnetic entities. The remaining terms, $dE/dt|_n$ and $dH/dt|_n$ (Eqs. (14) and (18)) require the normal component of the photospheric plasma velocity ($v_n$).

Kusano et al. (2002) have proposed to deduce $v_n$ using observations of $\vec{B}$ and $\vec{u}$ on $Sp$ together with the induction equation. The cross-calibration of observations taken in different wavelengths is still a research domain, so presently vertical gradients of $\vec{B}$ and $\vec{u}$ are not available. This restricts the use of the induction equation to its normal (vertical) component (which involves only horizontal derivatives):

$$\frac{\partial B_n}{\partial t} = \vec{\nabla} \times (\vec{v}_n \times \vec{B}_t + \vec{v}_t \times \vec{B}_n)\,, \tag{19}$$

$$= \vec{\nabla}_t (v_n \vec{B}_t - B_n \vec{v}_t)\,, \tag{20}$$

where $\vec{\nabla}_t()$ is the divergence including only the tangential (horizontal) derivatives. The left term, $\partial B_n/\partial t$, can be estimated from successive co-align magnetograms, while $\vec{B}$ comes from vector magnetograms (Section 3.1) and $\vec{u}$ is used as a proxy for $\vec{v}_t$ (Section 3.2). Then, Kusano et al. (2002) have proposed a method to derive the only unknown function of Eq. (20): $v_n$. This approach gives both $\vec{v}_t$ and $v_n$, so the three components of the velocity vector are deduced from the time evolution of only one component of the magnetic field ($B_n$) and the horizontal gradients of the other components ($\vec{B}_t$) !

At this point it is worth noting that any emergence of a flux tube contributes also to the observed velocity $\vec{u}$ because as the flux tube rises its photospheric foot-points separate. We can then ask whether the emergence contributes to both flux terms (tangential and normal) in a way that these contributions can be separated. Or, on the other hand, do observations mix these contributions so that considering both terms is redundant ? Let us analyze a simple example to start answering this question: a magnetic configuration that is ideally transported by plasma motions across the boundary $Sp$ with only a normal velocity component $v_n$ (the plasma velocity $\vec{v}_t$ is supposed to be null). By a simple geometric argument, we see that the photospheric foot-point of any field line is moving on $Sp$ with the tangential velocity $\vec{u}_f$ (see Figure 1):

$$\vec{u}_f = -\frac{v_n}{B_n}\vec{B}_t \, . \tag{21}$$

Let's further suppose that, as in observed cases, the magnetic field is clustered in several flux tubes on $Sp$. Then, either tracking method (TIE or LCT, see Section 3.1) would detect a tangential velocity $\vec{u} = \vec{u}_f$ (provided that all flux tubes are well separated and resolved). That is to say, we have two complementary views to analyze the photospheric flux of magnetic energy and helicity. From a "theoretical" point of view only the vertical fluxes $dE/dt|_n$ and $dH/dt|_n$ (Eqs. (14) and (18)) are present since we set the evolution with no tangential plasma velocity $\vec{v}_t$. The fluxes through $Sp$ are then:

$$dE/dt|_{Sp} = dE/dt|_n \quad \text{and} \quad dH/dt|_{Sp} = dH/dt|_n \, . \tag{22}$$

However, from an "observational" point of view, the tracking methods find the tangential motion $\vec{u} = \vec{u}_f$ of the flux tubes. Then Eq. (21) implies that (using $\vec{u}$ as a proxy of $\vec{v}_t$):

$$dE/dt|_t = dE/dt|_n \quad \text{and} \quad dH/dt|_t = dH/dt|_n \, . \tag{23}$$

We conclude that, for both energy and helicity, all the flux is present in the tangential fluxes determined by tracking methods ! This example clearly shows that we **should not** add the normal fluxes.

Indeed, in general, plasma motions have both components $\vec{v}_t$ and $v_n$. The first transports the magnetic flux tubes tangentially to $Sp$, while the second adds the tangential velocity $\vec{u}_f$ (Eq. 21). Tracking methods are designed to follow the photospheric foot-points of flux tubes so they measure a velocity, $\vec{u}$ which is the sum of both velocities:

$$\vec{u} = \vec{v}_t - \frac{v_n}{B_n}\vec{B}_t \, . \tag{24}$$

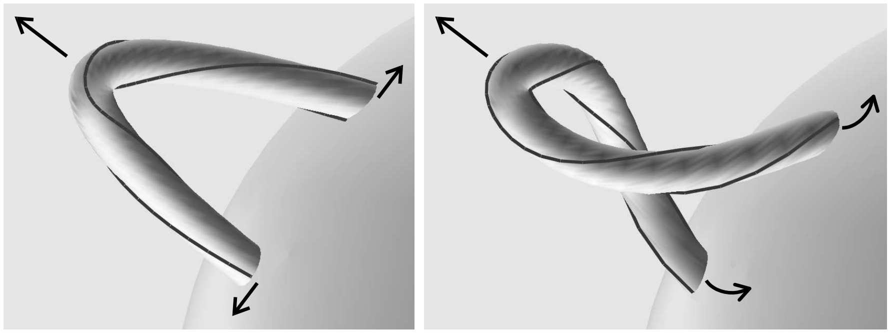

*Figure 2.* Two examples of rising loop structures. Field lines are drawn to show the internal twist. The arrows at the surface represent the global motions of the loop footpoints (there are also footpoint motions associated to the twist). On the left side the loop has internal twist but straight legs (planar $\Omega$-loop), while on the right side the loop has writhe as well as twist.

In general, tracking methods provide the transverse velocities which are the only ones needed to compute the full energy and helicity flux across the photosphere:

$$\left.\frac{dE}{dt}\right|_{Sp} = \left.\frac{dE}{dt}\right|_{t} + \left.\frac{dE}{dt}\right|_{n} = -\frac{1}{\mu_0}\int_{Sp}(\vec{B}_t.\vec{u})B_n dS\,, \qquad (25)$$

$$\left.\frac{dH}{dt}\right|_{Sp} = \left.\frac{dH}{dt}\right|_{t} + \left.\frac{dH}{dt}\right|_{n} = -2\int_{Sp}(\vec{A}_P.\vec{u})B_n dS\,. \qquad (26)$$

The use of $v_n$ (as deduced from Doppler measurements or as proposed by Kusano et al., 2002) would only duplicate part of the fluxes already included in the tracking velocity $\vec{u}$. This result is independent of the complexity of the magnetic configuration analyzed.

We conclude this section with two illustrative examples in which $\Omega$-loops are formed by a twisted flux tube with a finite cross-section. The first one has its main axis located in a plane (as a classical planar $\Omega$-loop, see Figure 2, left side), while the main axis of the second has a helix-like shape (Figure 2, right side) so that it has both twist and writhe helicities (see López Fuentes et al., 2003 for observed examples of such flux tubes forming active regions). As the top part of the planar $\Omega$-loop crosses the photosphere, the upward motion induces horizontal photospheric foot-point motions as given by Eq. (24), as well as energy and helicity flux as given by Eqs. (25)-(26). For illustration, let us simply suppose that the legs are vertical. If the flux tube has no twist, all the field lines are vertical in the legs and no foot-point motions are present ($\vec{u} = 0$), so Eq. (26) implies $dH/dt|_{Sp} = 0$ (the same result is given by Eqs. (17)-(18) since $\vec{v}_t = 0$ and $\vec{B}_t = 0$), a logical result since the flux tube has no magnetic helicity. If the flux tube has a finite twist, the upward motion implies a rotation for the photospheric footpoints of the field-lines (or elementary flux tubes) and one can directly verify that the injection of helicity is given by Eq. 26). However, the observations of such vortex motions may be not obvious, in particular if the observed magnetic flux distribution is not split into several flux tubes that tracking method can follow (e.g. in a sunspot, see next Section for a further discussions of the observational limits). In the case of a non-planar loop (Figure 2, right side), even if the tracking method is not able to follow the internal motions (thus losing the associate twist helicity), the tracking of both photospheric polarities gives the writhe-helicity flux.

### 3.4. Uncertainties on the derived fluxes

While we show above that tracking methods are able, in principle, to provide the full energy and helicity flux across the photosphere, they certainly have limitations, which we analyze further. Some of these limitations are linked to:

- the spatial resolution,
- the regions with absence of significant contrast,
- undetectable motions,
- filtering of velocities.

The spatial resolution is certainly the main concern since the tracking methods are explicitly supposed to track individual magnetic flux tubes, which can be as thin as 100 km outside sunspots, while the actual pixel size of magnetograms is typically 10 times larger (e.g. about the resolution of MDI/SoHO in its "high-resolution" mode). In such an extreme case, where an ensemble of unresolved flux tubes are observed as a nearly continuous pattern, one may wonder about the significance of the deduced velocities (they will include, at most, only the global motion of the flux tubes).

The second point may also have a considerable impact in the magnitude of the computed fluxes since it concerns the sunspots, where a large fraction of the magnetic flux of active regions is present (at least during their early period). This can be amplified by the non-linear response ("saturation") of the magnetograph at high field values: for example MDI gives about the same response when the measured strength is larger than 1300 G (Berger & Lites, 2002). Then tracking methods are expected to face great difficulties in regions where the field distribution has only low spatial variation since the observations do not provide enough informations to follow individual flux tubes.

The third point, while linked to the above ones, has also its own specific relevance (e.g. the kind of motions really present). Rewriting Eq. (20) using Eqs. (24), we get:

$$\frac{\partial B_n}{\partial t} = -\vec{\nabla}_t (B_n \vec{u}) \,. \tag{27}$$

This shows that a part of the photospheric transverse velocity will have no effect on the $B_n$ distribution, so it will not be detectable. This corresponds to the curl-free contribution to $B_n \vec{u}$; i.e., the part that can be written as $\vec{\nabla} U \times \hat{n}$, where $U$ is an arbitrary function. This includes, for example, motions along the isocontours of $B_n$. In practice, possible $U$ functions should give maximum velocities not larger than $\approx$ few km.s$^{-1}$ at the photospheric level (velocities larger than the maximum ones ever observed are very suspicious). This still gives a large freedom for undetectable transverse velocities, in particular it includes rotational motions in observed magnetic entities and such motions are very efficient to transfert both magnetic energy and helicity ! The possible implication on the fluxes is expected to decrease as the magnetic field is better resolved but this clearly needs a detailed analysis.

The last point is linked to the temporal resolution (so the ability to follow fast motions), but also more specifically to the LCT method as follows. The LCT correlates portions of the magnetograms, so it determines only an average velocity for the defined portions. This has an effect that is similar to the finite resolution of observations (first above point), but is rather intrinsic to the method and can have stronger implications (because these portions are typically larger than the spatial resolution by a factor $\approx 5$).

On top of the velocity uncertainties, there are uncertainties coming from the magnetic field measurements (see e.g. Wang et al., 1992, Bao et al., 2000). The estimated random errors are typically a factor 10 lower on the longitudinal field component, $B_\parallel$, than on the transverse one, $\vec{B}_\perp$. Important calibration problems can be present not only on $\vec{B}_\perp$, but also on $B_\parallel$ as recently detected on MDI/SoHO magnetograms by Berger and Lites (2002). They find that the MDI longitudinal field is underestimated by a factor that is in the range $[0.64, 0.69]$. This implies a correction factor slightly above 2 to the helicity fluxes computed by Chae (2001), Chae et al. (2001), Démoulin et al. (2002), Green et al. (2002), Kusano et al. (2002), Moon et al. (2002a,2002b), Nindos & Zhang (2002). This correction factor can be as high as 4 if most of the flux is in strong field regions (where $B_\parallel \geq 1300$ G). Finally, the spatial resolution of magnetograms is also important for deriving good magnetic field measurements. However, this is less important in longitudinal magnetograms (which measure approximately the magnetic flux per pixel) than in transverse magnetograms (as one needs to resolve flux tubes to measure significant values for $B_t$).

Finally it is worth recalling that from observations we can only deduce physical quantities which are weighted averaged values across a photospheric layer of finite vertical extension while all quantities have strong vertical gradient. For example, the magnetic field strength $B$ is decreasing approximately with the height $z$ as $\exp -z/2H$ where $H$ is the gravitational scale height of the plasma (see e.g. Solanki et al., 1999). So the thickness of the observed layer should be much smaller than $H$ to be able to compute fluxes through a surface. Moreover, variation of the plasma opacity can also make the observed region depart from a local plane. Such limitations should be further analyzed when the formulae of this paper are used with observations. Nevertheless, since a large fraction of the photospheric field is nearly vertically oriented, the limitations, discussed at the beginning of this section, are more serious ones.

## 4. Conclusion

We have revisited the theoretical equations for the energy and helicity fluxes with the aim to clarify which kind of observations are needed to estimate them. With present available observations, the fluxes can be computed only at the photospheric level (with a possible extension to the chromospheric level in a near future, e.g. see Choudhary et al., 2001). The estimation of both fluxes require magnetic and velocity field measurements. While the way magnetic field observations should be used was clear, it was not the case for photospheric velocities. We show that tracking methods, which track the photospheric displacement of magnetic flux tubes, are the most suited to estimate the fluxes since they provide the total fluxes (within their accuracy that still remains to be fully quantified). More precisely they do include the fluxes associated to horizontal motions, as well as the fluxes associated to magnetic flux emergence and to vertical plasma motions.

Here it is worth noting that the photospheric helicity flux (Eq. 26) requires only the normal field component $B_n$ ($\vec{A}_P$ is computed from $B_n$ on $Sp$, see end of Section 2.1), while the energy flux (Eq. 25) requires the vector field $\vec{B}$ on $Sp$. Then, the helicity flux is not only easier to estimate from observations, but also more precise, than the energy flux (there is typically a larger noise on the transverse magnetic field component than on the longitudinal one).

Moreover, while magnetic energy is transformed in the corona to various different forms of energy, there is a negligible dissipation of magnetic helicity. This implies that an estimation of the budget of magnetic helicity (see Eq. 15) can be determined in a much less complex way, and therefore in a more accurate way, than the equivalent budget of magnetic energy (Eq. 11). Moreover techniques are presently available (and/or under development) to estimate both the coronal helicity and its outward flux (via CMEs). With present estimations, there is a large deficit (around a factor 10) of photospheric helicity flux (Démoulin et al., 2002; Green et al., 2002; Nindos & Zhang, 2002). The logical way to continue in this line leads to clearing up these discrepancies in the helicity budget (which can presently come from any one of the included terms). When this is achieved to a reasonable level of precision, we can be confident that the tracking methods, which measure transverse photospheric velocities, are reliable enough. Then, we will be in position to advance towards the determination of the energy flux across the photosphere, then next, the evaluation of the energy budget.

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