Improved computational methods for non-linear stellar pulsations

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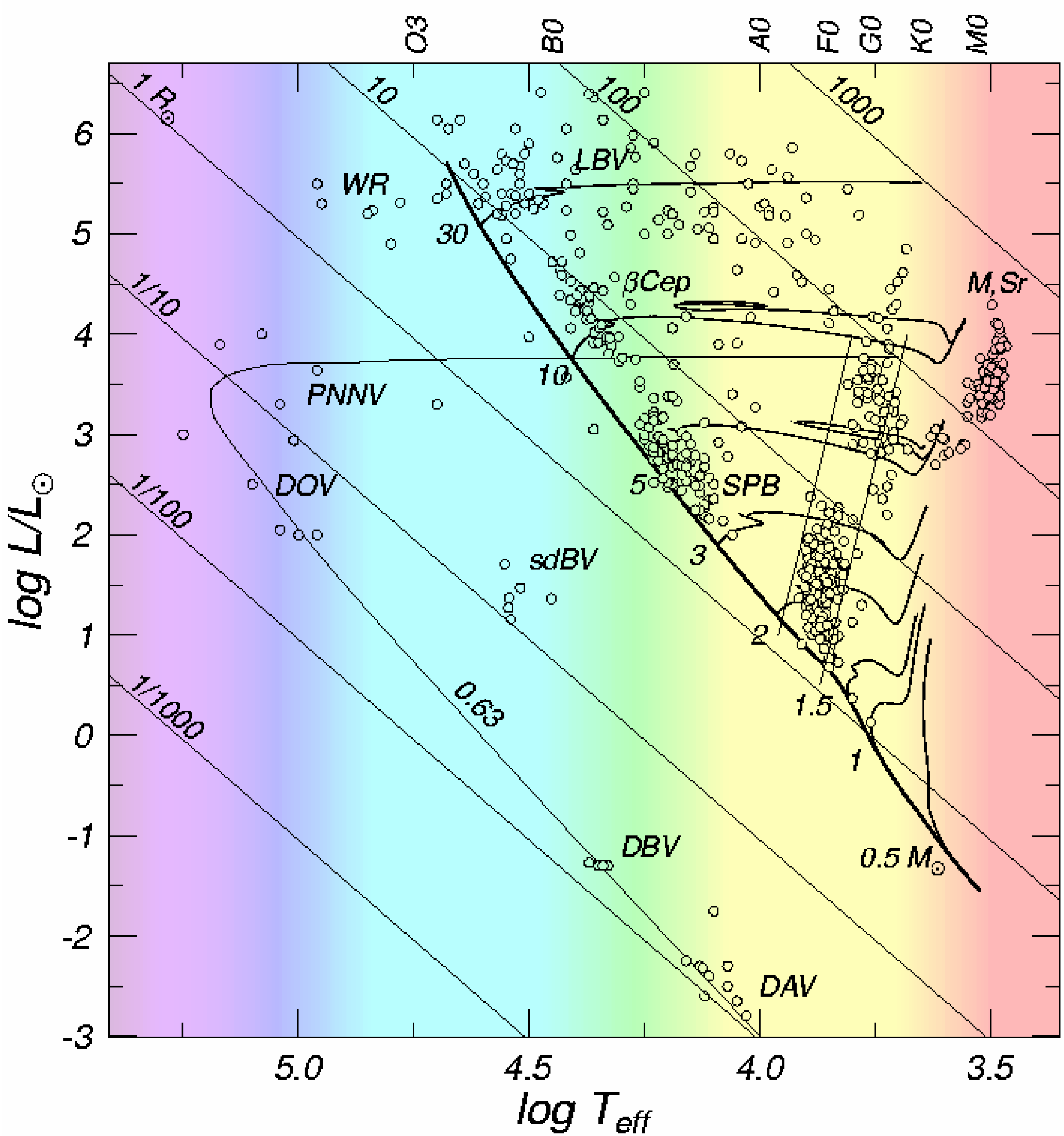
Outline

- Introduction
- Numerical requirements
- Numerical improvements
- Some results on stellar pulsations
- Conclusions and outlook



Adopted from
Gautschy & Saio
1996

Computational Requirements

- Resolve relevant features within one single computation like driving zone, ionization zones, opacity changes, shock waves, stellar winds, …
- Kinetic energy is only small fraction of the total energy of the pulsation star
- Steep gradients within the stellar atmosphere and/or possible changes of the atmospheric stratification due to energy deposition may change boundary conditions
- Long term evolution of stellar pulsations, secular changes on thermal time scales, i.e. $t_{KH} >> t_{dyn}$
- Solve full set of **Radiation Hydrodynamics** (RHD), problem: detailed properties of **convection**

Computational RHD properties

- All variables depend on time and radius, X=X(r,t)
- Equations are discretized in a conservative way, i.e. global quantities are conserved, correct speed of propagating waves
- Adaptive grid to resolve steep features within the flow
- Implicit formulation, large time steps are possible, solution of a non-linear system of equations at every new time step
- Flexible approach to incorporate also new physics

Adaptive conservative RHD

$$\frac{\partial}{\partial t}\left[\int_{V(t)} X\, dV\right] + \oint_{\mathcal{O}(t)} X\,\mathbf{u}^{\mathrm{rel}}\cdot d\mathbf{f} = \int_{V(t)} \left[X_{\mathrm{source}} - X_{\mathrm{sink}}\right] dV$$

- Integration over finite but time-dependent volume *V(t)* due to moving grid points
- Advection terms calculated from fluxes over cell boundaries
- Relative velocities between mater and grid motion: $u^{rel} = u - u^{grid}$

Implicit RHD

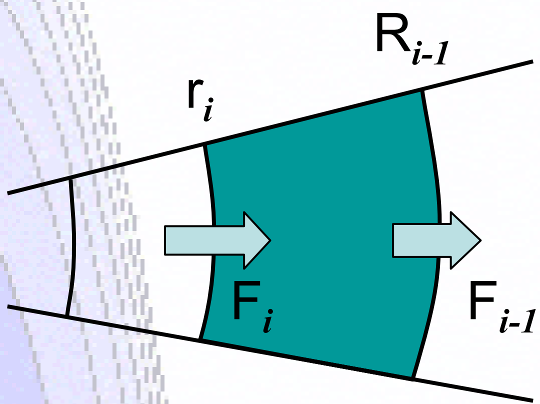
- Advantage: Time step Δt_{CFL} not limited by the Courant-Friedrichs-Levy (CFL)-condition:

$$\Delta t_{\mathrm{CFL}} \leq \min_i \frac{\Delta x_i}{|u_i| + c_{\mathrm{S}}}$$

- Advantage: Time step is given by accuracy conditions, e.g. maximum variation allowed per time step
- Advantage: Solution can be followed over long times
- Disadvantage: At every new time step a large non-linear system of equations (= 8N, N ... grid points) has to be solved by an iteration (8N x 8N Jacobi-matrix), i.e. mass changes inside a cell :

$$\delta\left(\rho V\right) + 4\pi\delta t\,\Delta\left(r^2 u^{\mathrm{rel}}\tilde{\rho}\right) = 0$$

Conservative RHD

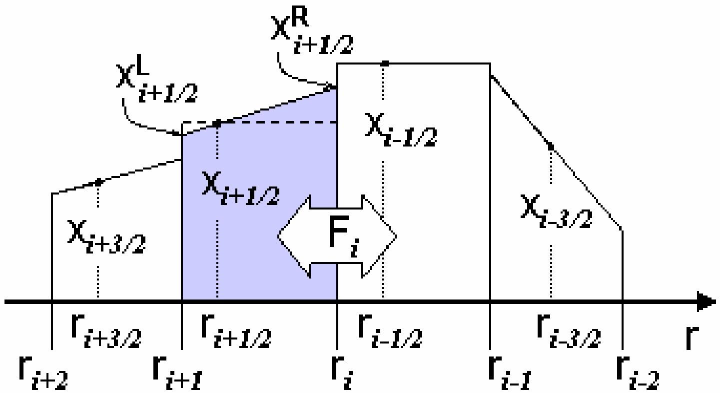


$$F_i = 4\pi\delta t\, r_i^2 u_i^{\rm rel}\tilde{\rho}_i = \left[4\pi\delta t\, r_i^2 u_i - \frac{4\pi}{3}\delta(r_i^3)\right]\tilde{\rho}_i$$

- Essential property for numerical schemes
- Discrete equations are written in as finite volume, i.e. calculate fluxes $F_{\rm i}$ across cell boundaries to advance integrated quantities
- No specialized numerical scheme (e.g. Riemann-solver, etc.) to obtain these fluxes

Advection (I)

$$r^3_{i+\frac{1}{2}} = \frac{1}{2}\left(r^3_i + r^3_{i+1}\right)$$

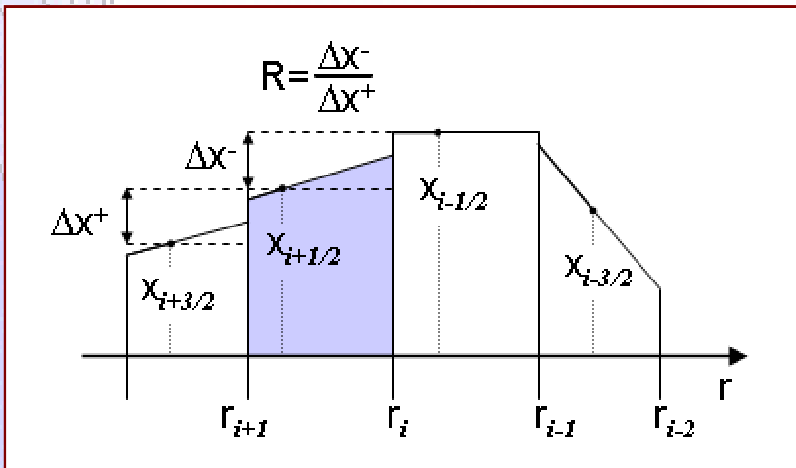


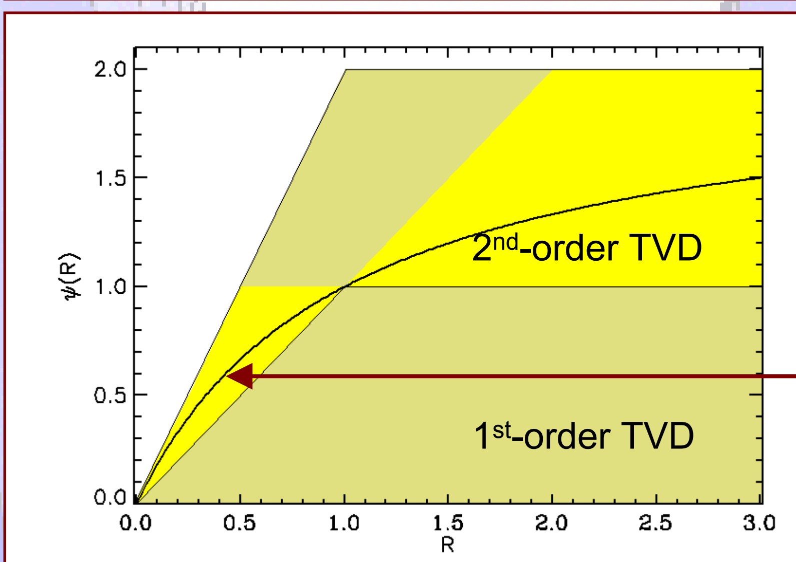
$$x^{\mathrm{L}}_{i+\frac{1}{2}} = x_{i+\frac{1}{2}} - \frac{1}{2}\psi(\theta_{i+\frac{1}{2}})\Delta_i(x)$$

$$x^{\mathrm{R}}_{i+\frac{1}{2}} = x_{i+\frac{1}{2}} + \frac{1}{2}\psi(\theta_{i+\frac{1}{2}})\Delta_i(x)$$

- Transport through moving shells as accurate as possible
- Usage of a staggered mesh, i.e. variables located at cell center or cell boundary
- Fulfil accuracy as well as stability criteria for sub- and supersonic flow
- Avoid numerical oscillations, so-called TVD-schemes
- Ensure correct propagation speed of waves

Advection (II)

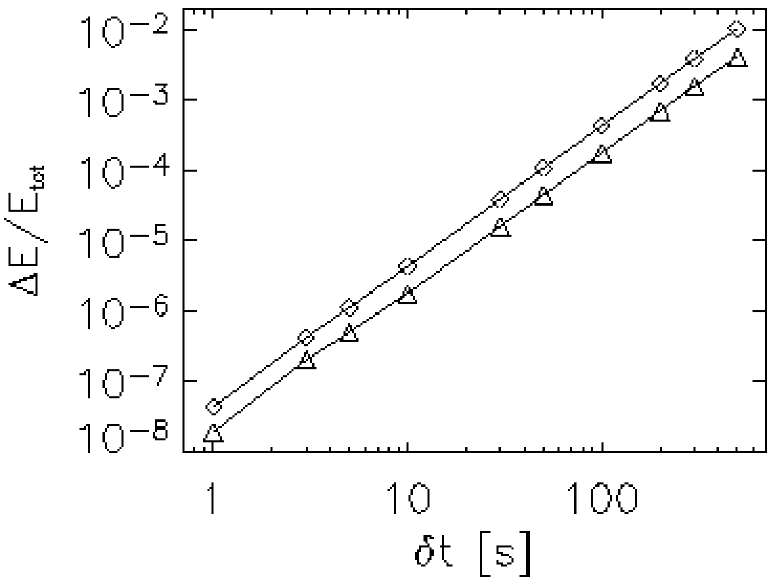




- TVD-schemes are based on monotonicity criteria of the consecutive ratio R
- Correct propagation speed of waves requires $\psi(1)=1$
- Monotonic advection scheme according to van Leer (1979) essential for stellar pulsations:

$$\psi(R) = \begin{cases} \frac{2R}{1+R}, \text{ if } \quad R \geq 0 \\ 0, \text{ otherwise} \end{cases}$$

Temporal discretization



$$\frac{\partial \vec{x}}{\partial t} = H(\vec{x})$$

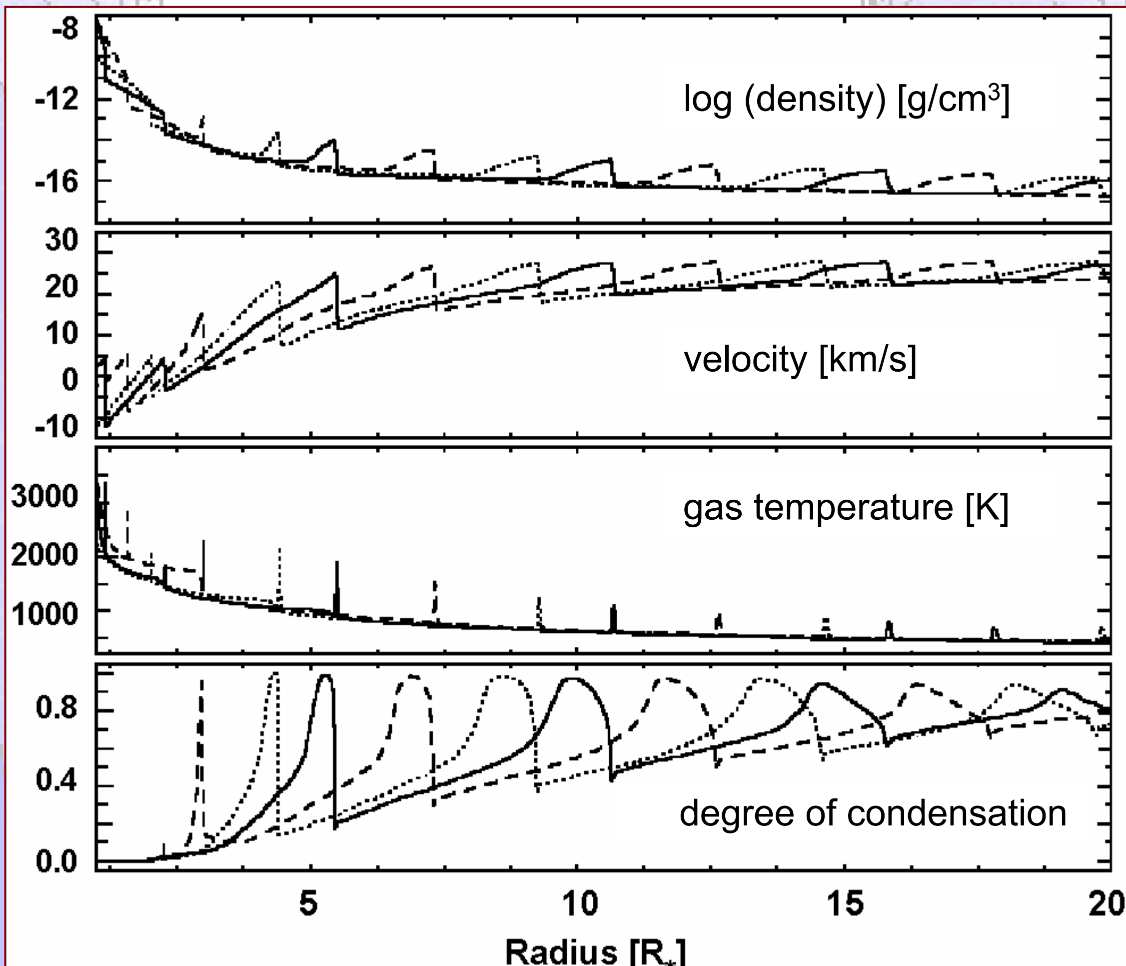
- 2^{nd}-order temporal discretization to reduce artificial damping of oscillations
- Smallest errors in case of time-centered variables

$$\frac{\vec{x}^{(n+1)} - \vec{x}^{(n)}}{\delta t} = (1-\theta)\, H(\vec{x}^{(n)}) + \theta\, H(\vec{x}^{(n+1)})$$

$$\frac{\vec{x}^{(n+1)} - \vec{x}^{(n)}}{\delta t} = H(\bar{\vec{x}}) = H((1-\theta)\vec{x}^{(n)} + \theta\vec{x}^{(n+1)})$$

Adaptive Grid

PISTON

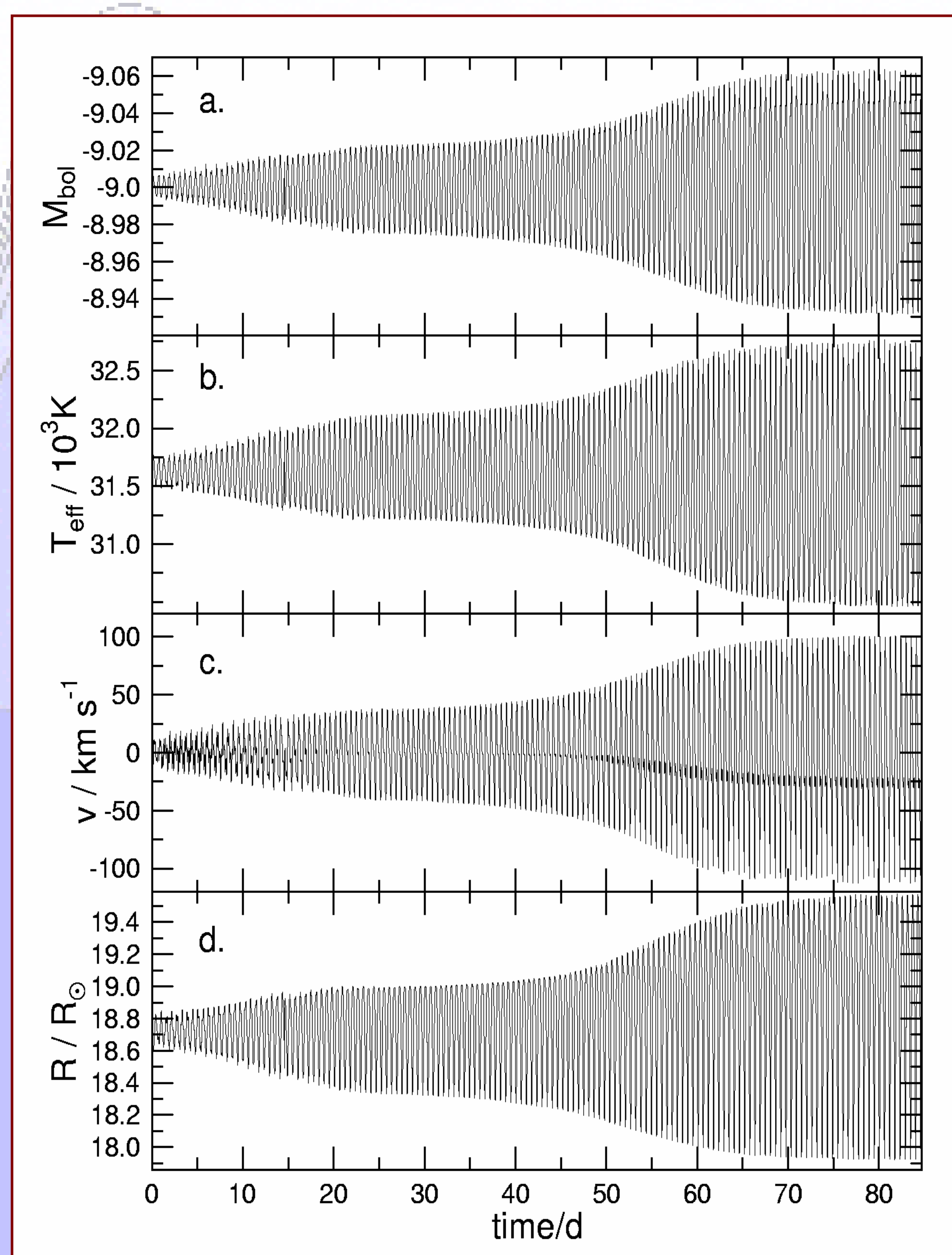


Red Giant: v = 25 km/s, $\dot{M} = 2 \cdot 10^{-6}\ M_\odot$/yr

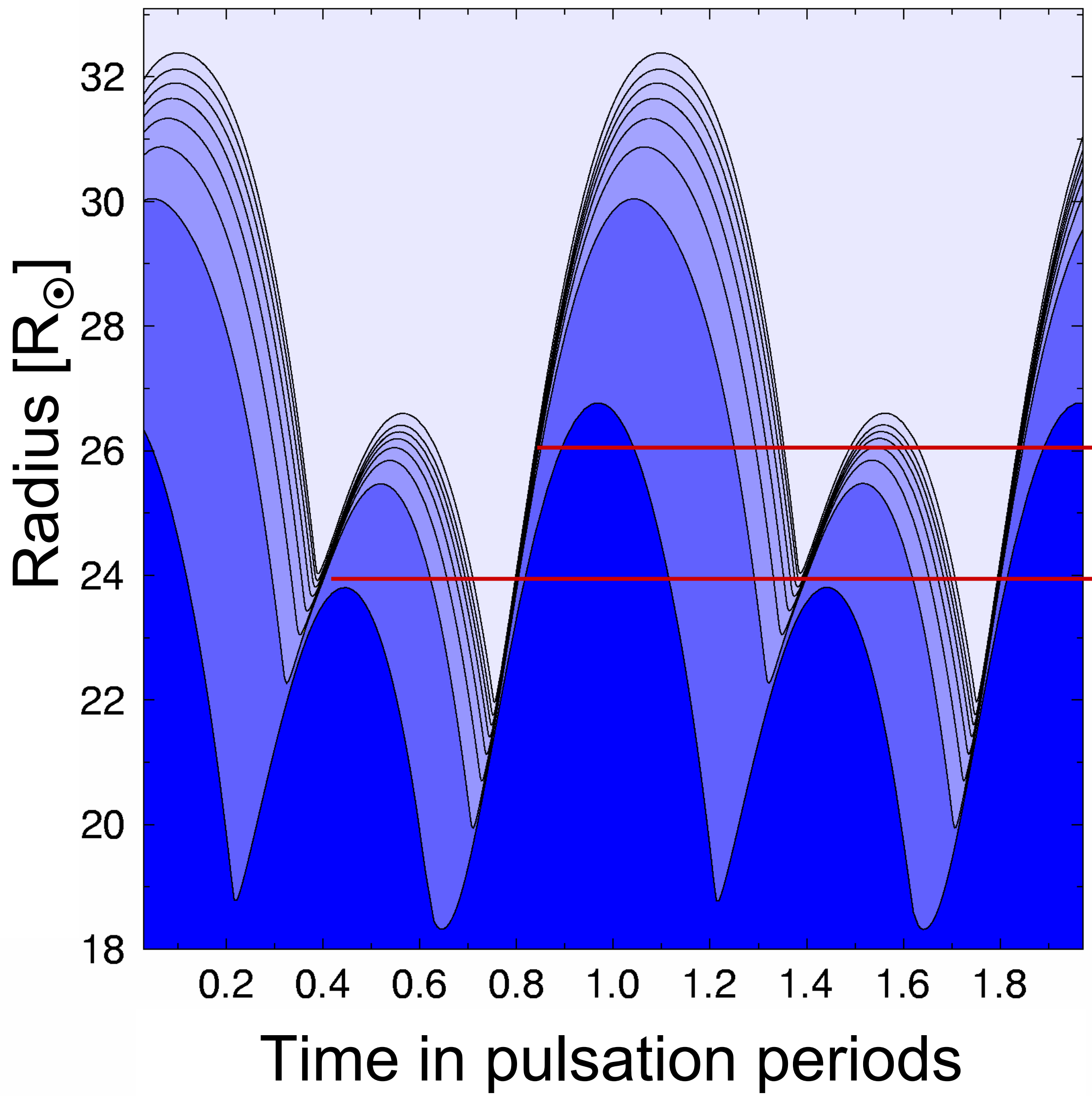
- Freely moving grid in one dimension
- Define the quantities (density, opacity,...) to be resolved on the grid, i.e. the desired resolution
- Clustering around steep gradients, smoothing
- Solutions are more stationary on the moving grid

Growth of pulsations

- Pulsations initiated by a small random perturbation
- Initial linear growth, atmosphere adjusts on a different time scale
- Final amplitude when the kinetic energy becomes constant
- Example: $M = 30M_\odot$, T_{eff}= 31600 K, L = 316 200L☉



LBV-Atmosphere with Shock waves



$M = 60\,M_\odot$

$L = 933\,000\,L_\odot$

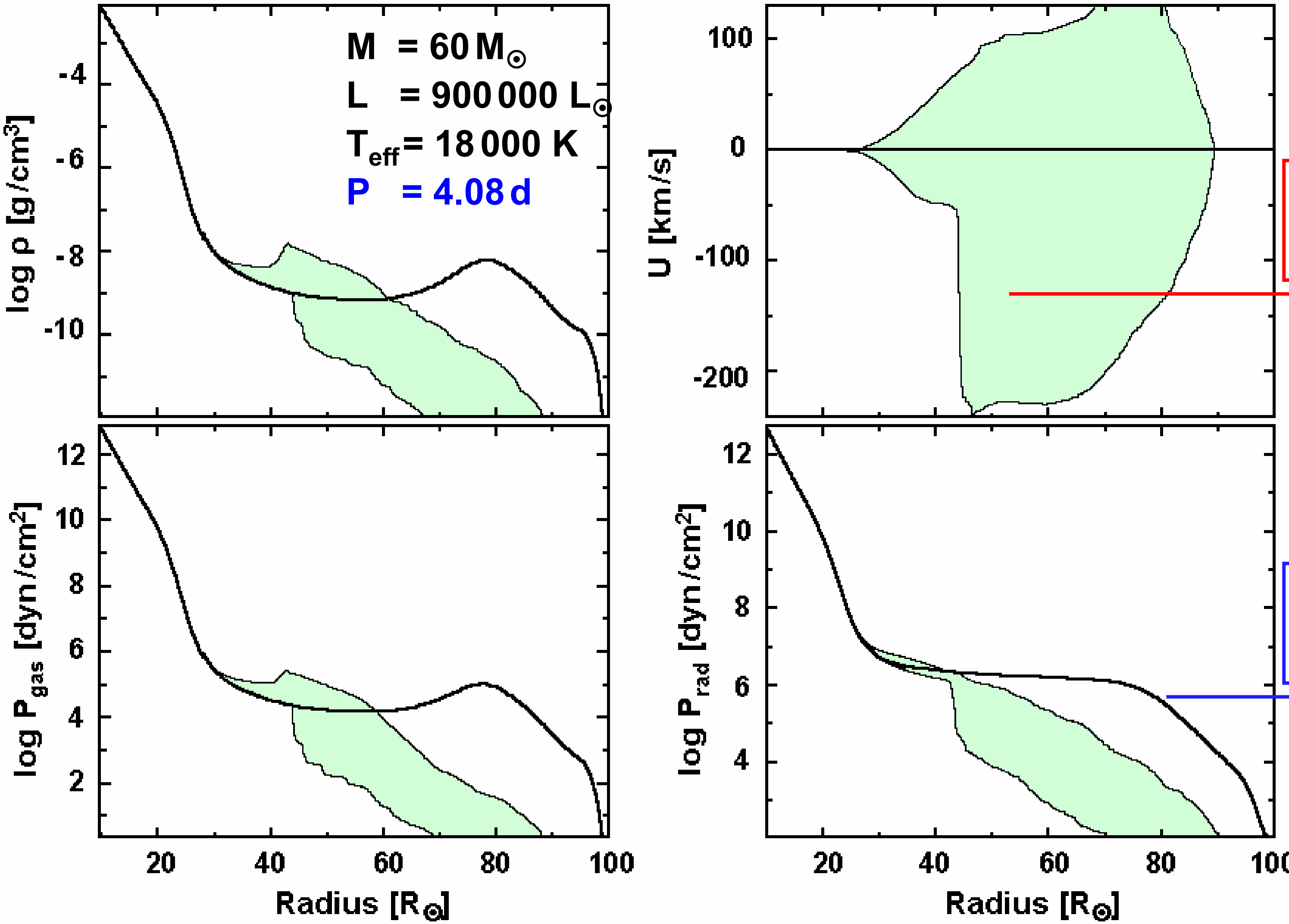
$T_{eff} = 34\,680\,K$

$P = 1.61$ days

Shockwaves

Ballistic motions on the scale of t_{ff}

Observation of stellar parameter



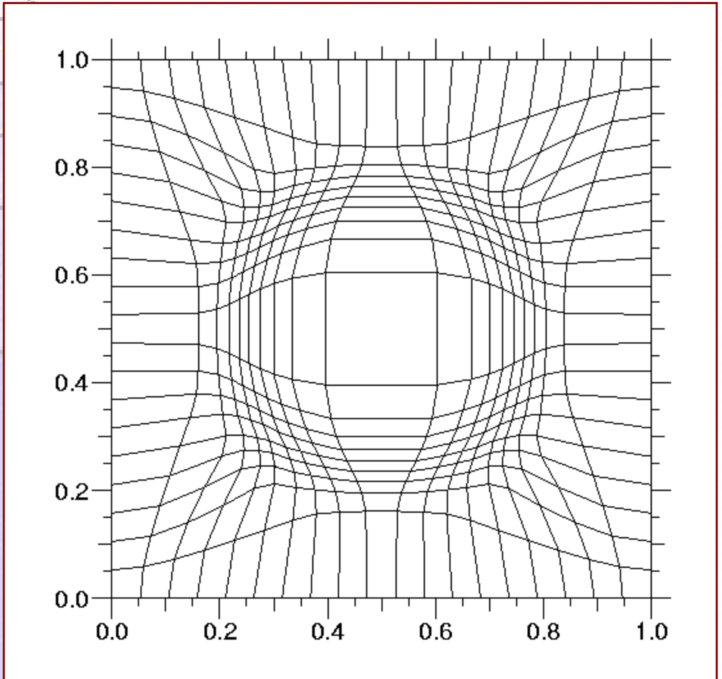
hydrodynamical structure

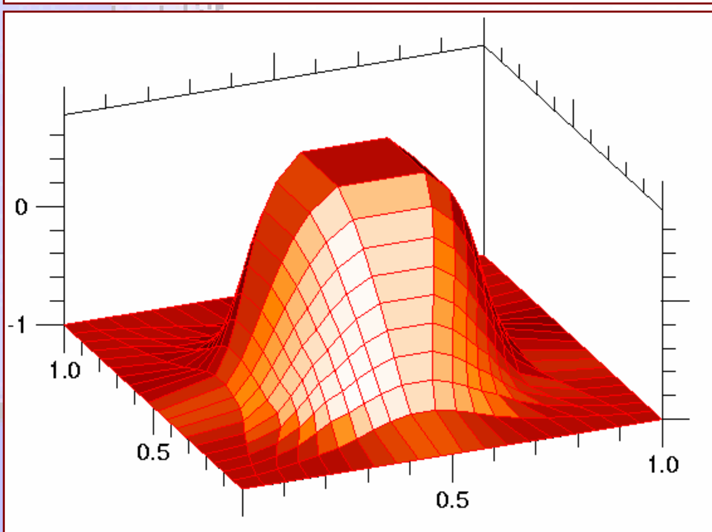
ΔT = 2700 K

hydrostatic structure

Conclusions

- Workhorse for non-linear stellar pulsations should be soon available
- Cumbersome programming and testing necessary
- All tables have to be provided together with their (smooth) derivatives (opacity, EOS,…)
- Improved accuracy:
  - Geometrical approach to advection
  - Finite volume interpolation and discretization
  - Monotonic advection on non-equidistant grids
  - New definition of grid velocity enables smooth transitions from a Lagrangian to a moving mesh





Courtesy of A. Stökl

Outlook

- Include better description of convective energy and momentum transport into the code
- Non-grey radiative transport on a small number (about 50) of frequency points
- 2-dimensional adaptive, implicit calculations based on the same numerical methods