 Constraints on Saturn’s G Ring from the Voyager 2 Radio Astronomy Instrument

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We have reanalyzed the data acquired by the planetary radio astronomy (PRA) experiment during the passage of Voyager 2 through the outer part of Saturn’s G ring, originally published by Aubier et al. (1983. Geophys. Res. Lett. 10, 5–8). This study closely parallels the reanalysis of the Voyager 1 PRA data during the E-ring passage (Meyer-Vernet et al. 1996. Icarus 123, 113–128). The instrument detected dust grain impacts on the spacecraft in a region of ≈1000 km vertical extent around the ring plane with a maximum at ring plane crossing. The signal is mainly produced by grains of radius of a few micrometers. We find a size distribution less steep than the $r^{-6}$ law inferred for submicrometer grains by Showalter and Cuzzi (1993. Icarus 103, 124–143) from photometric data. These results can be reconciled if the slope of the size distribution flattens above 0.5 $\mu$m. Assuming a rough continuity between the distributions deduced from the two data sets and an $r^{-q}$ law for the grains detected by PRA, we infer that the differential power law index $q < 3.5$ for grain radii between about a half micrometer and a few micrometers. From the observed vertical profile, we deduce an effective ring vertical thickness $H \approx 1200/ (q - 1)$ km. When $q$ varies in the range 3.5–2, $H$ varies in the range 500–1200 km and the geometric cross section per unit area is a few times $10^{-6}$. © 1998 Academic Press

Key Words: Saturn rings; planetary radio astronomy.

1. INTRODUCTION

In this paper, we reexamine the data of the Voyager 2 radio astronomy instrument acquired during the G-ring traversal, first published by Aubier et al. (1983), in the light of more recent and accurate calibrations and ephemeris, in order to deduce revised constraints on the G-ring structure and composition. This study closely parallels a recent reanalysis of the corresponding Voyager 1 data during the E-ring crossing (Meyer-Vernet et al. 1996).

Saturn’s G ring is believed to contain a relatively large population of dust particles, as most other ethereal planetary rings (Burns et al. 1984). Interest in this ring increased recently in the context of determining a safe trajectory for the Cassini orbiter, which is programmed to traverse this region.

The faintness of the G ring makes it difficult to observe. Apart from an absorption signature on Pioneer 11 detectors (Van Allen 1983), most knowledge on its structure comes from two kinds of measurements, (1) optical observations made on Voyager 1 and 2 (Smith et al. 1981, Smith et al. 1982, Showalter and Cuzzi (1993)) and also with ground-based telescopes and with the Hubble telescope during the Earth’s recent passage through Saturn’s ring plane (Nicholson et al. 1996, de Pater et al. 1996, Throop and Esposito 1998 and references therein), and (2) in situ detection of dust impacts by the Voyager 2 radio astronomy (Aubier et al. 1983) and plasma wave (Gurnett et al. 1983, Tsintikidis et al. 1994) instruments.

The Voyager radio astronomy (PRA) and plasma wave (PWS) instruments are not conventional dust detectors, but they can record dust impacts on the spacecraft body and its appendages. This is because each impacting grain is vaporized and ionized, producing charges a fraction of which is recollected by the target and is thus detected on the...
electric antennae. The interpretation of the recorded signal is difficult, because these instruments were designed to record radio and plasma waves, so that they were not calibrated for dust measurement. The PRA data are inherently easier to interpret than PWS data because the antennae are operated as monopoles, thus responding in a straightforward manner to pulses of the spacecraft potential.

Section 2 describes the observations. Section 3 summarizes the data analysis technique and discusses some physical points which were not considered in the E ring because the grain sizes detected were smaller and had narrow distributions. In Section 4 we infer some constraints on the ring structure and composition. In Section 5 we compare our results with those derived from spectral and photometric data, which further constrains the ring properties. In Section 6 we comment on the PWS results. Finally, Section 7 presents final comments and a summary.

Unless otherwise stated, we use the international system of units. All times are spacecraft event UT time.

2. VOYAGER 2 PLANETARY RADIO ASTRONOMY INSTRUMENT OBSERVATIONS

2.1. The Instrument and Its Calibrations

The planetary radio astronomy (hereafter, PRA) instrument is described in detail by Warwick et al. (1977). It consists of a pair of orthogonal monopoles, loaded against the conductive structure of the spacecraft and connected to a very sensitive broadband receiver. The monopoles are cylinders of length \( L = 10 \text{ m} \) and radius \( a = 0.63 \text{ cm} \). The receiver is swept through the full frequency range (1.2 kHz–40.5 MHz) every 6 s, dwelling at each of the 198 frequency channels for \( \Delta t = 25 \text{ ms} \). This study uses the channels below 1 MHz of the low-frequency band, whose spacing is 19.2 kHz and bandwidth 1 kHz.

The most recent calibration of the instrument, obtained by two independent methods, is described by Meyer-Vernet et al. (1996). The voltage power spectral density on one monopole, \( V^2 \) is given by

\[
4V^2 = 10^{-17} \times 10^{\nu/10} = V_0^2,
\]

where \( x \) is the telemetered and calibrated signal (in dB) measured at the input of the PRA receiver. We used this form because for the measurements for which the receiver was designed, the receiver gain is equal to 4, owing to the antenna base capacitance. In the present study, however, the receiver is used to measure voltages on the spacecraft whose capacitance is very large, so that the gain of the receiver is roughly equal to 1 instead of 4 (Meyer-Vernet et al. 1996). This calibration holds for signals much above the receiver noise, which is the case in the present study; it slightly improves the calibration used by Aubier et al. (1983).

2.2. Observations

The upper panel of Fig. 1 is a PRA spectrogram showing the observed voltage spectral density \( V_0^2 \) displayed as frequency versus time, with relative intensity indicated by increasing darkness, during the passage of Voyager 2 through the G ring on August 26, 1981.

The bottom panel of Fig. 1 shows the corresponding spacecraft trajectory projected in a meridian plane of Saturn. Near the equatorial plane crossing, the spacecraft velocity in cylindrical coordinates centered at Saturn is

\[
v_p = 6.4 \text{ km/s}, \quad v_z = -11.1 \text{ km/s}, \quad v_\phi = 19.8 \text{ km/s}.
\]

The voltage has a peak at \( \approx 4:18:17 \) (\( \pm 3 \text{ s} \)) undistinguishable from the equatorial plane crossing, which takes place at \( \approx 2.85 R_S \) from Saturn (with Saturn’s radius, \( R_S = 60330 \text{ km} \)).

In this region, the plasma (\( \text{O}^+ \)) density measured was \( n \approx 100 \text{ cm}^{-3} \) (Lazarus and McNutt 1983) with a cold electron temperature \( T \lesssim 1 \text{ eV} \) (Sittler et al. 1983) and a strongly depleted suprathermal electron population. With these parameters, the plasma quasi-thermal noise produced by plasma particle impacts and Langmuir waves (Meyer-Vernet and Perche 1989) or Bernstein waves (Meyer-Vernet et al. 1993) is smaller than the observed level by several orders of magnitude.

A key to the origin of the signal is the voltage power spectrum. Figure 2 shows this spectrum at the peak of the signal and also 12 s before and after, corresponding, respectively, to a vertical distance of 130 km above and below the equatorial plane. The spectral shapes are very close to \( V_0^2 \propto f^{-4} \), which is just the spectral index expected above a few \( 10^4 \text{ Hz} \) for dust grain impacts (Aubier et al. 1983, Meyer-Vernet 1985). Similar spectral indexes were also observed in the ring planes of Uranus (Meyer-Vernet et al. 1986) and Neptune (Pedersen et al. 1991) and interpreted as dust impacts.

3. THEORETICAL SPECTRUM PRODUCED BY DUST IMPACTS

3.1. Grain Impact Ionization

When a dust grain impacts a solid target at a velocity higher than a few km/s, it undergoes a strong shock compression which vaporizes and ionizes it, producing an expanding plasma cloud whose residual ionization can serve to detect the impinging grain. Basically, in traditional impact ionization detectors (see Fechtig et al. 1978), one measures the charge \( Q \) carried by the residual electrons (or ions) by separating them and recollecting one species; the grain mass \( m \) is then deduced from laboratory calibrations of the relation \( Q(m) \), which varies strongly with the impact velocity \( v_G \) and depends also on the materials involved.
The velocity of dust grains in circular Keplerian orbits at $R \approx 2.85 \, R_s$ from Saturn is $(M_s G/R)^{1/2} = 14.8$ km/s. (Here, $M_s$ is Saturn’s mass and $G$ is the gravitational constant.) Assuming equatorial (prograde) circular orbits and using the spacecraft velocity given in Section 2, we deduce the grain velocity in the spacecraft frame

$$v_G = 13.7 \text{ km/s}.$$
and the above mass range for which the calibrations hold corresponds to the size range $0.06 < r_e < 3$. For grains much larger, $Q$ may vary with $m$ with an exponent smaller than one (Fechtig et al. 1978).

### 3.2. Voltage Produced by One Grain Impact

Since the PRA instrument operates the antennae as monopoles, it records directly the pulses of spacecraft potential induced when the spacecraft conductive structure collects the charge produced by a grain impact. In order for the spacecraft to collect the whole charge $Q_e$ at least two conditions must be met: (i) the electrons and ions in the cloud must be decoupled, and (ii) the charge collected must not be cancelled by the charges coming from the ambient plasma (see Meyer-Vernet et al. 1986, Oberc 1994).

We approximate the plasma cloud produced by one impact as a homogeneous sphere of radius $R_C$ expanding with velocity $v_{ex}$ (somewhat smaller than $v_G$), which is admittedly a very rough approximation. After time $t$, the cloud has a radius $R_C = v_{ex}t$ and a plasma density

$$n_C \sim \frac{Q/e}{4\pi R_C^3/3} \propto t^{-3}. \quad (3)$$

The electrons and ions become decoupled when the cloud’s radius $R_C$ becomes smaller than its proper Debye length

$$L_{DC} = (e_0 KT_C/n_e e^2)^{1/2} \propto t^{3/2}, \quad (4)$$

where $T_C \approx 1$ eV is the temperature of the coldest species in the cloud (Hornung and Drapatz 1981), which determines its Debye length.

At this time, since $T_C \approx T$, the ambient electron temperature, condition (ii) above requires that the plasma density $n_C$ in the cloud must still be larger than the ambient value $n$. From (3) and (4), the inequalities $R_C < L_{DC}$ and $n_C > n$ translate into

$$\frac{3Qe}{4\pi e_0 KT_C} < R_C < \left(\frac{3Q}{4\pi ne}\right)^{1/3}, \quad (5)$$

which requires $Q < 4\pi (e_0 KT_C)^{3/2} (3n^{1/2}e^2)$. Substituting expression (2) of $Q$ with the relevant parameters, this yields the condition

$$r_e < 2. \quad (6)$$

It is important to note that, due to the crudeness of the model, this estimate is an order-of-magnitude one. Hence, we will assume that the spacecraft collects the whole charge $Q$ given in (2) if the impinging grain has a radius smaller than 2 micrometers having a specific density comparable to that of ice (1 g/cm$^3$), this yields

$$Q/m = 0.7 \text{ Cb/g}$$

for metallic targets. (This value is equal to the value used by Meyer-Vernet et al. (1996) scaled to the proper velocity and is close to the value used by Aubier et al. (1983)).

For grains of radius $r_e$ micrometers having a specific density comparable to that of ice (1 g/cm$^3$), this yields

$$Q \approx 3 \times 10^{-12} \times r_e^3 \text{ Cb} \quad (2)$$

and

$$S = 1 \text{ m}^2.$$
than a few micrometers. For much larger sizes, the charge collected should be smaller.

Note that we do not have to consider shielding by the ambient plasma, nor constraints due to the spacecraft size. This is because with $T_C \approx T$, the inequality $n_C > n$ yields $L_{DC} < L_D$, so that if $R_C < L_{DC}$, then $R_C < L_D$. In addition, since $L_D \approx 0.7$ m, the spacecraft size is still larger. Note also that the constraint (6) on $r_u$ was largely met for the E-ring crossing, given the plasma parameters and grain size involved (Meyer-Vernet et al. 1996). We do not consider magnetic field effects since the particle gyroradii are larger than the relevant scales.

If condition (6) holds in order of magnitude, the charge $Q$ released by one impact is collected by the conductive structure of the spacecraft, producing a time variation of the voltage detected by a PRA monopole, of amplitude

$$V_{\text{max}} \approx Q/C,$$

where the spacecraft capacitance estimated from rheographic measurements (R. Manning, personal communication 1994) is

$$C \approx 3 \times 10^{-10} \text{ F}.$$

As in our previous papers, we approximate the signal $V(t)$ produced by a particle impact as increasing to $V_{\text{max}}$ with the rise time $\tau_r$, and decaying with the time constant $\tau_d \gg \tau_r$. At frequencies $f \gg 1/2\pi\tau_r$, the Fourier transform is determined by the discontinuity of the derivative in the rising part (see Meyer-Vernet 1985) and is given by

$$|V(\omega)| = V_{\text{max}}/\tau_r\omega^2$$

or, using (1),

$$V_{\text{max}} = \frac{Q}{C\tau_r\omega^2}.$$  

(7)

For E-ring grain impacts, Meyer-Vernet et al. (1996) estimated the rise time $\tau_r$ from physical constraints on the cloud expansion and charge separation in the context of the simple model above. However, with the parameters relevant in the present paper, this does not constrain sufficiently $\tau_r$. In the present case, an important additional constraint comes from the fact that the observed spectrum varies as $f^{-3}$ for frequencies $f > f_{\text{min}} \approx 2 \times 10^4$ Hz. Hence $\tau_r > 1/(2\pi f_{\text{min}}) \approx 8 \times 10^{-6}$ s. Another constraint comes from the PWS instrument if one assumes that the rise time of dust-induced signals are similar for both instruments. (Although the detection mechanisms are different (see Section 6), this assumption seems reasonable because both rise times are determined by the dynamics of the charges in the plasma cloud.) The waveform observations (Gurnett et al. 1983) exhibit a signal rise time roughly equal to the time resolution of the instrument which is about 30 $\mu$s. Thus, one expects that $\tau_r < 3 \times 10^{-5}$ s.

We deduce from the above inequalities

$$\tau_r \sim 2 \times 10^{-5} \text{ s}$$

(within a factor of about two), which is the same value as used in our E-ring study (Meyer-Vernet et al. 1996).

3.3. Voltage Spectrum Produced by Grain Impacts

If all the grains were identical, with an impact rate of $R$ (uncorrelated impacts) per second, the voltage power spectral density would be for $R \gg 1$

$$V^2 = 2R|V(\omega)|^2.$$  

(8)

Substituting expression (7) of $|V(\omega)|$ with the parameters determined above, we get

$$V^2 = \frac{2RQ^2}{C^2\tau_r^2\omega^4} \sim 3.2 \times 10^2 R\rho_r^3/f^4.$$  

(9)

The impact rate can be expressed as a function of the grain number density $n_G$ as

$$R = n_Gv_GS \sim 1.4 \times 10^4 n_G.$$  

(10)

Using calibration (1), we deduce from (9) and (10) the theoretical PRA level produced by grain impacts

$$V_0^2 \sim 1.8 \times 10^7 n_G\rho_r^3/f^4.$$  

(11)

for $f > 2 \times 10^4$ Hz and $r$ not much larger than a few micrometers (with $V_0^2$ in $V^2/$Hz$^{-1}$, $n_G$ in m$^{-3}$ and $f$ in Hz). To simplify the notation, from now on $r$ stands for $r_u$, the grain radius in micrometers. Since the G-ring grains have a size distribution of nonzero width (Showalter and Cuzzi 1993), Eq. (11) should be rewritten as

$$V_0^2 \sim 1.8 \times 10^7 n_G\langle \rho^3 \rangle/f^4,$$  

(12)

where the brackets stand for a mean over the size distribution of grains contributing to the signal. Let $dn_G/\text{dr}$ be the number of grains per unit volume of the ring with radii between $r$ and $r + \text{dr}$ (as said above, $r$ is now in $\mu$). Then

$$n_G\langle \rho^3 \rangle = \int_{r_{\text{min}}}^{r_{\text{max}}} \frac{dn_G}{\text{dr}} r^3 \text{dr},$$  

(13)

where $r_{\text{min}}$ and $r_{\text{max}}$ bracket the radii of the grains detected. The largest detected radius $r_{\text{max}}$ is determined by the finite individual measurement time $\Delta t = 0.025$ s of the instrument; it can be estimated by assuming a Poisson statistic and writing that the probability of having one impact with radius larger than $r_{\text{max}}$ is $1/e$, which gives using (10)
Let us now assume a power-law size distribution, i.e.,
\[ \frac{dn_G}{dr} = A r^{-q} \quad \text{with } 1 < q < 7. \]  
(15)

In this case, if \( r_{\text{min}} \ll r_{\text{max}} \), Eqs. (13) and (14) yield, respectively,
\[ n_G(r^0) \approx \frac{A}{7 - q} r_{\text{max}}^{-q} \]  
(16)
\[ A \approx 2.9 \times 10^{-3} (q - 1) r_{\text{max}}^{-q-1}. \]  
(17)

Using (16) and (17), the voltage (12) transforms into
\[ V_0^2 \sim \frac{5.2 \times 10^4}{f^2} \frac{q - 1}{f_{\text{max}}} r_{\text{max}}^6 \]  
(18)
\[ \sim \frac{5.2 \times 10^4}{f^2} \frac{q - 1}{f_{\text{max}}} \left[ \frac{A}{2.9 \times 10^{-3} (q - 1)} \right]^{6(q - 1)}. \]  
(19)

4. INFERRING DUST PROPERTIES

4.1. Dust Parameters in the Ring Plane

To infer the dust parameters at the observed maximum at ring plane crossing, we use the data plotted in Fig. 2, restricted to the frequency range 30–300 kHz, for which the spectrum is \( \propto f^{-4} \) and the signal produced by the dust is much above the background noise, a part of which is due to Saturn’s radio emissions (Kaiser et al. 1984). The best fitted \( f^{-4} \) spectrum is
\[ V_0^2 \approx 1.5 \times 10^9 / f^4 V^2 Hz^{-1} \]  
(20)
(with an uncertainty of a factor of about 2). Compared with the theoretical level (12) produced by dust impacts, we find
\[ n_G(r^0) \sim 80 \text{ m}^{-3} \times \mu^6. \]  
(21)

This corrects the value initially found by Aubier et al. (1983), where the estimated values for the spacecraft effective area and capacitance were incorrect.

With the power-law size distribution (15), we deduce from (18), (19), and (20)
\[ r_{\text{max}} = 5.5 \left( \frac{7 - q}{q - 1} \right)^{1/6} \]  
(22)
\[ A \approx 2.9 \times 10^{-3} (q - 1) 5.5^{q-1} \left( \frac{7 - q}{q - 1} \right)^{(q - 1)/6}. \]  
(23)

The maximum detected radius \( r_{\text{max}} \) is not extremely sensitive to the value of the exponent \( q \); for example, with \( q \) varying from 2 to 6, \( r_{\text{max}} \) varies from 7 to 4. Note that these radii have the same order of magnitude as the maximum size of a few micrometers for which our model applies. On the contrary, \( A \) is very sensitive to \( q \); when \( q \) varies from 2 to 6, \( A \) varies from 0.021 to 19.

4.2. Ring Vertical Profile and Thickness

Figure 3 shows the measured voltage power spectrum normalized to the \( f^{-4} \) spectral shape, i.e., the quantity \( V_0^2 \times (f/10^3)^4 \) in the frequency range 30–300 kHz, plotted versus the distance \( z \) to the equatorial plane. This profile can be fitted by the law
\[ V_0^2 \times (f/10^3)^4 \approx 9 \times 10^{-4} e^{-|z|/H_V} \]  
(24)
\[ H_V \approx 1.7 \times 10^{-3} R_S \approx 100 \text{ km} \]
in the region of about 1000 km vertical extent, where the signal due to the ring is larger than the background noise.

In order to deduce the actual vertical profile of the ring, we must (i) estimate the effect of the inclination of the spacecraft trajectory to the vertical direction (i.e., of the
variation in $\rho$) and (ii) to transform the voltage profile into a profile concerning a physical property of the grains.

When the spacecraft moves southward over a vertical distance of $H_N$, its radial distance $\rho$ increases by about 60 km ($\approx 10^{-3} R_S$). Since this radial displacement is negligible compared to the ring radial range of about 7000 km (Showalter and Cuzzi 1993), we will neglect the effect of the radial variation on our measured profile. We will also assume that the ring is axisymmetric.

From (24) we then deduce the vertical profile of the parameter $A$ which determines the grain number density (as defined in (15)), by assuming that the shape of the size distribution (i.e., $q$) does not change with $z$. Since from (19), $A \propto (V_0^2 q^{1/6})$, we have

$$A(z) \propto e^{-|z|/H_A}$$

$$H_A \approx H_N \times 6/(q - 1). \quad (26)$$

The effective ring thickness (spatial density integrated normal to the ring plane divided by the peak value) is thus

$$H = 2H_A \approx 200 \times 6/(q - 1) \text{ km.} \quad (27)$$

When $q$ varies from 2 to 6, $H$ varies from $\approx 1200$ to 250 km.

5. COMPARISON WITH PHOTOMETRIC DATA AND CONSEQUENCES

In this section, we compare our results with a photometric analysis by Showalter and Cuzzi (1993) and discuss them in the context of a recent reanalysis by Throop and Esposito (1998) including spectra acquired during the 1995–1996 ring plane crossing by the Earth.

5.1. Size Distribution Integrated both Vertically and Radially

Equation (18) shows that the PRA signal $V_0^2$ is mainly determined by the largest grains that can be detected during the finite measurement time of the instrument. We have seen that the corresponding radius $r_{\text{max}}$ is of the order of a few micrometers at ring plane crossing; it is somewhat smaller outside the equator since $r_{\text{max}} \propto (V_0^2)^{1/6}$, but remains larger than one micrometer in the whole region where we can detect the ring. This size range is above the 0.03–0.5 $\mu$m range involved in the photometric analysis by Showalter and Cuzzi (1993).

Although the size ranges of both data sets are disjoint, it is interesting to determine whether the size distributions are similar. To put the PRA results in the same form as the photometric ones, let us integrate our measured size distribution both normal to the ring plane and radially across the ring. With the radial profile obtained by Showalter and Cuzzi (1993), our size distribution integrated along $z$ and $\rho$ is

$$N(\rho) = A \rho^{-q} \times H \times \alpha \Delta \rho, \quad (28)$$

where $\alpha \Delta \rho \approx 17,000$ km is the effective ring radial width $\Delta \rho$ times the ratio $\alpha$ of the distribution at $\rho = 168,000$ km (where the optical radial profile peaks) to its value at the radial distance of the Voyager encounter. With $A$ given in (23) and $H$ given in (27) (putting $H$ and $\Delta \rho$ in SI units), this yields

$$N(\rho) \approx 5.9 \times 10^{10} \times 5.5^{q-1} \left(\frac{7 - q}{q - 1}\right)^{(q-1)/6} \rho^{-q} \mu^{-1} \text{ m}^{-1}. \quad (29)$$

If $q$ were equal to 6 as found by Showalter and Cuzzi (1993) for submicrometer grains, this would yield

$$N(\rho) \approx 8.10^{13} \rho^{-6} \mu^{-1} \text{ m}^{-1}, \quad (30)$$

which is larger by a factor of about $10^7$ than the value of $N(\rho)$ deduced by these authors. This huge discrepancy shows that the distribution found by PRA for grains of size a few micrometers cannot be similar to that deduced by Showalter and Cuzzi (1993) for submicrometer grains.

5.2. Composite Size Distribution?

We conclude that if the power-law slope is actually $q = 6$ for $r < 0.5 \mu$m as proposed by Showalter and Cuzzi (1993), then it must change somewhere between this radius and a few micrometers. We assume that this transition takes place at a radius $r_0$ (thus necessarily in the range 0.5$\mu$m to a few $\mu$m), and that

- $N(\rho) \propto C_{\text{SC}} r^{-6}$ for $0.03 < r < r_0$ with $C_{\text{SC}} = 5.8 \times 10^9 \mu^{-1} \text{ m}^{-3}$ as found by the above authors,
- $N(\rho) \propto C_{\text{PRA}} r^{-q}$ for $r_0 < r < r_{\text{max}}$ with $C_{\text{PRA}}$ defined in (29), and $r_{\text{max}} \equiv r_{\text{max}}$, the maximum radius detected by PRA (Section 3), i.e., a few micrometers,
- the shape of this composite distribution (i.e., $q$ and the ratio $C_{\text{SC}}/C_{\text{PRA}}$) does not change within the ring.

Imposing that this composite distribution must be continuous at $r_0$, we deduce

$$r_0^{6-q} \propto C_{\text{SC}}/C_{\text{PRA}} \approx 0.1 \times 5.5^{1-q} \times \left(\frac{q - 1}{7 - q}\right)^{(q-1)/6} \quad (31)$$

This relation shows that the inequality $r_0 \geq 0.5 \mu$m implies $q \leq 1.5$. However, this inequality is certainly too stringent, given the simplifications of the model. In particular, the uncertainty on the determination of $C_{\text{SC}}/C_{\text{PRA}}$ is substan-
tional. We estimate that the parameter $Q/(mC\tau_s)$ might be off by a factor of about 10, due to the crudeness of the modeling of impact ionisation and that Eq. (17) may be off by a factor of 3 due to sampling statistics. Taking into account the uncertainty on the measured PRA level given in Eq. (20), this translates (using Eqs. (9) and (19)) into an uncertainty of a factor of $3 \times (10\sqrt{2})^{(q-1)/3}$ on $A$ and thus on $C_{\text{PRA}}$. In addition, the transition between the two power laws should actually be smooth, and the above inequality $r_0 = 0.5 \mu$ may be too strict since the photometric results are weakly dependent on the extremes of the size distribution. Allowing for a supplementary factor of about 5 for these two latter effects, we find that in the worst case the above inequality on $q$ is replaced by

$$q < 3.5$$  

(32)

(this is equivalent to allowing for the possibility that the first numerical factor in (31) be larger by a factor of about 100.) Note also that from (31), $q > 1$ requires $r_0 < 0.6 \mu$, so that the transition radius $r_0$ is expected to be rather close to $0.5 \mu$.

5.3. Discussion

Our results can only be made compatible with those of Showalter and Cuzzi (1993) if the grain size distribution flattens above $0.5 \mu$, so that the power index $q < 3.5$ in this range. However, there may be another way to resolve this contradiction. Throop and Esposito (1998) have recently reanalyzed the G-ring optical data using the recent modeling by Canup and Esposito (1997) of the steady-state size distribution of ejecta produced from meteoroid impacts and subjected to removal due to plasma drag. Using constraints from spectral observations acquired during the recent passage of the Earth through Saturn’s ring plane, in addition to the Voyager phase curve, they find a power law exponent in the range $1.5 < q < 3.5$ above 0.03 $\mu$.

With such a small value of $q$, our results yield a size distribution integrated vertically and radially which may be compatible with the photometric data. Indeed, Eq. (29) yields $N(r) \approx C_{\text{PRA}}/r^q$ with $C_{\text{PRA}} \approx 4 \times 10^{11} \mu^{-1} \text{ m}^{-1}$ for $q = 2$ and $2 \times 10^{12}$ for $q = 3$. These values of $C_{\text{PRA}}$ are much smaller than the result given in Eq. (30) (which corresponds to $q = 6$), whereas the value of $C_{\text{SC}}$ found by Showalter and Cuzzi (1993) increases as $q$ decreases; both effects act in the same sense to reduce the discrepancy between $C_{\text{PRA}}$ and $C_{\text{SC}}$.

5.4. Geometric Cross Section per Unit Area

We estimate the geometric cross section per unit area of the PRA grain distribution at the center of the ring $\kappa = n_{\text{cr}}(z = 0) \times \pi r^2 H$ by integrating $\pi r^2 N(r)$ over the size distribution and dividing by the ring effective width $\Delta \rho \approx 3500 \text{ km}$

$$\kappa \approx (1/\Delta \rho) \int_{r_0}^{r_0} N(r)(r \times 10^{-6})^2 \, dr$$  

$$\approx 5 \times 10^{-8} \times 5.5^{q-1} \left( \frac{7 - q}{q - 1} \right)^{(q-1)/6} \times \left[ (r_0^3)^q - (r_0^3)^q \right]_{q=3}. \tag{34}$$

(In the limit $q \rightarrow 3$, the factor in brackets is to be replaced by $\ln(r_0^3/r_0)$.) When $q$ varies in the range 1.5 to 3.5, with $r_0 \approx 0.5 \mu$ and $r_M \approx r_{\text{max}}$ defined in (22), $\kappa$ varies in the range

$$\kappa \approx 2 \times 8 \times 10^{-6}, \tag{35}$$

which is not very sensitive to the precise values of $r_0$ and $r_M$.

6. COMPARISON WITH RESULTS FROM THE PLASMA WAVE INSTRUMENT

In this section we compare our results with those deduced from the PWS instrument. PWS provides potentially more information because it can detect individual impulses (since it uses a wide band receiver), although the quantitative interpretation is difficult due to an automatic gain control which changes continuously the response of the system.

Unfortunately, as already noted in the Introduction, PWS data are inherently more difficult to interpret than those of PRA because this instrument operates the antennae as dipoles, whereas PRA uses them as monopoles. Consequently, PRA detects the difference of potential between the antenna booms and the spacecraft, so that it measures in a straightforward manner the variations in spacecraft potential induced when its surface collects the impact produced charges (see Meyer-Vernet 1985). On the contrary, PWS detects the difference of potential between the two antenna booms.

As a consequence, the PWS potential produced by a grain impact may a priori stem from several different processes:

1. straightforward response to the charge collected by one antenna boom (Gurnett et al. 1983),
2. indirect response to the charge collected by the spacecraft (this response—the so-called common mode rejection—exists only with a nonsymmetrical dipole configuration),
3. detection of the electric field produced by charge separation in the impact produced plasma cloud (Obere 1994).

All these PWS responses are difficult to calculate, but they are expected to be much smaller than the PRA response. For process (1) this is due to the small surface of the antenna booms, for process (2) this is caused by the
small dissymmetry of the antenna system (Gurnett et al.
1987), and for process (3) this is due to the large distance
between the impact plasma cloud and the booms.
Without taking into account this small response, Gurnett
et al. (1983) found that PWS detected particles in the range
0.3–3 μ with a size distribution \(dn_G/dr \propto r^{-7}\) and estimated
a ring north–south thickness of 106 km. Tsintikidis et al.
1994 have revised these values by using an empirical PWS
response based on the ratio between the PRA and PWS
observed voltage power spectra. They find that PWS detec-
ted particles in the range \(r = 11 – 16 \mu\) at ring plane
crossing and that the north–south thickness is about 960
km. It is certainly true that the initial PWS estimate of
particle sizes was too low since the actual response of the
instrument was overestimated. Oberc (1994) reached a
similar general conclusion—albeit with different quantita-
tive values.
According to the revised estimation of Tsintikidis et al.
1994, the grains detected by PWS have a mass typically 10
times larger than those detected by PRA. This is due to the
reduced sensitivity of the PWS (dipole) configuration and
means that the two experiments are sensitive to differ-
ent parts of the grain mass distribution.

The revised PWS ring vertical width can be used to
deduce a constraint on the size distribution index if this
index is similar for the PWS and PRA size ranges. The
PWS ring width was directly deduced by Tsintikidis et al.
(1994) from the observed impact rate profile, without tak-
ing into account the fact that the size threshold for de-
tecting the particles decreases with \(|z|\) because of the auto-
matic gain control. Indeed, with the power-law size
distribution \(15\), the observed PWS impact rate \(R\) should
be proportional to \(A \int_{r_{\text{min}}}^{r_{\text{max}}} dr/r^q\), i.e., \(R \propto A/r_{\text{min}}^{q-1}\) with \(r_{\text{min}}\)
decreasing with distance from the ring plane. Hence, the
PWS impact rate \(R\) should decrease less quickly with dis-
tance than \(A\), since the instrument becomes more sensitive.
This suggests that the vertical profile of \(A\), thus of the
grain number density, could be narrower than that of the
PWS impact rate \(R\), so that the actual width \(H\) could be
smaller than the PWS determination of \(H_{\text{PWS}} = 960\) km.

This gives a further constraint on the power index of
the grains detected by PRA if we suppose that the actual
ring thickness is the same for the different grain masses
detected by both instruments. Substituting \(H \leq 960\) km
into Eq. (27), we obtain

\[ q \geq 2.2. \quad (36) \]

7. FINAL REMARKS AND SUMMARY

In this paper, we have analyzed the results of the PRA
measurements acquired when Voyager 2 crossed the G
ring. This analysis differs from our previous study of the
e ring with the Voyager 1 Saturn data (Meyer-Vernet et al.
1996) due to three main reasons. First, the voltage power
spectrum acquired in the G ring showed a \(f^{-4}\) slope, indicat-
ing that only grains contributed to the acquired signal, not
the ambient plasma. Second, the G ring exhibited a wide
grain-size distribution, which complicated the analysis. The
third difference is that the G ring was crossed rapidly by
Voyager 2, due to the narrow vertical extension. As a
consequence, we were not able to acquire a histogram of
grain impact induced voltages, preventing us from directly
deducing the grain size distribution as we did for the E
ring. However, we were able to constrain the G ring grain-
size distribution by comparing our results with related
plasma and optical measurements.

Below, we summarize the results presented in this paper.

- The signal recorded is mainly produced by grains with
  radius \(r_g\) of the order of a few micrometers. Larger grains
  cannot be detected because of the small measurement time
  and the reduced response to large grain impacts due to
  Debye shielding.
- At the peak value, which takes place at ring plane
crossing, \(n_G(r_e) \sim 80\) m\(^{-3}\) μ\(^6\), where \(n_G\) is the grain spatial
density (number per m\(^3\)) and the brackets denote a mean
over the size distribution of the grains able to be detected.
Due to the large uncertainties on the impact ionization
parameters and to the crudeness of the modeling, we esti-
mate that this result might be off by a factor of about 10\(^2\).
- As the size of the largest grains that can be detected
during the measurement time increases with the grain con-
centration, the amplitude of the PRA signal varies more
steeply than the grain concentration itself, so that the ring
vertical profile is wider than that of the PRA signal. For
a power-law differential grain size distribution \(\propto r^{-q}\), we
find that the effective ring thickness (spatial density inte-
grated normal to the ring plane divided by the peak value)
is \(H \sim 1200/(q - 1)\) km.
- Using the ring radial profile measured by Showalter
  and Cuzzi (1993) from photometry of Voyager images, we
deduce the grain-size distribution integrated both radially
and vertically, \(N(r)\), as a function of the size distribution
index \(q\) (Eq. (29)). With \(q = 6\) as proposed by these authors
for submicrometer grains, the PRA results would give a
value of \(N(r)\) roughly 10\(^4\) greater than the value deduced
by these authors. Since this discrepancy is much greater
than the uncertainties, we conclude that the distribution
found by PRA for grain sizes of a few micrometers is less
steep than that deduced by Showalter and Cuzzi (1993)
for submicrometer grains.
- Showalter and Cuzzi’s and PRA results can be re-
ciled if the size distribution flattens above 0.5 μ. Assuming
a rough continuity between both distributions and using a
further constraint from PWS Voyager data, we infer that
\(2.2 < q < 3.5\) for grain radii between about half micrometer
and a few micrometers.
• Our conclusion that the G-ring size distribution is not steep, at least above 0.5 \( \mu \), agrees with the optical data acquired during the last Earth passage through Saturn’s rings. Indeed, Nicholson et al. (1996) indicate that their brightness measurements at 2.26 \( \mu \) do not appear to be consistent with a steep size distribution. Furthermore, a recent reanalysis of the G ring including spectral measurements acquired during the last Earth’s passage (in addition to the Voyager phase curve) finds a range of indices \( q \approx 1.5 \sim 3.5 \) above 0.03 \( \mu \) (Throop and Esposito 1998). With such a small value of \( q \), the discrepancy on the amplitudes of the distributions deduced from the PRA data and by Showalter and Cuzzi (1993) should be strongly reduced.

• In the range of indices \( q \approx 2 \sim 3.5 \), the effective ring vertical thickness \( H \approx 1200 \sim 500 \) km and the geometric cross section per unit area is found to be a few times \( 10^{-6} \).

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