CAPTURE OF GRAINS INTO RESONANCES THROUGH POYNTING-ROBERTSON DRAG

B. SICARDY

Observatoire de Paris, DAEC-EUROPA, 92195 Meudon Cédex Principal, France, Université Pierre et Marie Curie, 75005 Paris, France

C. BEAUGÉ and S. FERRAZ-MELLO

Instituto Astrónomico e Geofísico, Univ. de São Paulo, CP9638, São Paulo, SP, Brazil

D. LAZZARO

Observatório Nacional, DAF, Rio de Janeiro, RJ, Brazil, Observatoire de Paris, DAEC-EUROPA, 92195 Meudon Cédex Principal, France

and

F. ROQUES

Observatoire de Paris, DAEC-EUROPA, 92195 Meudon Cédex Principal, France

Abstract. We review here some relevant problems connected to the evolution of circumstellar dust grains, subjected to Poynting-Robertson (PR) drag, and perturbed by first-order resonances with a planet on a circular orbit. We show that only outer mean motion resonances are able to counteract the damping effect of PR drag. However, the high orbital eccentricities reached by the particle lead to orbit crossings with the planet. This is a serious difficulty for a permanent trapping to be achieved. In any case, we show that the time spent in the resonance is long enough for statistical effects (accumulation at the resonant radius) to be significant. We underline some difficulties associated with this problem, namely, the non-adiabaticity of motion in the resonance phase space and the existence of close encounters with the planet at high eccentricities.

Key words: Circumstellar dust grain – Poynting-Robertson drag – resonance

1. Introduction

Dissipative effects, when combined to resonance phenomena, may lead to complex and often counter-intuitive, dynamical behaviours. One of the first examples of this kind was given by the locking of natural satellites in mean motion resonances through the slow and irreversible tidal evolution (Goldreich, 1964, and see the review by Peale, 1986). Tidal effects can then account for most of the orbit-orbit and spin-orbit resonances now observed among planets and satellites.

Another field of interest is the influence of gas drag on planetesimal orbits in the primordial solar system. Such a drag, when combined to the perturbations of a jovian planet, can cause some trapping and eccentricity pumping of the planetesimal orbits near resonance radii (Greenberg, 1978, Weidenschilling and Davis, 1985, Patterson, 1987).

Planetary rings also exhibit resonance-dissipation coupling phenomena. It is now widely accepted that inelastic collisions, combined to collective effects and resonant perturbations from satellites, can lead to sharp edges in rings, confining of ringlets, opening of gaps, etc... (see the reviews by Goldreich and Tremaine, 1982, Borderies, Goldreich and Tremaine, 1984, Meyer-Vernet and Sicardy, 1987).

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In this paper, we will review a new kind of dissipative process, acting together with the resonant perturbation of a planet. Namely, we investigate the effect of Poynting-Robertson (PR) drag on particles orbiting the Sun (or another star), while they enter the region of influence of a mean-motion resonance with the planet, in the course of their slow orbital decay towards the central body. Because the radiation forces responsible for the PR drag essentially act on particles with radius comparable to the wavelength (see section 2), dynamical effects will be mainly noticeable for μm-sized dust particles.

In contrast with the previous studies quoted above, PR drag vs. resonance dynamics is a relatively new subject and the full implications of the interplay between these two effects is far from being fully analyzed. Consequently, the aim of this paper is to present some review of what has been done to our knowledge on that topic, but also to underline some of the difficulties of the problem and possible future directions of investigation.

Pioneering work in this field has been achieved by Gonczi, Froeschlé and Froeschlé (1982), and by Jackson and Zook (1989). The former authors studied the passage of dust grains through the inner 2:1 mean motion resonance with Jupiter and pointed out that this passage may cause large variations in the particle osculating elements. The latter showed that particles may be trapped in outer resonances with the Earth. Jackson and Zook (1992) analyzed the orbital evolution of dust particles originating from comets or asteroids, while Dermott *et al.* (1992) reviewed the effects of planetary perturbations on the zodiacal dust cloud. Also, Scholl, Roques and Sicardy (1992) examined the collective response of a circumstellar disk perturbed by Earth-like or jovian planets showing that the global morphology of the disk may be strongly affected by resonance effects, with potential applications to the structure of the β-Pictoris circumstellar dust disk.

Without being exhaustive here, we may think of several situations of interest, where PR drag and resonances have significant dynamical consequences. First, we must note here that PR drag is a quite "unavoidable" effect, since it appears as soon as photons hit a small particle. This contrasts with gas drag, which pertains to the primordial (and thus largely unknown) solar system. Actually, PR drag is an on-going process, effective at any time around any star. Among the possible fields of investigation, we may quote:

- The evolution of dust in the solar system:
  - Zodiacal dust bands in the asteroid belt.
  - Cometary dust.
  - rings around planets.
- Cosmogonic implications of resonance trapping:
  - Confining or clearing of dusty material in the early solar system.
  - Comparison with other confining or clearing mechanisms.
- On-going processes around other stars:
  - Effect of a hypothetical planet on a circumstellar disk subjected to PR drag.

- Inverse problem: derivation of a planet mass and orbital elements from its effects on a circumstellar disk.

Although the PR drag problem bears some resemblance with the topics described at the beginning of this Section, it allows, nevertheless, the particle to reach very high orbital eccentricities, with correlated difficulties, like the crossing of the planet orbit and the non validity of the usual expansions of the disturbing potential.

We will mainly deal in this paper about orders of magnitude and also about some relevant mechanisms, even though the formalism we use is not necessarily valid for high eccentricities. We will restrict ourselves to the simplest case, i.e. a test particle orbiting a star and perturbed by a planet with circular orbit, with no mutual inclination (planar circular restricted problem), in the presence of PR drag. We will be concerned more precisely with the following points:

- Derive some order of magnitude results on the PR drag itself.
- Present a simple dynamical formulation which describes both the effect of the resonance and that of the PR drag.
- Look in this frame for the existence of periodic orbits, deriving some constraints on the planet mass.
- Discuss the difficulties inherent to PR drag (high eccentricities, orbit crossing, no adiabaticity of the motion).
- Discuss some future directions of investigation.

2. Order of Magnitude Considerations

We give here some figures which will be useful in the rest of the paper. Most of them come from the seminal paper of Burns, Lamy and Soter (1979, thereafter referred to as BLS) on radiation forces in the solar system.

The radiation force acting on a particle is:

$$\mathbf{F}_R = \beta F_g \left[(1 - \dot{r}/c)\hat{S} - \mathbf{v}/c \right], \tag{1}$$

where $\beta = SA_SQ_{RP}/cF_g$ is the (constant) ratio of the force SA_SQ_{RP}/c due to the pressure of radiation, and the gravitational force F_g due to the central star. Here, S is the stellar flux at the particle, A_S is the geometrical cross section of the latter, c is the speed of light, Q_{RP} is the radiation pressure efficiency, $\mathbf{v}$ and $\dot{r}$ are the total and radial velocities of the particle, and $\hat{S}$ is the unit vector of the incident radiation. The radiation force may be split into a main radial component, $\beta F_g \hat{S}$ (the "pressure of radiation"), and a velocity dependent component (the "Poynting-Robertson drag").

The net effect of the radiation pressure component $\beta F_g \hat{S}$ is to replace the central star of mass M_* by an "apparent" mass M_*':

$$M_*' = (1 - \beta) M_*. \tag{2}$$

Numerical calculations (BLS) show that $\beta \sim 5.7 \times 10^{-7} Q_{RP}/\rho s$ for a spherical particle orbiting the Sun, where the radius s and the density ρ are expressed in cgs units. For typical materials of density $\rho = 3$ g cm^{-3} and in the frame of geometrical optics (Q_{RP}= 1, a reasonable assumption at this stage), one gets:

$$\beta \sim \frac{0.2}{s_{\mu m}}, \tag{3}$$

where $s_{\mu m}$ is the radius expressed in μm. For a star of luminosity L, and with a spectrum comparable to that of the Sun, one would get $\beta \sim (0.2/s_{\mu m}) \times (L/L_\odot)$, where $L_\odot$ is the Sun luminosity.

An important consequence of Eq. (2) is that the mean motion, and thus the corresponding resonant semi-major axis of a particle, are changed by the radiation pressure term. This is far from being negligible, since β is a substantial fraction of unity for μm-sized particles.

The average decay rates of semi-major axis a and eccentricity e due to PR drag are (BLS):

$$\begin{cases} < \dot{a}/a >= -\frac{\eta}{a^2}\frac{2+3e^2}{(1-e^2)^{3/2}} \\ < \dot{e}/e >= -\frac{5\eta}{2a^2}\frac{1}{(1-e^2)^{1/2}}, \end{cases} \tag{4}$$

where $\eta = \beta G M_*/c$. It is worth emphasizing that when we consider only the part of the drag proportional to **v** in Eq. (1), the results are different. The term $< \dot{a}/a >$ is affected by $\mathcal{O}(e^2)$ and the numerical coefficient of $< \dot{e}/e >$ becomes 2 (instead of $\frac{5}{2}$). In this review the drag proportional to $\dot{r}$ is also considered since the effects of the two terms of the Poynting-Robertson drag cannot be physically separated. For almost circular orbits ($e \ll 1$), one gets:

$$\begin{cases} < \dot{a}/a >= -4C/5 \\ < \dot{e}/e >= -C, \end{cases} \tag{5}$$

where the the damping coefficient C is given by $C = 5\eta/2a^2$. The characteristic decay time around the Sun is thus:

$$t_{decay} = \frac{1}{C} \sim 3200 R_{AU}^2 s_{\mu m} \qquad years, \tag{6}$$

where R_{AU} is the orbital radius in astronomical radius. This time must be multiplied by $L_\odot/L$ for another star of mass comparable to that of the Sun. Note that the decay time is short compared to stellar, or planetary, ages for typical R of a few astronomical units. Only for particles of a few mm in radius, located beyond Uranus's orbit, is the decay time comparable to the age of the solar system.

Another quantity of interest is the "fractional loss" per orbit, i.e. the dimensionless ratio of C to the mean motion n of the particle:

$$\frac{C}{n} \sim \frac{5 \times 10^{-5}}{s_{\mu m}\sqrt{R_{AU}}}, \tag{7}$$

for a circumsolar grain. Thus, the relative damping of eccentricity during one revolution is small compared to unity. This contrasts with other problems like collisions in a dense ring, or gas drag in the early solar system, where the damping per orbit may be a substantial fraction of unity. This smallness allows typical dust particles to get quite high orbital eccentricities, with the associated problems discussed in Section 5.

3. Dynamical Model of PR Drag-Resonance Interaction

In order to simplify as much as possible our problem, we will consider here a massless particle orbiting a star of mass M_*, perturbed by a planet of mass M_p with circular orbit, near a first order resonance $m : m+1$, i.e. $(m+1)n_p - mn \sim 0$, where n (resp. n_p) is the particle (resp. the planet) mean motion. Furthermore, all the orbits are coplanar. Close to the resonance, the dynamics of the motion is governed by the terms in the planet perturbing potential which are slowly varying with time (averaging principle).

The analysis in this Section and the following is based on the work of Greenberg (1978). However, we extend his study by taking into account the variation of the semi-major axis a associated to the variation of the eccentricity e (while Greenberg assumed that the semi-major axis is fixed during the trapping in the resonance). The simultaneous variation of a and e is actually required by the conservation of the Jacobi constant. This results in a dependence of the particle mean motion on the eccentricity (see Eq. (15)), and allows one to obtain the phase space diagrams shown in Figs. 3 and 4. We assume here that the orbital eccentricity of the particle is small, which allows us to keep only the term to first order in eccentricity in the perturbing potential, Φ_p, due to the planet (see, e.g., Brouwer and Clemence, 1961):

$$\Phi_p = a_0^2 n_0^2 \epsilon e A.cos(\Psi_L), \tag{8}$$

where a_0 and n_0 are the reference values of the semi-major axis and mean motion of the particle at the resonance, ϵ is the (small) ratio $\epsilon = M_p/M_*$, Ψ_L is the critical argument of the (Lindblad) resonance, and A is a dimensionless coefficient which depend on the Laplace coefficients $b_{1/2}^{(m)}$:

$$A = \frac{\alpha}{2}\left[2(m+1)b_{1/2}^{(m+1)} + \alpha\frac{db_{1/2}^{(m+1)}}{d\alpha}\right] \tag{9}$$

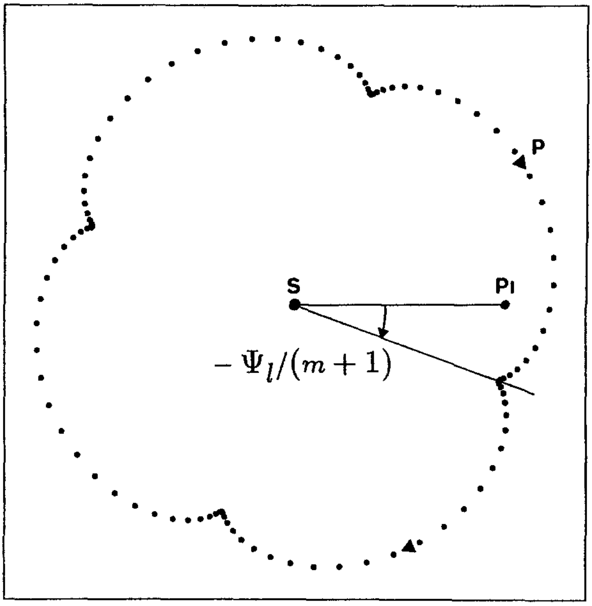

Fig. 1. The geometrical interpretation of the Lindblad resonance critical argument. The orbit of the particle **P** around the star **S** is plotted is the frame rotating with the planet **Pl**. This motion corresponds to the 5:4 outer Lindblad resonance ($m = -5$, see the text).

where $\alpha = a/a_p$ is the ratio of the particle semi-major axis to that of the planet. For numerical purposes, one can note that A is of order m. For large values of m, A/m tends to 0.802...

The critical argument Ψ_L is given by:

$$\Psi_L = (m+1)\lambda_p - m\lambda - \tilde{\omega}, \tag{10}$$

where λ is the mean longitude, $\tilde{\omega}$ is the longitude of periapsis of the particle and the subscript p refers to the planet.

The term proportional to $\epsilon e.cos(\Psi_L)$, in Eq. (8), couples the elements of the particle orbit to the gravitational influence of the planet. The angle Ψ_L can be interpreted as the orbital phase lag of the particle as observed *from the planet* (Fig. 1). This lag allows the planet to exchange energy and angular momentum with the particle and, in particular, to pump up the eccentricity of the latter (Sicardy, 1991).

If the planet orbit is elliptic, another term, proportional to the planet orbital eccentricity e_p, will appear in the perturbing potential Φ_p. This term contains a new critical argument $\Psi_c = (m+1)\lambda_p - m\lambda - \tilde{\omega}_p$, corresponding to a corotation resonance (see the discussion by Sicardy, 1991, for the geometrical and physical interpretation of Lindblad vs. corotation resonances). This more complete approach is followed by Beaugé and Ferraz-Mello (1992) who take into account the planet orbital eccentricity and, in the case of PR drag, use an asymmetric expansion of the perturbing potential valid for very high eccentricities (see Ferraz-Mello and Sato, 1989).

It is convenient to define the eccentricity (h, k) vector as:

$$\begin{cases} h = & e.cos(\Psi_L) \\ k = & e.sin(\Psi_L), \end{cases} \tag{11}$$

and the "distance", in mean motion, to the exact resonance by:

$$\Delta n = (m+1)n_p - mn. \tag{12}$$

The classical techniques of perturbation theory then yields:

$$\begin{cases} \dot{h} = & -\Delta n.k \\ \dot{k} = & \Delta n.h + \epsilon An, \end{cases} \tag{13}$$

where the dots stand for time derivatives.

It is important to note that hereafter the quantities A and n are considered as *constant*. They actually represent the values around which the equations are expanded.

From the above system, one can derive $de^2/dt = 2\epsilon Ank$. From a physical point of view, this stems from the exchange of angular momentum between the planet and the particle, at a rate which is proportional to $k = e.sin(\Psi_L)$. In other words, the torque applied to the particle orbit is proportional to the distortion e of the orbit, and to the sine of the phase lag Ψ_L (Fig. 1).

Energy is also exchanged between the two bodies, so that Δn varies at a rate:

$$\dot{\Delta n} = -3m^2 n^2 \epsilon Ak. \tag{14}$$

This ensures that the quantity:

$$J = 3m^2 ne^2 + 2\Delta n \tag{15}$$

is conserved. Actually, J is nothing but the expansion in e^2 and Δn of the Jacobi constant, or more precisely, the Jacobi constant averaged over one sidereal period. From $J = cste$, one can derive:

$$\dot{a} = -ma\frac{de^2}{dt}. \tag{16}$$

This shows in particular that for a particle orbiting outside the planet orbit (outer Lindblad resonance, $m < 0$), $\dot{a}$ and de^2/dt have the same sign. Thus, if a particle gains energy, its eccentricity is also increased. The opposite is true for inner Lindblad resonances. This has an important consequence when dissipative processes are included: a particle at an outer Lindblad resonance will receive energy from the planet, thus increasing its eccentricity, which can compensate in principle for the damping effect of dissipation. At an inner Lindblad resonance, the planet will act to decrease the particle orbital eccentricity as it provides energy, so that no equilibrium is a priori possible. This is a particular case of a more general phenomenon, which tends to push the particle away from the planet orbit as soon as some collective or irreversible effects are introduced in the system (the so-called "shepherding mechanism", see the reviews by Goldreich and Tremaine, 1982, and Meyer-Vernet and Sicardy, 1987).

The secular effect of gas drag can be derived from Eqs. (5), assuming again small eccentricity for the particle. The complete set of equations (PR drag + resonance) then reads:

$$\begin{cases} \dot{\Delta n} = & -\frac{3mn}{2}\left[2mn\epsilon Ak + \frac{4C}{5}\right] \\ \dot{h} = & -\Delta n.k - Ch \\ \dot{k} = & \Delta n.h + \epsilon An - Ck. \end{cases} \tag{17}$$

4. Evolution of the Particle in the Resonance

4.1. Stationary Orbits

One may ask whether the system (17) has fixed points, corresponding to stationary orbits. By setting all the time derivatives to zero, one can easily show that there are actually two fixed points:

$$\begin{cases} h_f = & -\epsilon An\frac{\Delta n}{C^2+\Delta^2 n} \\ k_f = & \epsilon An\frac{C}{C^2+\Delta^2 n}, \end{cases} \tag{18}$$

where Δn has the two opposite values defined by:

$$\Delta^2 n = -\left[\frac{5}{2}mn^2A^2\epsilon^2 + C^2\right]. \tag{19}$$

Since $\Delta^2 n \geq 0$, we retrieve the necessary condition that m should be negative, i.e. the particle should be in an *outer* Lindblad resonance for a fixed point to exist. Assuming from now on that $m \leq 0$, we get the final expressions for h_f and k_f:

$$\begin{cases} h_f = & \pm\sqrt{\frac{2}{5|m|}}\sqrt{1 - \frac{2}{5|m|}\left(\frac{C}{\epsilon An}\right)^2} \\ k_f = & \frac{2}{5|m|}\left(\frac{C}{\epsilon An}\right), \end{cases} \tag{20}$$

where the sign + (resp. −) applies to Δn positive (resp. negative). This shows that the fixed points correspond to a "universal eccentricity" $e_f = \sqrt{h_f^2 + k_f^2}$, which is *independent* of the damping coefficient C:

$$e_f = \sqrt{\frac{2}{5|m|}}. \tag{21}$$

A more exact calculation following what was done by Beaugé and Ferraz-Mello (1992) shows that the universal eccentricity is given by the root of the equation $|m|(1-e^2)^{3/2} - (|m|-1)(1+1.5e^2) = 0$. Actually, one can see that Eq. (21) is a good approximation of the root of the latter equation. For $|m|$=2 and $|m|$= 3, one gets e_f= 0.45 and 0.36 respectively, while the exact equation yields 0.48 and 0.37, respectively. For large $|m|$, the exact equation gives $e_f \sim 1/\sqrt{3|m|}$, close to the result of Eq. (21). The critical angles corresponding to each of the fixed points are:

$$\Psi_{Lf} = atan\left(-\frac{C}{\Delta n}\right), \quad \Psi_{Lf} = atan\left(-\frac{C}{\Delta n}\right) + \pi, \tag{22}$$

for Δn positive and negative, respectively, where the argument $atan$ is always taken between $-\pi/2$ and $\pi/2$.

The independence of the eccentricity e_f with respect to dissipation may appear puzzling a priori. Physically, it comes from the fact that when C increases, the critical argument Ψ_{Lf} gets closer to $-\pi/2$, i.e. farther away from the symmetry values 0 or π, so that more energy may be exchanged between the planet and the particle, everything equal besides. Thus, although more energy is dissipated through PR drag, the eccentricity corresponding to the fixed point may remain the same.

4.2. Stability of the Fixed Points

The linear stability of the fixed points is determined by the eigenfrequencies λ's of the system (17) near h_f, k_f and Δn_f. The characteristic equation of the system (17) reads:

$$(\lambda + 2C)(\lambda^2 + 3mn\Delta n_f/2) = -\left(\epsilon A n/e_f\right)^2 \lambda. \tag{23}$$

For $\Delta n_f \leq 0$, the left-hand side of the equation is a third degree polynomial in λ with only one, negative, real root ($-2C$). The right hand side is a linear function of λ, with negative slope. Thus, the real roots of the complete Eq. (23) are bracketed by $-2C$ and 0. Since the sum of the three roots of Eq. (23) is $-2C$, this requires that all the real parts of the roots are negative, which ensures the linear stability of the corresponding fixed point. On the contrary, if $\Delta n_f \geq 0$, the left hand side has a positive real root and has a negative value at $\lambda = 0$. This shows that Eq. (23) has at least one positive root and, thus, that the corresponding fixed point is linearly unstable.

One can note from Eqs. (22) that the solution corresponding to $\Delta n_f \leq 0$ has Ψ_{Lf} closer to π (apocentric librator), and the solution corresponding to $\Delta n_f \geq 0$ has Ψ_{Lf} closer to zero (pericentric librator). Thus, the apocentric librating resonant orbit is linearly stable, while the pericentric one is unstable. The value of Ψ_{Lf} closer to π provides a "protection mechanism", in which the conjunction of the planet and the particle tend to occur when the two bodies are farthest away (see Fig. 1). This result is known in the conservative case and appears to be valid also in the presence of PR drag.

4.3. Minimum Planetary Mass for Stationary Orbits

A consequence of Eqs. (22) is that the critical argument corresponding to the stable stationary orbits tends to π for vanishing C, and tends to $-\pi/2$ for increasing C. This is a classical result of forced oscillators in the presence of damping: while they are in phase with the forcing term in the conservative case, they tend to be in quadrature for high dissipation. This is because more and more energy has to be provided by the perturber.

Beyond a certain value of C, however, the condition (19) cannot be achieved any more. Then, the planet is unable to provide enough energy to the particle through the Lindblad resonance, even though $\Psi_{Lf} = -\pi/2$. The condition for (19) being possible is thus:

$$\epsilon \geq \frac{C}{n|A|}\sqrt{\frac{2}{5|m|}}. \tag{24}$$

Numerical applications for a star of luminosity L yields:

$$\epsilon \gtrsim \frac{3.2 \times 10^{-5}}{|m|^{3/2}} \frac{1}{s_{\mu m}} \frac{1}{\sqrt{R_{AU}}} \frac{L}{L_{\odot}}. \tag{25}$$

It is interesting to note that this order of magnitude calculation provides minimum masses which lie in the range of typical planetary masses, for μm-sized particles at a few or several astronomical units from the star (recall that $\epsilon \sim 3 \times 10^{-6}$ for the Earth, $\sim 5 \times 10^{-5}$ for Uranus and Neptune and $\sim 10^{-3}$ for Jupiter).

5. Limitations and Difficulties

5.1. High Eccentricities

A first limitation of our approach is that the eccentricity corresponding to the stationary orbit, $\sqrt{\frac{2}{5|m|}}$, is *not* small compared to unity for small values of $|m|$. However, we have seen that the order of magnitude of the universal eccentricity derived with our approach was the same as that derived following Beaugé and Ferraz-Mello (1992), once second order terms in eccentricity are taken into account in the perturbing potential.

At that point, a second difficulty arises, because such a high eccentricity will cause the particle to *cross* the planet orbit. The distance in semi-major axis between the two bodies is of order $2a/3|m|$, so that orbit crossing will occur for $ae_f \geq 2a/3|m|$, i.e. $|m| \leq\sim 8/9$. In other words, *all* the outer Lindblad resonances yield stationary orbits which cross the planet orbit. Thus, although these orbits are interesting as benchmarks, which define possible attractors for the particle motion, they are nevertheless difficult to reach. More precisely, a particle may collide with the planet before it settles in the stationary orbit. This depends on the efficiency of the protection mechanism which should keep the value of Ψ_L close to π.

This situation is depicted in Fig. 2, in which we plot the osculating eccentricity e of a particle vs. its osculating semi-major axis a. The particle is assumed to orbit the star β-Pictoris ($\sim$ 1.5 solar mass, $L/L_\odot \sim 6$) and to have a radius of 3.6 μm, corresponding to a value of $\beta = 0.33$. Finally, it is perturbed by a planet of mass $\epsilon = 10^{-4}$, orbiting at a_p= 20 AU from the star. The continuous curve is the set of points verifying $a(1-e) = a_p$. Above that curve, the pericenter of the particle orbit lies inside the planet orbit, so that the two orbits intersect.

The motion of the particle is calculated exactly, through a 4^{th} order Runge-Kutta integrator, with the full perturbing force of the planet taken into account, superimposed to the perturbing radiation force (Eq. (1)). In a first stage, the particle semi major axis decreases, due to PR drag. Then, the eccentricity increases, while the semi-major axis is locked at the value corresponding to the 2:1 outer Lindblad resonance. On its way to reach the periodic orbit defined by Eqs. (20), the particle orbit crosses that of the planet. A more detailed analysis of the motion shows that the particle escapes the libration region of the resonance and then has a close encounter with the planet, which explains its sudden removal from the resonance region. We have performed several numerical integrations, with various initial conditions. Although the particles may stay for a rather long time in the resonance region (1.6 Myears in the case of Fig. 2), they eventually all have a close encounter with the planet.

5.2. Non adiabaticity of the motion

Before reaching the periodic orbit, the particle must first be captured in a stable libration. Then, a slow evolution of the phase space, due to PR drag, must increase the particle orbital eccentricity to e_f, while avoiding collision with the planet by an appropriate range of values for Ψ_L. In the mean time, the particle may escape the resonance region by crossing the separatrix between libration and circulation.

This problem is much more difficult to solve than merely determining the fixed points in Eqs. (17). This requires numerical integrations rather than an analytical approach. We note here that an adiabatic invariant method could in principle be used

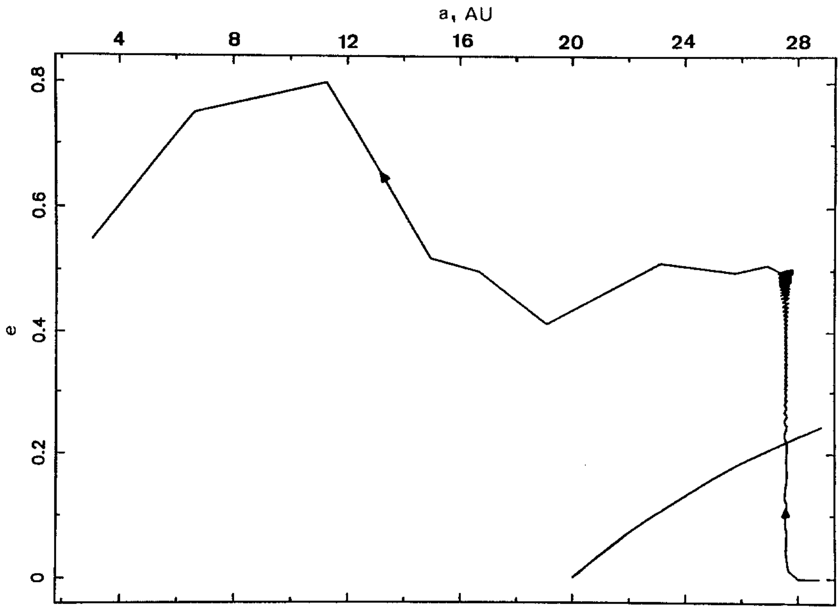

Fig. 2. The osculating eccentricity e of a particle is plotted against its semi-major axis a. The particle is perturbed by a planet of mass $\epsilon = 10^{-4}$ stellar mass, revolving on a circular orbit at 20 A.U. from the star β-Pic (1.5 $M_{\odot}$). The time step beween two points is 100 planetary revolutions, i.e. 7300 years. The particle spends 1.6 $\times 10^6$ years in the resonance region. The curve in the lower right of the figure is the set of points where the pericenter of the particle orbit intersect the planet orbit. Above that curve, the two orbits cross. See the text for a more detailed description.

to follow the particle motion in some suitable phase space, since the dissipation term is small (Eq. (7)). Unfortunately, we will now show that this is *not* the case for a typical μm-sized particle perturbed by a typical planet.

More specifically, we want here to compare the libration period of the eccentricity vector, and the time scale over which the phase space itself evolves, due to PR drag. We turn back to the system (17), and we examine the conservative case, i.e. setting C to zero. Then, it is easy to show that besides the Jacobi constant $J = 3m^2ne^2 + 2\Delta n$, there is a second integral of motion, K:

$$e^4 - \mathcal{J}e^2 + \epsilon\mathcal{A}h = K, \tag{26}$$

where the constant coefficients are defined by:

$$\begin{cases} \mathcal{J} = 2J/3m^2n = 2(e^2 + 2\Delta n/3m^2n) \\ \mathcal{A} = -8A/3m^2. \end{cases} \tag{27}$$

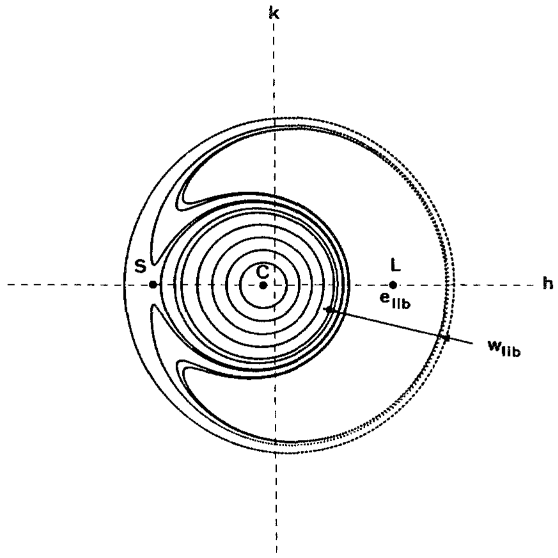

Fig. 3. Definitions of the various quantities used in the eccentricity vector (h, k) space. The quantities h and k are defined in Eqs. (11).

Note that because we are dealing with outer Lindblad resonances ($m \leq 0$), we have $\mathcal{A} \geq 0$, since A and m have the same sign. Also, because $A \sim m$, $\mathcal{A} \sim -8/(3m)$.

Eq. (26) entirely defines the trajectory followed by the eccentricity vector, once $\mathcal{J}$ and K are fixed. These curves are the projections on the (h, k) plane of the surfaces, in the (h, k, z) space, defined by $z = e^4 - \mathcal{J}e^2 = cste$ and $z = \epsilon\mathcal{A}h = cste$. The first surface is axisymmetric around the z axis, and the second surface is a plane parallel to the h axis, with a very small inclination with respect to the (h, k) plane, since ϵ is very small. A complete analysis of the morphology of the trajectories is given, e.g., by Ferraz-Mello (1985).

In particular, this analysis shows that the critical points corresponding to periodic orbits are given by $k = 0$ and $h^4 - \mathcal{J}h^2 + \epsilon\mathcal{A}h$ extremum, i.e.:

$$4h^3 - 2\mathcal{J}h + \epsilon\mathcal{A} = 0. \tag{28}$$

This third degree polynomial equation yields the classical equilibrium points that one can see in Fig. 3. One can note the stable libration point L, surrounded by bean-shaped or banana-shaped trajectories, themselves enclosed in the separatrix which connect itself through the unstable saddle point S (at least if $\mathcal{J} \geq 0$). Then, in the inner circulating region lies the third, stable, equilibrium point, C.

Due to faintness of the factor ϵ, it is easy to show that the stable libration point L is given by $h_{lib} \sim -\sqrt{\mathcal{J}/2}$, corresponding to an eccentricity of:

$$e_{lib} = |h_{lib}| = \sqrt{\mathcal{J}/2}. \tag{29}$$

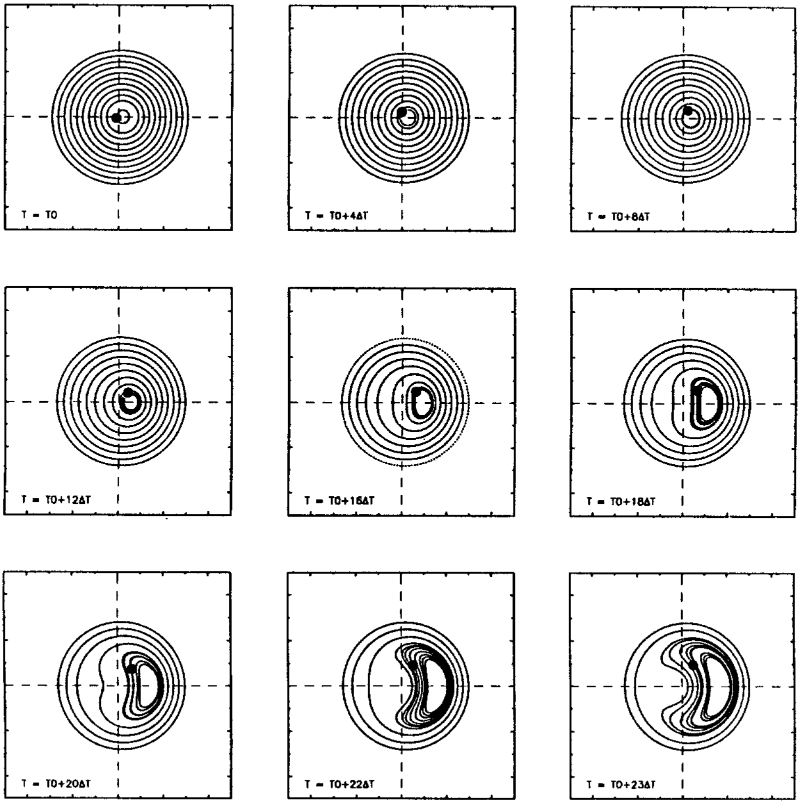

Fig. 4. The evolution of the eccentricity vector (h, k) of a particle around β-Pic, with β= 0.3, perturbed by a planet of mass $\epsilon = 10^{-5}$, orbiting at 20 A.U. from the star. The diagram corresponds to a 2:1 outer Lindblad resonance. The position of the particle is these diagrams is plotted as a dot. The time step ΔT corresponds to 10 planetary revolutions, i.e. 730 years. The units for h and k are arbitrary. See the text for details.

Moreover, the maximum width of the libration region enclosed in the separatrix is:

$$w_{lib} \sim \sqrt{2\mathcal{A}\epsilon/e_{lib}}. \tag{30}$$

Finally, a classical analysis of libration motion close to a stable equilibrium point provides the following libration period around L:

$$T_{lib} \sim \frac{4T_{orb}}{3m^2\sqrt{2\mathcal{A}\epsilon e_{lib}}}, \tag{31}$$

where T_{orb} is the orbital period of the planet.

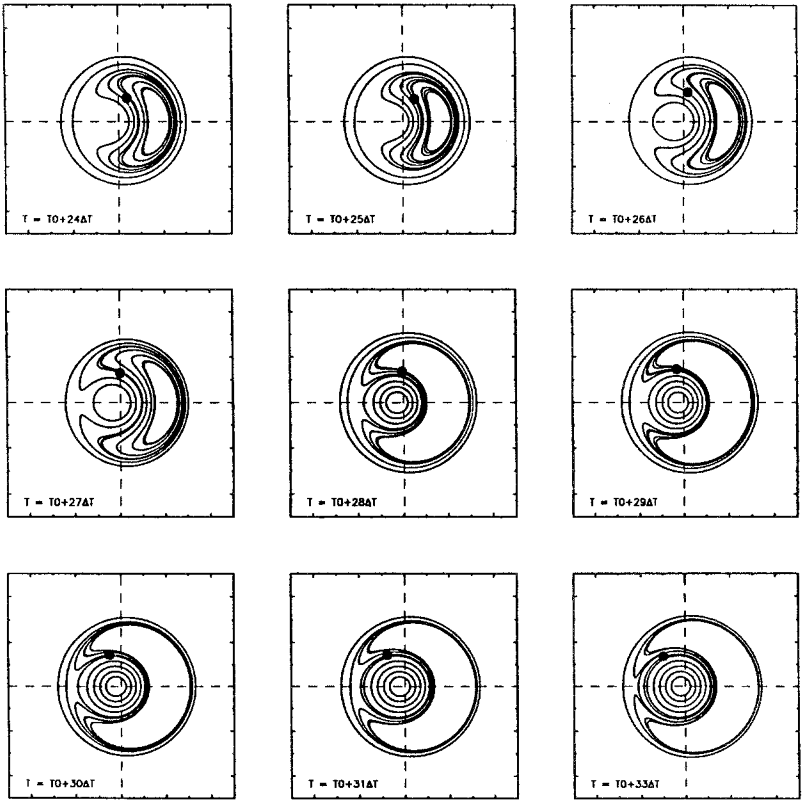

Fig. 5. Continuation of Fig. 4

Due to PR drag, the quantity $\mathcal{J}$ will slowly evolve, and so will the position of the instantaneous stable libration point L. Reintroducing the value of $\mathcal{J}$ as defined in Eq. (27), in the complete system (17) with dissipation, one can derive:

$$\dot{\mathcal{J}} \sim \frac{8C}{5|m|}, \tag{32}$$

in the approximation $e^2 \ll 1$. Since the stable libration point L corresponds to $\mathcal{J} = 2e_{lib}^2$, the forced eccentricity e_{lib} varies like:

$$e_{lib} \sim \sqrt{\frac{4Ct}{5|m|}}, \tag{33}$$

with time t. This result is interesting, because it tells us that the time scale to build up a significant eccentricity is $\sim m/C$, i.e. of the same order of magnitude as the decay time calculated in (6), $1/C$. In other words, even though the trapping in the resonance is not permanent, it is nevertheless long enough for the particles to statistically accumulate at the resonance. In this case, particles ejected through close encounters with the planet are replaced by new particles brought into the resonance from the outer regions of the system.

For the motion to be adiabatic, the libration period calculated in Eq. (31) should be very small compared to the time scale of evolution T_{evol} of the phase space itself. The latter time scale can be defined by the time it takes to the forced eccentricity e_{lib} to increase its value by an amount corresponding to the width w_{lib} of the libration region. The comparison of Eqs. (30) and (33) shows that:

$$T_{evol} \sim \frac{5|m|}{2C}\sqrt{2\mathcal{A}\epsilon e_{lib}}, \tag{34}$$

so that the motion is adiabatic if:

$$\frac{T_{lib}}{T_{evol}} \sim \frac{4CT_{orb}}{15|m|^3\mathcal{A}\epsilon e_{lib}} \ll 1. \tag{35}$$

The combination of Eq. (7) and $\mathcal{A} \sim -8/(3m)$ yields:

$$\frac{T_{lib}}{T_{evol}} \sim \frac{1}{m^2}\frac{3\times 10^{-5}}{s_{\mu m}\sqrt{R_{AU}}}\frac{1}{\epsilon e_{lib}}. \tag{36}$$

Because m is a few times unity, ϵ is in the range $10^{-6} - 10^{-3}$, and e_{lib} is small compared to unity at the entrance in the resonance, this means that for μm-sized grains, the adiabaticity condition (35) is *not* satisfied for typical situations of interest.

This problem is illustrated in Figs. 4 and 5. We consider a particle orbiting the star β-Pictoris ($L/L_{\odot} \sim 6$), with the coefficient β set to 0.3 (corresponding to a radius of ~ 4 μm, according to Eq. (3)). The particle is perturbed by a hypothetical planet of mass $\epsilon \sim 10^{-5}$, orbiting at 20 A.U. from the central star. The motion of the particle is integrated near the 2:1 outer Lindblad resonance, by retaining only the resonant term in the planet perturbing potential. At each time step, the "osculating conservative phase space" was plotted, according to the instantaneous value of $\mathcal{J}$ assumed by the particle (whose position in the phase space is plotted as a dot). Thus all the curves, in a given diagram in Fig. 4, or Fig. 5, have the same value of $\mathcal{J}$, but different values of K.

Following the motion of the particle, it can be seen that it lies first in the circulation region, then enters the libration zone, and finally escapes the resonance by passing in the inner circulation zone. An important feature to be noted is that the time of libration of the particle is comparable to the time of evolution of the libration zone itself. From the analysis of such a figure, it is possible to conclude that the crossing of a resonance, even if smooth, cannot be considered as adiabatic, at least with the parameters used in this case.

6. Conclusions

We have restricted ourselves to a rather specific problem, namely a massless particle orbiting a star and perturbed by (i) a planet on a circular orbit and (ii) radiation forces, including PR drag, with no mutual inclination between the particle and the planet. A simple approach, summarized by the equations of motion (17), allows us to draw several qualitative and quantitative conclusions:

- Only outer Lindblad resonances are able to counteract the dissipative effect of PR drag.
- For a given dissipation coefficient C (see Eq. (7)), there exists one linearly stable periodic orbit where the energy dissipated by the drag is balanced by the energy provided by the resonance.
- However, the eccentricity of this periodic orbit is of order $\sqrt{2/5|m|}$, where m refers to the $m : m+1$ resonance. Besides the technical difficulties associated to this high eccentricity (expansion of the perturbing potential), this means that the particle orbit will cross that of the planet at some point, a highly precarious situation for keeping the particle in the resonance region.
- A closer look at the equations of motion at small eccentricities shows that the eccentricity first increases like $\sqrt{4Ct/5|m|}$, which means that the particle may spend a large time in the resonance before colliding with the planet. This may account for statistical accumulation of particles at the resonance radii.
- For typical μm-sized particles perturbed by typical planets with masses $\sim 10^{-6} - 10^{-3}$ stellar mass, the motion of the particle eccentricity vector is not adiabatic in the phase space. This prevents the classical techniques of adiabatic invariant being used to derive the probability of capture into the resonance, the time of escape from the resonance, etc...

Several points remain to be studied at that point. The periodic orbits deserve a study of their own. After all, they could be reached directly by particles on elliptic orbits from the beginning (cometary origin). They could avoid close encounters with the planet through an efficient protection mechanism, associated to special initial conditions. Also, statistical methods, using many numerical integrations, are now necessary to derive the probability of capture in resonance of randomly distributed particles. Third, a more detailed description of the close encounters with the planet should be presented. In particular, one would like to know the percentage of particles ejected in the outer regions of the system, or sent onto the central star, during such encounters. Finally, the full problem, which takes into account the planet orbital eccentricity, as well as mutual inclinations of the planet and the particles, could exhibit a much more complicated dynamics, through the appearance of new kinds of resonances (corotation and inclined resonances).

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