intergalactic magnetic fields that these fields should not perturb them. Their arrival directions should thus point back to their sources in the sky, which does not appear consistent with the available observations. All these difficulties are so serious that they pose a challenge to standard particle physics and cosmology [4].

8.2.2 Rudiments of the acceleration of particles

Let us examine how the particles may be accelerated. A basic requirement for particles to be accelerated to energies much above thermal is that the energy be not shared between all constituents of the medium. Namely, collisions and other equilibration mechanisms must be rare. This usually requires the medium to be dilute. Plasmas, however, share two properties that facilitate particle acceleration:

- the particles carry an electric charge and are thus subject to the electromagnetic field,
- the collisional free path is proportional to the energy squared and thus extremely large for energetic particles.

Since the subject of particle acceleration in plasmas may fill several volumes, we only consider a few basic acceleration mechanisms.

First of all, because the Lorentz force (Section 2.2) does not change the energy of a particle if the electric field vanishes, any acceleration mechanism relies ultimately on the electric field.

Consider the simple case of a static electric field. If the electric field amplitude is such that the energy gained by a particle of charge e over a mean collisional free path $eE \times l_f$ is greater than the thermal energy, then since the free path increases rapidly with speed, all the suprathermal particles of the velocity distribution (whose free path is still greater) will be accelerated without any impediment, and will thus run away from the bulk of the distribution. Because large-scale electric fields of large amplitude are rare in plasmas because of their large electrical conductivity, this acceleration process works generally only at small scales, for example at sites of magnetic field reconnection.

In practice, particles are generally accelerated by varying magnetic fields, which produce electric fields via Maxwell's equation $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$. They are ubiquitous in the Universe on large scales, and on small scales too, due to MHD waves and turbulence.

Rigidity and radius of gyration

We saw in Section 2.2 that the dynamics of charged particles in a magnetic field is governed by two basic parameters:

- the rigidity $R = pc/q = (v/c)(W/q) = (v/c)[(W/e)/Z]$,
- the radius of gyration $r_g \sim p/qB = R/Bc$,

where v is the particle speed, W is the energy, $p = vW/c^2$ the relativistic momentum, $Z = q/e$ the number of elementary charges and B the magnetic field strength. In practical units, for a particle made of A nucleons, this may be written

$$R \simeq \frac{v}{c} \times \frac{A}{Z} \times \frac{W/e}{A} \equiv \frac{v}{c} \times \frac{A}{Z} \times \text{energy per nucleon (in eV)} \qquad (8.5)$$

$$r_g \sim \frac{R}{Bc} \sim \text{energy per nucleon (in eV)} \times \frac{v}{c} \times \frac{A}{Z} \times \frac{3 \times 10^{-9}}{B(\mathrm{T})} \text{ m.} \qquad (8.6)$$

Particles of different charges and masses but with the same rigidity have the same dynamics in a given magnetic field configuration (Section 2.2). If two particles have the same velocity, and therefore the same Lorentz factor γ, they have the same energy per nucleon ($W/A \simeq \gamma m_p c^2$), and their rigidity depends only on their mass to charge ratio A/Z; whereas $A/Z = 1$ for protons, we have seen that for most heavy elements stripped of all their electrons, $A/Z \simeq 2$, so that all completely ionised elements have the same value of A/Z to a factor of two.

The gyroradius depends on the ratio of the rigidity to the magnetic field strength. It determines the minimum scale of a magnetic field structure capable of affecting the dynamics of the particle. From this argument, the galactic magnetic field $B \sim 10^{-10}$ T of spatial scale about 10^{19} m is expected to confine the bulk of cosmic rays of energy at least up to the 'knee' of Fig. 8.3; some observations are consistent with the production of these cosmic rays by shocks produced by the remnants of explosions of massive stars (the so-called *supernovae*; see Fig. 8.11 below) in the Galaxy, by a mechanism we shall examine later.

Betatron acceleration

Consider a particle in a uniform magnetic field, and let the magnetic field increase slowly compared to the particle gyration, so that the magnetic moment of the particle (the first adiabatic invariant) is conserved (Section 2.2). The momentum perpendicular to $\mathbf{B}$ thus increases as $p_\perp^2 \propto B$, whereas the parallel momentum $p_\parallel$ remains constant, so that the total particle energy increases.

This *betatron acceleration*, however, is a reversible process. Sooner or later the magnetic field should decrease, and when it returns to the initial value, the particle loses its energy gain. Assume, however, that before this happens, some irregularities scatter the particle faster than the gyration so that the magnetic moment is not conserved, whereas energy is conserved. This tends to isotropise the velocity distribution, making the parallel energy increase at the expense of the perpendicular one. If the magnetic field now decreases and returns to its original value, the perpendicular energy decreases accordingly, but the particle keeps the parallel energy gained during the stochastic part of the cycle, producing a net energy gain during the cycle.

This basic acceleration process appears under many guises, and is known as *magnetic pumping*. It is an instructive example of the interplay between a reversible effect and stochasticity.

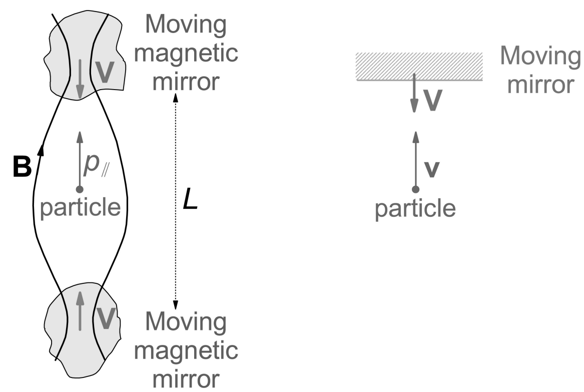

Figure 8.6 Fermi acceleration of a particle trapped between two magnetic mirrors (left), and the equivalent acceleration of a particle upon head-on reflection by a moving mirror (right).

A simple form of betatron acceleration occurs in planetary magnetospheres, when time variations enable particles to move from a large distance where the planetary magnetic field is small, to a small distance where it is large. This may increase the particle perpendicular energy by a factor of the order of the ratio of the magnetic fields, i.e. the cube of the size of the magnetosphere expressed in planetary radius.

Magnetic pumping is expected to take place at small scales, in low-frequency waves.

Fermi acceleration

Fermi acceleration, originally proposed by Fermi in 1949 to explain the acceleration of cosmic rays, is at the basis of most acceleration mechanisms thought to act in astrophysics; as the betatron mechanism, it requires a stochastic process to act. Fermi acceleration is based on the scattering of particles on large clumps of plasma that distort the magnetic field, producing magnetic mirrors which reflect the particles. The particle's energy is not changed in the mirror's rest frame, but if the mirror is moving towards the incident particle, the particle gains energy upon reflection, just as does a tennis ball pushed by a racket. Repeated scattering of the particles by randomly moving 'mirrors' (irregularities of the turbulent magnetic field) produces a net transfer of energy from the moving irregularities to the individual charged particles.

Let us study this in more detail. Just as betatron acceleration, Fermi acceleration may be understood from the conservation of an adiabatic invariant (Section 2.2), in this case the second invariant. Consider a particle trapped between two *magnetic mirrors*, which may be formed by plasma clouds of large magnetic field (Fig. 8.6, left), and of mass much greater than the one of the particle itself. Let the distance L between the mirrors decrease slowly compared to the particle parallel motion, so that the second adiabatic invariant is conserved. The parallel momentum thus increases as $p_{\parallel} \propto 1/L$, making the total energy increase. Note that, as in the case of magnetic pumping, this acceleration cannot proceed indefinitely unless some isotropisation process acts, because the increase in $p_{\parallel}$ decreases the pitch angle of the particle, so that sooner or later it will no longer be reflected by the magnetic mirrors. Another difficulty is that the mirrors cannot approach each other indefinitely. If the mirrors are moving away instead of approaching each other, L increases, making $p_{\parallel} \propto 1/L$ decrease and return to its initial value after a cycle. A gain in energy thus requires the intervention of a stochastic process.

The acceleration by a magnetic mirror moving towards the particle is similar to that of a tennis ball by a racket. Consider the simple case of a particle of speed v impinging normally on a mirror (assumed infinitely massive) moving at speed $V \ll c$ towards the particle in the same direction (Fig. 8.6, right). Assume first the particle to be non-relativistic, so that in the frame of the mirror its speed is simply $v + V$. Upon elastic reflection, the particle's speed just changes of sign in this frame, becoming $-(v + V)$. Transforming back to the observer's frame, the particle's new speed is $-(v + V) - V = -(v + 2V)$. The particle has thus gained the speed $\Delta v = 2V$ upon reflection. Generalising the calculation to relativistic particle velocities, we find that upon head-on reflection, a particle of momentum p (at normal incidence) gains the energy $\Delta W \simeq 2Vp$ for $V \ll c$. Using $p = vW/c^2$, and generalising to different velocity directions, we find that the relative energy gain is

$$\Delta W/W = -2\,(\mathbf{v} \cdot \mathbf{V})\,/c^2 \tag{8.7}$$

since only the projection of $\mathbf{v}$ on $\mathbf{V}$ plays a role. This energy variation is positive for a head-on collision, and negative for a following collision, just as for a tennis ball.

This would be great, were it not for a severe problem: the particle sees mirrors that are moving towards it and away from it, which at first sight should cancel the effect. This is not exactly so, however, because, just as you get more rain on the front windscreen of your car than on the rear one, there are slightly more head-on reflections than tail-on ones. To calculate the balance, we note that the probabilities are proportional to the relative velocities of approach of the mirror and the particle, which are greater for head-on than for tail-on reflections. For $V \ll v$, the relative excess of head-on collisions over tail-on collisions is $2V/v$, so that the net relative energy gain per reflection is in average (considering reflection at normal incidence only)

$$\frac{\langle \Delta W \rangle}{W} \simeq \frac{2V}{v} \times \frac{2vV}{c^2} = 4\frac{V^2}{c^2}. \tag{8.8}$$

This energy gain is of second order in the small parameter V/c, whence its name: *second-order Fermi acceleration*, so that the acceleration rate is in practice very small. It is generally much smaller than the escape rate of the particles from the scattering region, making this acceleration process rather ineffective.

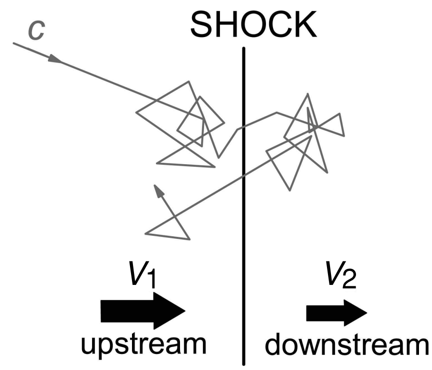

Figure 8.7 First-order Fermi acceleration of a particle near a (quasi-parallel) shock.

First-order Fermi acceleration at shocks

The original Fermi acceleration process is very slow because of the averaging between head-on and tail-on reflections, with only a small excess of head-on reflections, so that it is of second order in the small parameter V/c. The process would be much more effective if there were only head-on reflections. This is exactly what happens at shocks under adequate conditions [1]. The trick is that the medium undergoes a speed decrease from upstream to downstream at the shock, so that from the point of view of the medium on each side of the shock, the other side is moving towards it, whereas both the upstream and downstream regions are full of irregularities that scatter the particles and isotropise the velocity distributions in each frame.

To understand this, consider a plane shock, in which the velocity decreases from V_1 upstream, to $V_2 = V_1/n$ downstream, in the frame where the shock is at rest (Fig. 8.7). For a strong shock of adiabatic index $\gamma = 5/3$, we have seen in Section 2.3 that $n = 4$. In the frame of reference of the upstream medium, in which the velocity distribution of the particles is isotropic, the downstream medium (and its scattering irregularities) are coming head-on at speed $V_1 - V_2$ (the velocity difference between the scattering centres upstream and downstream); likewise, from the point of view of the downstream medium, the upstream medium (and its scattering irregularities) are coming head-on at the same speed. Therefore, each time an average particle (of velocity randomised by scattering on the irregularities) traverses the shock, it sees the plasma irregularities on the other side coming head-on at speed $V_1 - V_2$ in average. This is the basis of the diffusive acceleration at shocks, which is expected to have a great importance in astrophysics given the ubiquity of shocks in the Universe.

As an average particle of speed v traverses the shock downstream and back upstream, the total relative energy gain is given by twice the value given in (8.7), with the velocity $V = V_1 - V_2$, and $(\mathbf{v} \cdot \mathbf{V}) = -vV\cos\theta$, θ being the angle of incidence of the particle to the shock, which satisfies $0 < \theta < \pi/2$. For ultra-relativistic particles ($v \simeq c$), this yields

$$\frac{\Delta W}{W} \simeq 4\frac{V_1 - V_2}{c}\cos\theta. \tag{8.9}$$

We must average this expression over the directions of incidence. Since the rate of arrival of the particles to the shock is proportional to the normal component of the velocity ($\propto \cos\theta$), and the energy gain (8.9) is itself proportional to $\cos\theta$, we must average $\cos^2\theta$, which yields a factor $\langle\cos^2\theta\rangle_0^{\pi/2} = 1/3$. Hence, for a round trip across the shock and back again, the fractional energy increase of the particles is finally, on average

$$\frac{\langle\Delta W\rangle}{W} \simeq \frac{4}{3}\frac{V_1 - V_2}{c} \simeq \frac{4V_1}{3c} \times \frac{n-1}{n} \tag{8.10}$$

where $n = V_1/V_2$ is the shock compression ratio. To work out the resulting energy distribution of the accelerated particles, we must estimate their probability of escape from the shock. Ultra-relativistic particles of number density n_{CR} arrive on the shock at the rate $n_{CR} \times c/4$ (the factor 1/4 comes from averaging over the directions of arrival, as found in Section 7.2.2). On the other hand, the rate at which they are swept away from the shock (without returning) is, since this occurs downstream, $n_{CR}V_2 = n_{CR}V_1/n$. The escape probability of particles is thus $(V_1/n)/(c/4) = 4V_1/nc$. The ratio of the timescales of energy release and acceleration is therefore

$$\frac{4V_1/nc}{\langle\Delta W\rangle/W} \simeq \frac{3}{n-1} \tag{8.11}$$

where we have substituted (8.10). We deduce (Problem 8.4.2, or using the reasoning of Section 4.5.3) that the differential energy distribution is given by (4.35), at high energies, i.e. $dN/dW \propto W^{-(\kappa+1)}$ with $\kappa = 3/(n-1)$, i.e.

$$dN/dW \propto W^{-(n+2)/(n-1)}. \tag{8.12}$$

The particles thus emerge from the acceleration site with a power law spectrum, whose index depends on the shock compression ratio, and not on the shock speed, nor on the detailed geometry or the scattering process, as long as the shock may be considered as plane.[5] This makes this acceleration process universal.[6] For a strong shock with adiabatic index $\gamma = 5/3$, we have $n = V_1/V_2 \simeq 4$, so that the high-energy spectrum has the form $dN/dW \propto W^{-2}$, which agrees well with the observed cosmic ray spectrum (above 1 GeV and below the 'knee' of Fig. 8.3), when due account is made of particle propagation in the interstellar medium.

Three important comments are in order. First, the accelerated particles are expected to be confined within some distance from the shock, with a concentration that decreases farther away. Second, this mechanism requires the particles

[5] Namely, the radius of curvature of the shock must be much greater than the other scales.
[6] However, the timescale of acceleration does depend on the diffusion process.

to make multiple shock traversals. The scale of diffusion (and the particle gyroradius) must therefore be greater than the shock thickness. Since the shock thickness is of the order of magnitude of the gyroradius of the particles of the ambient medium, this condition means that the particles must already move faster than average, in order to pass freely between the upstream and downstream sides and be significantly accelerated.[7] Third, the average magnetic field has been implicitly assumed to be quasi-parallel to the shock normal (a so-called quasi-parallel shock; see Fig. 6.14, left).

Shock drift acceleration

In the opposite case, when the magnetic field makes an appreciable angle to the shock normal (a quasi-perpendicular shock; see Fig. 6.14, right), the motion of the plasma across the magnetic field produces an electric field $-\mathbf{V} \times \mathbf{B}$ (conserved upon shock traversal) in the frame of the shock. Furthermore, particles drift along the shock surface along $\mathbf{B} \times \nabla \mathbf{B}$ for positive charges (and the opposite for negative charges), perpendicular to both the magnetic field and its gradient (Section 2.2). Since we have seen that B increases upon traversing the shock from upstream to downstream (Section 6.3), the drift is in the same sense as the shock electric field for positive charges (and the opposite for negative charges), so that the particles are accelerated by the shock electric field whatever the sign of their charge. We shall see an application of this *shock drift acceleration* below.

A review of the physics of particle acceleration at shocks may be found in [13]. These mechanisms are responsible of a large part of particle acceleration in the Universe, from solar, heliospheric and planetary shocks, to distant astrophysical objects.

8.2.3 Modulation of galactic cosmic rays by solar activity

Galactic cosmic rays entering the heliosphere are subjected to:

- scattering by magnetic field irregularities due to turbulence, and by larger transient structures as the solar mass ejections,
- drifts due to the magnetic field gradient and the curvature of the field lines, determined by the three-dimensional structure of the heliospheric magnetic field, including the current sheet separating opposite magnetic polarities,
- outward convection and adiabatic deceleration as they follow the large-scale magnetic field of the expanding solar wind.

As a result, theoretical models have to consider the detailed structure of the magnetic turbulence, which we have seen to be far from understood, the

[7] These particles, however, may simply be the fast-speed particles of an ambient near-equilibrium velocity distribution.